Unit 3

3.1 Games with Mixed Strategies

In certain cases, no pure strategy solutions exist for the game. In other words, saddle point does not exist. In all such game, both players may adopt an optimal blend of the strategies called **Mixed Strategy** to find a saddle point. The optimal mix for each player may be determined by assigning each strategy a probability of it being chosen. Thus these mixed strategies are probabilistic combinations of available better strategies and these games hence called **Probabilistic games**.

The probabilistic mixed strategy games without saddle points are commonly solved by any of the following methods

Sl. No.	Method	Applicable to
1	Analytical Method	2x2 games
2	Graphical Method	2x2, mx2 and 2xn games
3	Simplex Method	2x2, mx2, 2xn and mxn games

Reduction by Dominance

1. Check whether there is any row in the (remaining) matrix that is dominated by another row (this means that it is ≤ some other row). If there is one, delete it.
2. Check whether there is any column in the (remaining) matrix that is dominated by another column (this means that it is ≥ some other column). If there is one, delete it.
3. Repeat steps 1 and 2 in any order until there are no dominated rows or columns

Quick Example

In the payoff matrix

$$A \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \overset{\displaystyle B}{\overset{\begin{array}{ccc} 1 & 2 & 3 \end{array}}{\begin{bmatrix} 3 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & -2 & 1 \\ 2 & 3 & 4 \end{bmatrix}}},$$

row 4 dominates both rows 2 and 3, so we eliminate both of these rows at once.

$$A \begin{array}{c} 1 \\ 4 \end{array} \overset{\displaystyle B}{\overset{\begin{array}{ccc} 1 & 2 & 3 \end{array}}{\begin{bmatrix} 3 & -1 & -1 \\ 2 & 3 & 4 \end{bmatrix}}}$$

Turning to the columns, we see that column 2 dominates column 3, so we eliminate column 3.

$$A \begin{array}{c} 1 \\ 4 \end{array} \overset{\displaystyle B}{\overset{\begin{array}{cc} 1 & 2 \end{array}}{\begin{bmatrix} 3 & -1 \\ 2 & 3 \end{bmatrix}}}$$

Looking again at the rows, we find that none of the rows is dominated by any of the others, and that the same is true for the columns. Thus the game cannot be reduced any further.

This process might go on until you are left with a (1x1) matrix. If this is the case, then you are lucky indeed, and are left with a very simple game.

H.W: reduce the following payoff matrix by dominance rule.

A \ B	1	2	3
1	0	−1	5
2	7	5	10
3	15	−4	−5
4	5	0	10
5	−5	−10	10

3.1.1 Analytical Method

A 2 x 2 payoff matrix where there is no saddle point can be solved by analytical method. Given the matrix

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Value of the game is

$$V = \frac{a_{11}\,a_{22} - a_{21}\,a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

With the coordinates

$$x_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} \quad , \quad x_2 = \frac{a_{11} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$y_1 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} \quad , \quad y_2 = \frac{a_{11} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

Alternative procedure to solve the strategy

- Find the difference of two numbers in column 1 and enter the resultant under column 2. Neglect the negative sign if it occurs.
- Find the difference of two numbers in column 2 and enter the resultant under column 1. Neglect the negative sign if it occurs.
- Repeat the same procedure for the two rows.

1. Solve

$$A \overset{B}{\begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix}}$$

Solution

It is a 2 x 2 matrix and no saddle point exists. We can solve by analytical method

$$A \overset{B}{\begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix}} \begin{matrix} 1 \\ 4 \end{matrix}$$
$$\begin{matrix} 3 & \quad 2 \end{matrix}$$

$$V = \frac{a_{11}\, a_{22} - a_{21}\, a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{20-3}{9-4}$$

V = 17 / 5

$S_A = (x_1, x_2) = (1/5, 4/5)$

$S_B = (y_1, y_2) = (3/5, 2/5)$

2. Solve the given matrix

$$A \overset{B}{\begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix}}$$

Solution

$$A \overset{B}{\begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix}} \begin{matrix} 1 \\ 3 \end{matrix}$$
$$\quad\; 1 \quad\; 3$$

$$V = \frac{a_{11}\,a_{22} - a_{21}\,a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{0-1}{2+2}$$

V = - 1 / 4

$S_A = (x_1, x_2) = (1/4, 3/4)$

$S_B = (y_1, y_2) = (1/4, 3/4)$

3.1.2 Graphical method

The graphical method is used to solve the games whose payoff matrix has

- 2 rows and n columns (2 x n)
- n rows and 2 columns (n x 2)

Algorithm for solving 2 x n matrix games

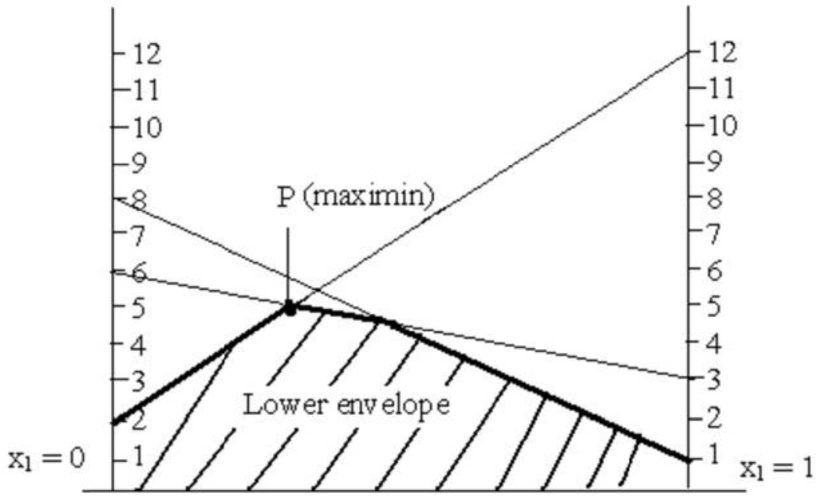
- Draw two vertical axes 1 unit apart. The two lines are x1 = 0, x1 = 1
- Take the points of the first row in the payoff matrix on the vertical line x1=1 and the points of the second row in the payoff matrix on the vertical line x1=0.
- The point a1j on axis x1 = 1 is then joined to the point a2j on the axis x1=0 to give a straight line. Draw 'n' straight lines for j=1, 2… n and determine the highest point of the lower envelope obtained. This will be the Maximin point.
- The two or more lines passing through the Maximin point determines the required 2 x 2 payoff matrix. This in turn gives the optimum solution by making use of analytical method.

Example 1

Solve by graphical method

	B1	B2	B3
A1	1	3	12
A2	8	6	2

Solution



	B2	B3	
A1	3	12	4
A2	6	2	9
	10	3	

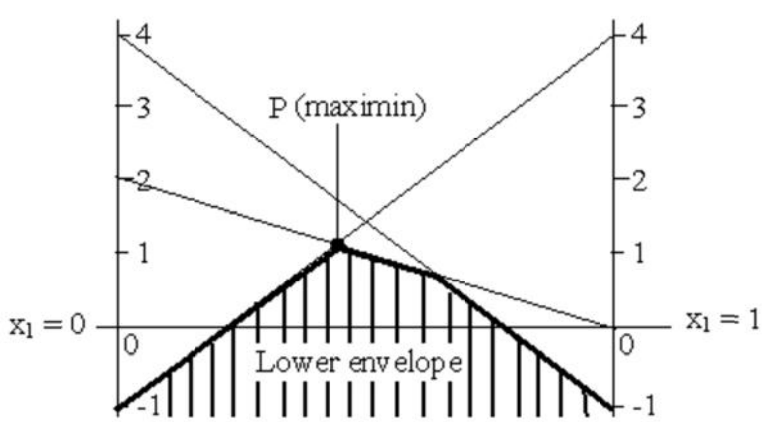
V = 66/13
SA = (4/13, 9 /13)
SB = (0, 10/13, 3 /13)

Example 2

Solve by graphical method

	B1	B2	B3
A1	4	-1	0
A2	-1	4	2

Solution



	B1	B3	
A1	4	0	3
A2	-1	2	4
	2	5	

V = 8/7
SA = (3/7, 4 /7)
SB = (2/7, 0, 5 /7)

Algorithm for solving n x 2 matrix games

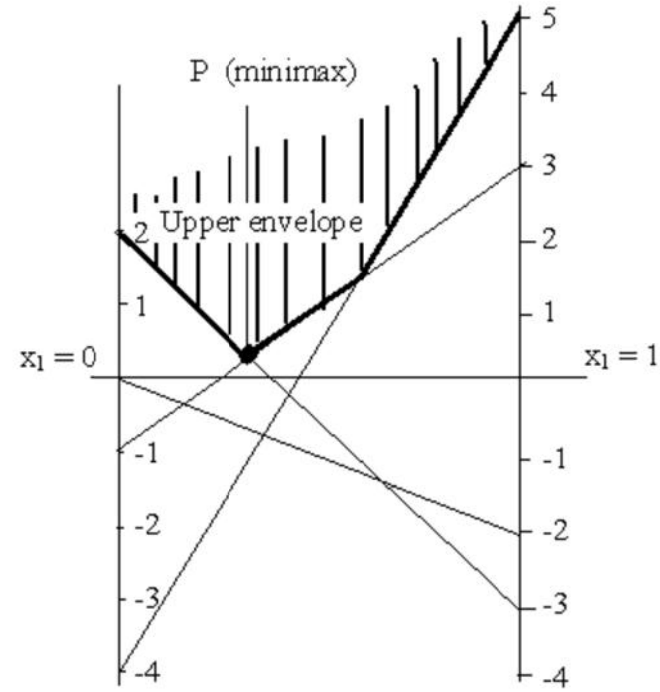
- Draw two vertical axes 1 unit apart. The two lines are x1 =0, x1=1
- Take the points of the first column in the payoff matrix on the vertical line x1=1 and the points of the second column in the payoff matrix on the vertical line x1=0.
- The point aj1 on axis x1=1 is then joined to the point aj2 on the axis x1=0 to give a straight line. Draw 'n' straight lines for j=1, 2… n and determine the lowest point of the upper envelope obtained. This will be the minimax point.
- The two or more lines passing through the minimax point determines the required 2 x 2 payoff matrix. This in turn gives the optimum solution by making use of analytical method.

Example 1

Solve by graphical method

	B1	B2
A1	-2	0
A2	3	-1
A3	-3	2
A4	5	-4

Solution



	B1	B2	
A2	3	-1	5
A3	-3	2	4
	3	6	

V = 3/9 = 1/3

SA = (0, 5 /9, 4/9, 0)

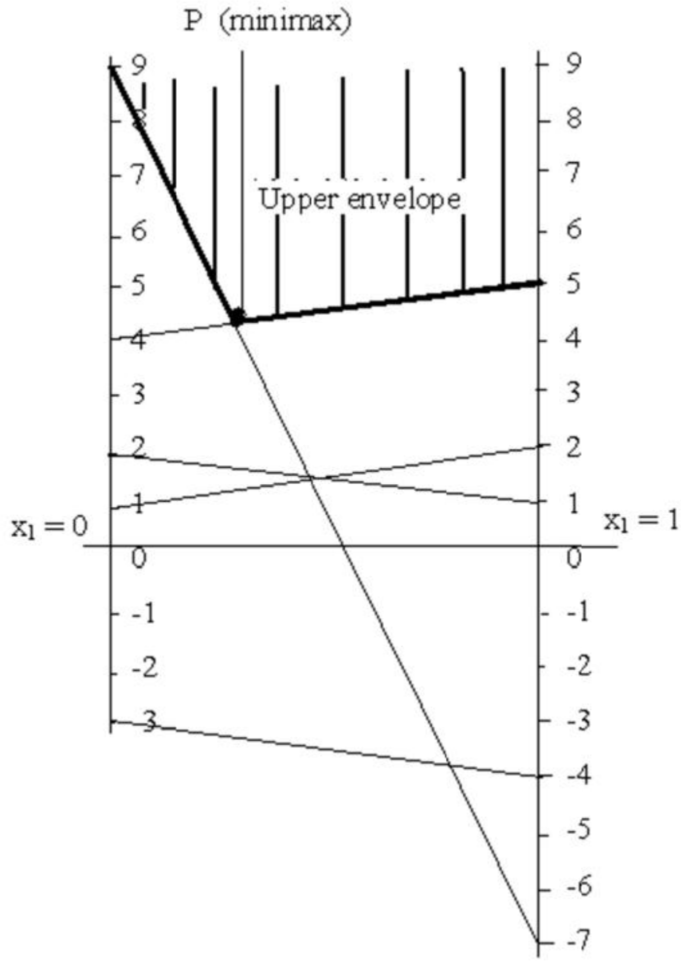
SB = (3/9, 6 /9)

Example 2

Solve by graphical method

	B1	B2
A1	1	2
A2	5	4
A3	-7	9
A4	-4	-3
A5	2	1

Solution



	B1	B2	
A2	5	4	16
A3	-7	9	1
	5	12	

V = 73/17

SA = (0, 16/17, 1/17, 0, 0)

SB = (5/17, 12 /17)

Links:

Dropbox:

https://www.dropbox.com/s/pg8fq2z7dkm9yx6/Industrial%20Engineering%20Lectures.pdf?dl=0

https://goo.gl/DUzKvd

4shared:

http://www.4shared.com/office/kmaLX3XKce/Industrial_Engineering_Lecture.html

http://goo.gl/t5uXQJ