Unit 2

Decision theory

2.1 Introduction to Game Theory

2.2 Properties of a Game

2.3 Characteristics of Game Theory

2.4 Classification of Games

2.5 Limitations of Game Theory

2.5 Solving Two-Person and Zero-Sum Game

2.1 Introduction to Game Theory

Game theory is a distinct approach to the study of human behavior. The disciplines most involved in game theory are mathematics, economics and the other social and behavioral sciences.

Game theory is a type of decision theory in which one's choice of action is determined after taking into account all possible alternatives available to an opponent playing the same game, rather than just by the possibilities of several outcome results. Game theory does not insist on how a game should be played but tells the procedure and principles by which action should be selected. Thus it is a decision theory useful in competitive situations.

Game is defined as an activity between two or more persons according to a set of rules at the end of which each person receives some benefit or suffers loss. The set of rules defines the **game**. Going through the set of rules once by the participants defines a **play**.

2.2 Properties of a Game

1. There are finite numbers of competitors called 'players'

2. Each player has a finite number of possible courses of action called 'strategies'

3. All the strategies and their effects are known to the players but player does not know which strategy is to be chosen.

4. A game is played when each player chooses one of his strategies. The strategies are assumed to be made simultaneously with an outcome such that no player knows his opponents strategy until he decides his own strategy.

5. The game is a combination of the strategies and in certain units which determines the gain or loss.

6. The figures shown as the outcomes of strategies in a matrix form are called 'pay-off matrix'.

7. The player playing the game always tries to choose the best course of action which results in optimal pay off called 'optimal strategy'.

8. The expected pay off when all the players of the game follow their optimal strategies is known as 'value of the game'. The main objective of a problem of a game is to find the value of the game.

9. The game is said to be 'fair' game if the value of the game is zero otherwise it s known as 'unfair'.

2.3 <u>Characteristics of Game Theory</u>

1. Competitive game
A competitive situation is called a **competitive game** if it has the following four properties
1. There are finite number of competitors such that n ≥ 2. In case n = 2, it is called a **two-person game** and in case n > 2, it is referred as **n-person game**.

2. Each player has a list of finite number of possible activities.

3. A play is said to occur when each player chooses one of his activities. The choices are assumed to be made simultaneously i.e. no player knows the choice of the other until he has decided on his own.

4. Every combination of activities determines an outcome which results in a gain of payments to each player, provided each player is playing uncompromisingly to get as much as possible. Negative gain implies the loss of same amount.

2. Strategy

The strategy of a player is the predetermined rule by which player decides his course of action from his own list during the game. The two types of strategy are

1. Pure strategy

2. Mixed strategy

Pure Strategy

If a player knows exactly what the other player is going to do, a deterministic situation is obtained and objective function is to maximize the gain. Therefore, the pure strategy is a decision rule always to select a particular course of action.

Mixed Strategy

If a player is guessing as to which activity is to be selected by the other on any particular occasion, a probabilistic situation is obtained and objective function is to maximize the expected gain. Thus the mixed strategy is a selection among pure strategies with fixed probabilities.

3. Number of persons

A game is called 'n' person game if the number of persons playing is 'n'. The person means an individual or a group aiming at a particular objective.

Two-person, zero-sum game

A game with only two players (player A and player B) is called a 'two-person, zero-sum game', if the losses of one player are equivalent to the gains of the other so that the sum of their net gains is zero.

Two-person, zero-sum games are also called rectangular games as these are usually represented by a payoff matrix in a rectangular form.

4. Number of activities

The activities may be finite or infinite.

5. Payoff

The quantitative measure of satisfaction a person gets at the end of each play is called a payoff

6. Payoff matrix

Suppose the player A has 'm' activities and the player B has 'n' activities. Then a payoff matrix can be formed by adopting the following rules

- Row designations for each matrix are the activities available to player A

- Column designations for each matrix are the activities available to player B

- Cell entry V_{ij} is the payment to player A in A's payoff matrix when A chooses the activity i and B chooses the activity j.

- With a zero-sum, two-person game, the cell entry in the player B's payoff matrix will be negative of the corresponding cell entry V_{ij} in the player A's payoff matrix so that sum of payoff matrices for player A and player B is ultimately zero.

7. Value of the game

Value of the game is the maximum guaranteed game to player A (maximizing player) if both the players uses their best strategies. It is generally denoted by 'V' and it is unique.

2.4 Classification of Games

Generally games are classified into

- Pure strategy games

- Mixed strategy games

The method for solving these two types varies. By solving a game, we need to find best strategies for both the players and also to find the value of the game.

Pure strategy games can be solved by **saddle point method.**

The different methods for solving a mixed strategy game are

- Analytical method

- Graphical method

- Dominance rule

- Simplex method

2.5 Solving Two-Person and Zero-Sum Game

Two-person zero-sum games may be deterministic or probabilistic. The deterministic games will have saddle points and pure strategies exist in such games. In contrast, the probabilistic games will have no saddle points and mixed strategies are taken with the help of probabilities.

Definition of saddle point

A saddle point of a matrix is the position of such an element in the payoff matrix, which is minimum in its row and the maximum in its column.

Procedure to find the saddle point

- Select the minimum element of each row of the payoff matrix and mark them with circles.

- Select the maximum element of each column of the payoff matrix and mark them with squares.

- If their appears an element in the payoff matrix with a circle and a square together then that position is called saddle point and the element is the value of the game.

Solution of games with saddle point

To obtain a solution of a game with a saddle point, it is feasible to find out

- Best strategy for player A

- Best strategy for player B

- The value of the game

The best strategies for player A and B will be those which correspond to the row and column respectively through the saddle point.

Examples

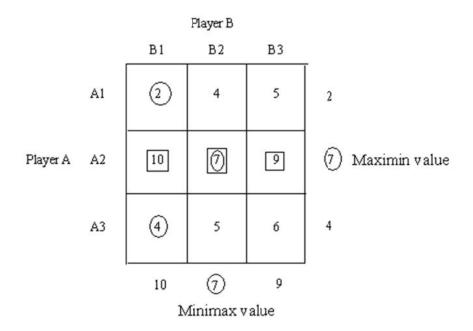**Solve the payoff matrix**

1.

Player B

		B1	B2	B3
Player A	A1	2	4	5
	A2	10	7	9
	A3	4	5	6

Solution

Strategy of player A – A2

Strategy of player B – B2

Value of the game = 7

2.

Player B

		I	II	III	IV	V
	I	-2	0	0	5	3
Player A	II	3	2	1	2	2
	III	-4	-3	0	-2	6
	IV	5	3	-4	2	-6

Solution

Strategy of player A – II

Strategy of player B - III

Value of the game = 1

3..

	B1	B2	B3	B4
A1	1	7	3	4
A2	5	6	4	5
A3	7	2	0	3

Solution

Strategy of player A – A2

Strategy of player B – B3

Value of the game = 4

4.

                              B's Strategy

		B1	B2	B3	B4	B5
	A1	8	10	-3	-8	-12
A's	A2	3	6	0	6	12
Strategy	A3	7	5	-2	-8	17
	A4	-11	12	-10	10	20
	A5	-7	0	0	6	2

Solution

Strategy of player A – A2

Strategy of player B – B3

Value of the game = 0

5.

$$\begin{bmatrix} 9 & 3 & 1 & 8 & 0 \\ 6 & 5 & 4 & 6 & 7 \\ 2 & 4 & 3 & 3 & 8 \\ 5 & 6 & 2 & 2 & 1 \end{bmatrix}$$

Solution

Value of the game = 4

Exercise

1. Explain the concept of game theory.

2. What is a rectangular game?

3. What is a saddle point?

4. Define pure and mixed strategy in a game.

5. What are the characteristics of game theory?

6. Explain two-person zero-sum game giving suitable examples.

7. Explain the following terms

   a. Competitive Game

   b. Strategy

   c. Value of the game

   d. Pay-off-matrix

   e. Optimal strategy

8. Explain Maximin and Minimax used in game theory

9. For the game with payoff matrix

$$
\text{Player A} \begin{array}{c} \text{Player B} \\ \begin{bmatrix} -1 & 2 & -2 \\ 6 & 4 & -6 \end{bmatrix} \end{array}
$$

Determine the best strategies for player A and B and also the value of the game.