# Unit 5 *Solution of LPP using Graphical method*

*5.1 Graphical solution Procedure*

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*5.3 Example Problems*

## 5.1 Graphical Solution Procedure

**The graphical solution procedure**

1. Consider each inequality constraint as equation.
2. Plot each equation on the graph as each one will geometrically represent a straight line.
3. Shade the feasible region. Every point on the line will satisfy the equation of the line. If the inequality constraint corresponding to that line is '≤' then the region below the line lying in the first quadrant is shaded. Similarly for '≥' the region above the line is shaded. The points lying in the common region will satisfy the constraints. This common region is called **feasible region.**
4. Choose the convenient value of Z and plot the objective function line.
5. Pull the objective function line until the extreme points of feasible region.
   a. In the maximization case this line will stop far from the origin and passing through at least one corner of the feasible region.
   b. In the minimization case, this line will stop near to the origin and passing through at least one corner of the feasible region.
6. Read the co-ordinates of the extreme points selected in step 5 and find the maximum or minimum value of Z.

## 5.2 Definitions

1. **Solution** – Any specification of the values for decision variable among ($x_1$, $x_2$… $x_n$) is called a solution.
2. **Feasible solution** is a solution for which all constraints are satisfied.
3. **Infeasible solution** is a solution for which atleast one constraint is not satisfied.
4. **Feasible region** is a collection of all feasible solutions of an inequality.

5. **Optimal solution** is a feasible solution that has the most favorable value of the objective function.
6. **Most favorable value** is the largest value if the objective function is to be maximized, whereas it is the smallest value if the objective function is to be minimized.
7. **Multiple optimal solutions** – More than one solution with the same optimal value of the objective function.
8. **Unbounded solution** – If the value of the objective function can be increased or decreased indefinitely such solutions are called unbounded solution.
9. **Corner point feasible solution** is a solution that lies at the corner of the feasible region.

## 5.3 Example problems

**Example 1**

Solve $3x + 5y < 15$ graphically

**Solution**

Write the given constraint in the form of equation i.e. $3x + 5y = 15$

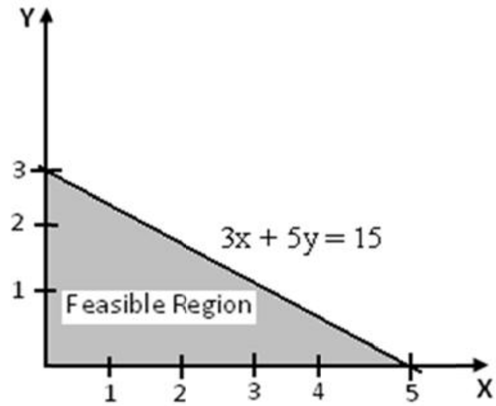
Put $x=0$ then the value $y=3$

Put $y=0$ then the value $x=5$

Therefore the coordinates are (0, 3) and (5, 0). Thus these points are joined to form a straight line as shown in the graph.

Put $x=0$, $y=0$ in the given constraint then

$0<15$, the condition is true. (0, 0) is solution nearer to origin. So shade the region below the line, which is the feasible region.




**Example 2**

Solve 3x + 5y >15

**Solution**

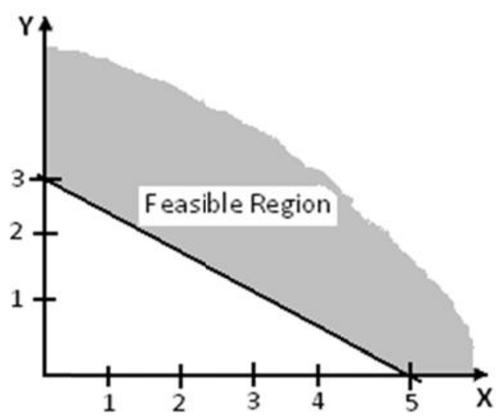
Write the given constraint in the form of equation i.e. 3x + 5y = 15

Put x=0, then y=3

Put y=0, then x=5

So the coordinates are (0, 3) and (5, 0)

Put x =0, y =0 in the given constraint, the condition turns out to be false i.e. 0 > 15 is false. So the region does not contain (0, 0) as solution. The feasible region lies on the outer part of the line as shown in the graph.



**Example 3**

Max $Z = 80x_1 + 55x_2$

Subject to

$4x_1 + 2x_2 \leq 40$

$2x_1 + 4x_2 \leq 32$

$x_1 \geq 0$ , $x_2 \geq 0$

**Solution**

The first constraint $4x_1 + 2x_2 \leq 40$, written in a form of equation

$4x_1 + 2x_2 = 40$

Put $x_1 = 0$, then $x_2 = 20$

Put $x_2 = 0$, then $x_1 = 10$

The coordinates are (0, 20) and (10, 0)

The second constraint $2x_1 + 4x_2 \leq 32$, written in a form of equation

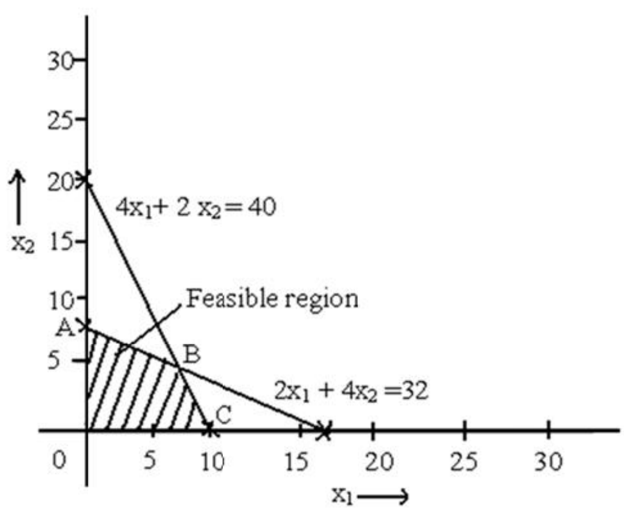
$2x_1 + 4x_2 = 32$

Put $x_1 = 0$, then $x_2 = 8$

Put $x_2 = 0$, then $x_1 = 16$

The coordinates are (0, 8) and (16, 0)

The graphical representation is




The corner points of feasible region are A, B and C. So the coordinates for the corner points are

A (0, 8)

B (8, 4) (Solve the two equations $4x_1 + 2x_2 = 40$ and $2x_1 + 4x_2 = 32$ to get the coordinates)

C (10, 0)

We know that Max $Z = 80x_1 + 55x_2$

At A (0, 8)

Z = 80(0) + 55(8) = 440

At B (8, 4)

Z = 80(8) + 55(4) = 860

At C (10, 0)

Z = 80(10) + 55(0) = 800

The maximum value is obtained at the point B. Therefore Max Z = 860 and $x_1 = 8$, $x_2 = 4$

**Exercise**

1. Using graphical method,

Minimize $Z = 10x_1 + 4x_2$

Subject to

$3x_1 + 2x_2 \geq 60$

$7x_1 + 2x_2 \geq 84$

$3x_1 + 6x_2 \geq 72$

$x_1 \geq 0$ , $x_2 \geq 0$

2. Solve by using graphical method

Max $Z = 4x_1 + 3x_2$

Subject to

$4x_1 + 3x_2 \leq 24$

$x_1 \leq 4.5$

$x_2 \leq 6$

$x_1 \geq 0$ , $x_2 \geq 0$

3. Solve graphically

Max $Z = 3x_1 + 2x_2$

Subject to

$x_1 + x_2 \leq 1$

$x_1 + x_2 \geq 3$

$x_1 \geq 0$ , $x_2 \geq 0$

4. Solve by graphical method

Max $Z = 3x_1 + 5x_2$

Subject to

$2x_1 + x_2 \geq 7$

$x_1 + x_2 \geq 6$

$x_1 + 3x_2 \geq 9$

$x_1 \geq 0$ , $x_2 \geq 0$