Many optical systems use circular apertures rather than slits. The diffraction pattern of a circular aperture, as shown in the lower half of Figure 38.13, consists of a central circular bright disk surrounded by progressively fainter bright and dark rings. Figure 38.13 shows diffraction patterns for three situations in which light from two point sources passes through a circular aperture. When the sources are far apart, their images are well resolved (Fig. 38.13a). When the angular separation of the sources satisfies Rayleigh's criterion, the images are just resolved (Fig. 38.13b). Finally, when the sources are close together, the images are said to be unresolved (Fig. 38.13c).

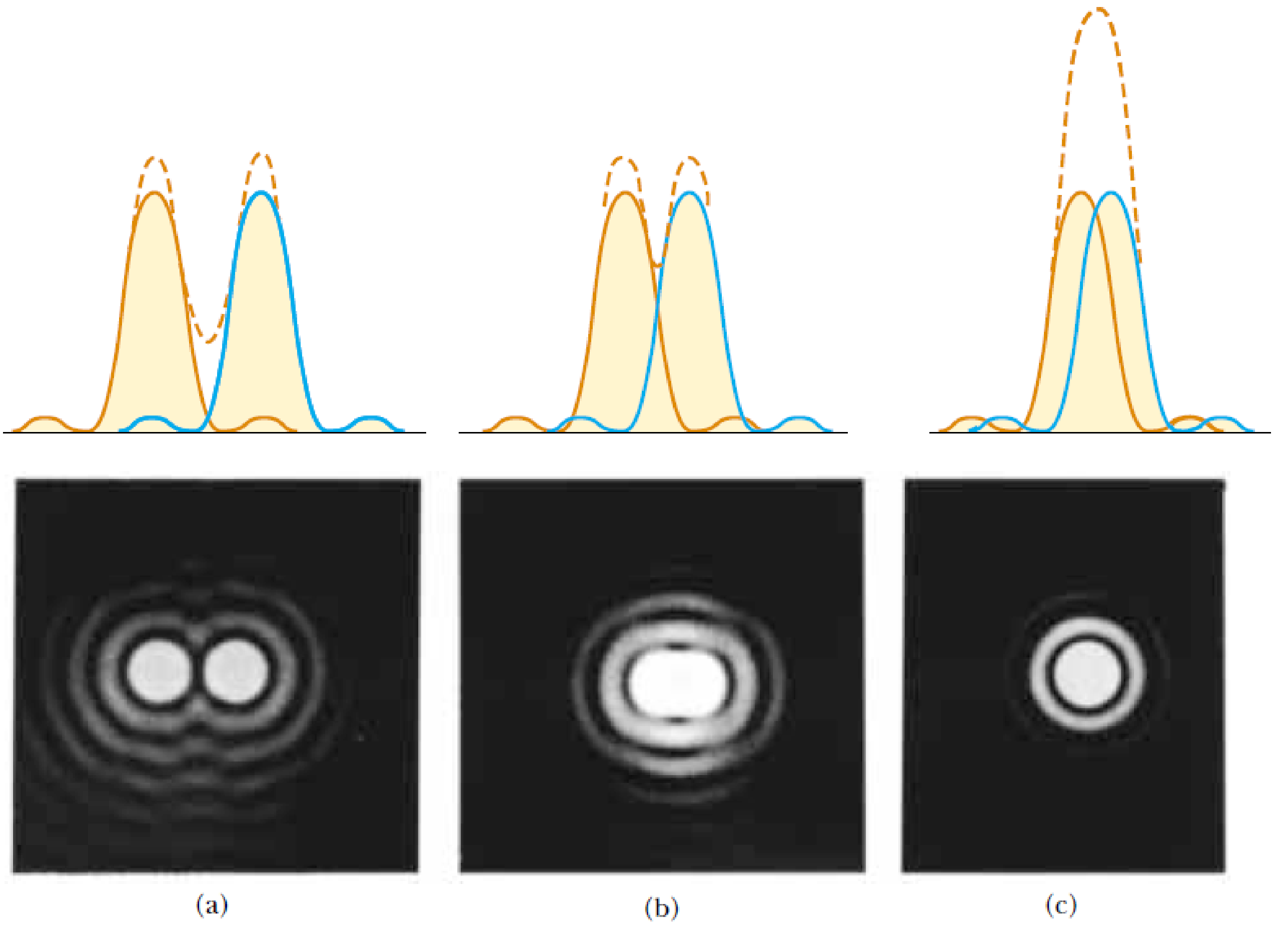

Figure 38.13 Individual diffraction patterns of two point sources (solid curves) and the resultant patterns (dashed curves) for various angular separations of the sources. In each case, the dashed curve is the sum of the two solid curves. (a) The sources are far apart, and the patterns are well resolved. (b) The sources are closer together such that the angular separation just satisfies Rayleigh's criterion, and the patterns are just resolved. (c) The sources are so close together that the patterns are not resolved.

Analysis shows that the limiting angle of resolution of the circular aperture is

$$\theta_{\min} = 1.22 \frac{\lambda}{D} \qquad (38.9)$$

Limiting angle of resolution for a circular aperture

where *D* is the diameter of the aperture. Note that this expression is similar to Equation 38.8 except for the factor 1.22, which arises from a mathematical analysis of diffraction from the circular aperture.

Example.3 **Limiting Resolution of a Microscope**
Light of wavelength 589 nm is used to view an object under a microscope. If the aperture of the objective has a diameter of 0.9 cm.

(A) what is the limiting angle of resolution?

Solution Using Equation 38.9, we find that the limiting angle of resolution is

$$\theta_{\min} = 1.22\left(\frac{589 \times 10^{-9}\ \mathrm{m}}{0.900 \times 10^{-2}\ \mathrm{m}}\right) = 7.98 \times 10^{-5}\ \mathrm{rad}$$

This means that any two points on the object subtending an angle smaller than this at the objective cannot be distinguished in the image.

(B) If it were possible to use visible light of any wavelength, what would be the maximum limit of resolution for this microscope?

Solution To obtain the smallest limiting angle, we have to use the shortest wavelength available in the visible spectrum. Violet light (400 nm) gives a limiting angle of resolution of

$$\theta_{\min} = 1.22\left(\frac{400 \times 10^{-9}\ \mathrm{m}}{0.900 \times 10^{-2}\ \mathrm{m}}\right) = 5.42 \times 10^{-5}\ \mathrm{rad}$$

Example.4 **Resolution of a Telescope**
The Keck telescope at Mauna Kea, Hawaii, has an effective diameter of 10 m. What is its limiting angle of resolution for 600nm light?

Solution: Because *D* = 10 m and $\lambda = 6.00 \times 10^{-7}$ m, Equation38.9 gives

$$\theta_{\min} = 1.22\,\frac{\lambda}{D} = 1.22\left(\frac{6.00 \times 10^{-7}\ \mathrm{m}}{10\ \mathrm{m}}\right)$$

$$= 7.3 \times 10^{-8}\ \mathrm{rad} \approx 0.015\ \mathrm{s\ of\ arc}$$

The **Rayleigh criterion** states that if the center of the bright spot of one star's image falls on the first dark ring of the second, the two images are at the limit of resolution. That is, a viewer will be able to tell that there are two stars rather than only one. If two images are at the limit of resolution, how far apart are the objects? Using the Rayleigh criterion, the centers of the bright spots of the two images are a distance of x_1 apart. Figure 1 shows that similar triangles can be used to find that $x_{obj}/L_{obj} = x_1/L$. Combining this with $x_1 = 1.22\lambda L/D$ to eliminate x_1/L, and solving for the separation distance between objects, x_{obj}, the following equation can be derived.

Rayleigh Criterion $$x_{obj} = \frac{1.22\lambda L_{obj}}{D}$$

The separation distance between objects that are at the limit of resolution is equal to 1.22, times the wavelength of light, times the distance from the circular aperture to the objects, divided by the diameter of the circular aperture.

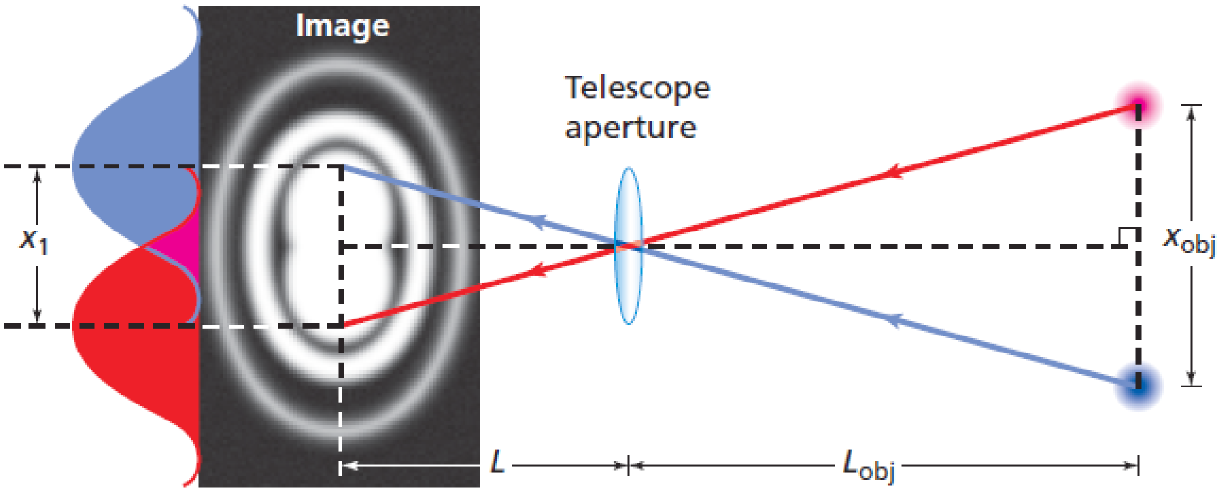

Figure 1. Similar-triangle geometry allows you to calculate the actual separation distance of objects. The blue and red colors are used only for the purpose of illustration

Rectangular Aperture

In the preceding sections the intensity function for a slit was derived by summing the effects of the spherical wavelets originating from a linear section of the wave front by a plane perpendicular to the length of the slit, i.e., by the plane of the page in Fig. ISC. Nothing was said about the contributions from parts of the wave front out of this plane. A more thorough mathematical investigation, involving a double integration over both dimensions of the wave front, * shows, however, that the above result is correct when the slit is very long compared to its width. The complete treatment gives, for a slit of width *b* and length *I,* the following expression for the intensity:

$$I \approx b^2 l^2 \frac{\sin^2 \beta}{\beta^2} \frac{\sin^2 \gamma}{\gamma^2} \qquad (15h)$$

where

$$\beta = (\pi b \sin \theta)/\lambda, \text{ as before, and } \gamma = (\pi l \sin \Omega)/\lambda.$$

The angles θ and Ω are measured from the normal to the aperture at its center, in planes through the normal parallel to the sides b and *l,* respectively. The diffraction pattern given. by Eq. (15h) when b and *l* are comparable with each other is illustrated in Fig. 15G. The dimensions of the aperture are shown by the white rectangle in the lower left-hand part of the figure.

Now for a slit having *l* very large, the factor $(\sin^2 \gamma)/\gamma^2$ in Eq. (15h) is zero for all values of Ω except extremely small ones. This means that the diffraction pattern will be limited to a line on the screen perpendicular to the slit and will resemble a section of the central horizontal line of bright spots in Fig. 15G.

When l approaches to zero, $(\sin^2 \gamma)/\gamma^2=1$,so the intensity in Eq. 15h become the intensity for single slit

$$I = I_o \frac{\sin^2 \beta}{\beta^2}$$

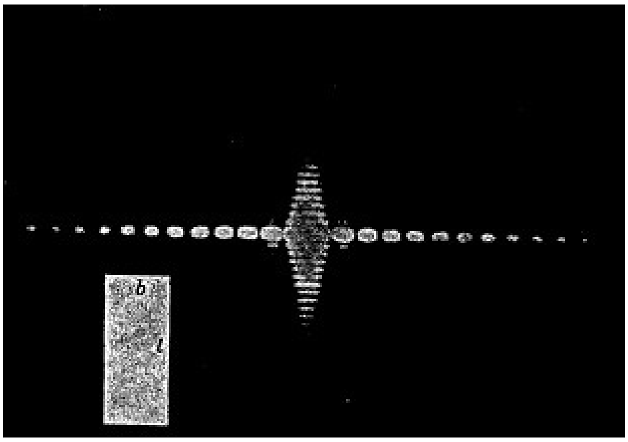

Figure 15G Diffraction pattern from a rectangular opening.

Resolving Power With A Rectangular Aperture

By the resolving power of an optical instrument we mean its ability to produce separate images of objects very close together. Using the laws of geometrical optics, one designs a telescope or a microscope to give an image of a point source which is as small as possible. However, in the final analysis, it is the diffraction pattern that sets a theoretical upper limit to the resolving power. We have seen that whenever parallel light passes through any aperture, it cannot be focused to a point image but instead gives a diffraction pattern in which the central maximum has a certain finite width, inversely proportional to the width of the aperture. The images of two objects will evidently not be resolved if their separation is much less than the width of the central diffraction maximum. The aperture here involved is usually that of the objective lens of the telescope or microscope and is therefore circular.

Figure 15H shows two plano-convex lenses (equivalent to a single double-convex lens) limited by a rectangular aperture of vertical dimension *b*. Two narrow slit sources S_1 and S_2 perpendicular to the plane of the figure form real images S_1' and S_2' on a screen. Each image consists of a single-slit diffraction pattern for which the intensity distribution is plotted in a vertical direction. The angular separation α of the central maxima is equal to the angular separation of the sources, and with the value shown in the figure is adequate to give separate images.

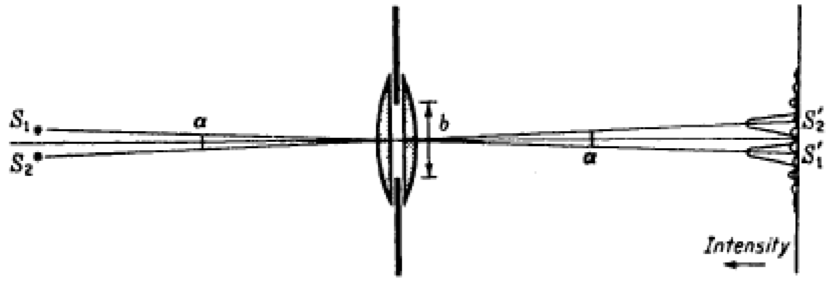

Figure 15H Diffraction images of two slit sources formed by a rectangular aperture.

The condition illustrated is that in which each principal maximum falls exactly on the second minimum of the adjacent pattern. This is the smallest possible value of α (which will give zero intensity between the two strong maxima in the resultant pattern. The angular separation from the center to the second minimum in either pattern then corresponds to $\beta = 2\pi$ (see Fig. 15D), or $\sin\theta \sim \theta = 2\lambda/b = 2\theta$, As α is made smaller than this, and the two images move closer together, the intensity between the maxima will rise, until finally no minimum remains at the center. Figure l5I illustrates this by showing the resultant curve (heavy line) for four different values of α.