where β is measured in radians. Combining this information with the previous expression gives

$$E_R = 2R\sin\frac{\beta}{2} = 2\left(\frac{E_0}{\beta}\right)\sin\frac{\beta}{2} = E_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]$$

Because the resultant light intensity I at a point on the screen is proportional to the square of the magnitude E_R, we find that

$$I = I_{\max}\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 \qquad (38.4)$$

Intensity of a single-slit Fraunhofer diffraction patter

where $I_{\max}$ is the intensity at $\theta = 0$ (the central maximum). Substituting the expression for β (Eq. 38.3) into Equation 38.4, we have

$$I = I_{\max}\left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right]^2 \qquad (38.5)$$

From this result, we see that *minima* occur when

$$\frac{\pi a\sin\theta_{\text{dark}}}{\lambda} = m\pi$$

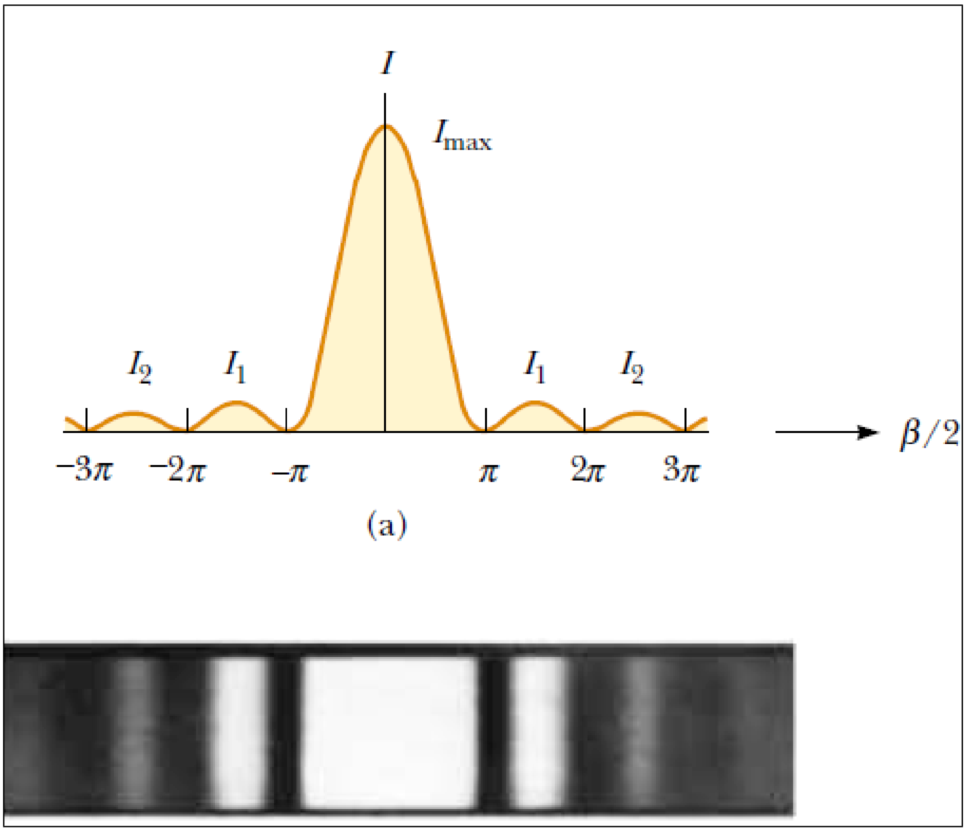


Figure 38.10 (a) A plot of light intensity *I* versus β/2 for the single-slit Fraunhofer diffraction pattern. (b) Photograph of a single-slit Fraunhofer diffraction pattern

Or

$$\sin \theta_{\text{dark}} = m \frac{\lambda}{a} \qquad m = \pm 1, \pm 2, \pm 3, \ . \ . \ .$$

Condition for intensity minima for a single slit

in agreement with Equation 38.1.

Figure 38.10a represents a plot of Equation 38.4, and Figure 38.10b is a photograph of a single-slit Fraunhofer diffraction pattern. Note that most of the light intensity is concentrated in the central bright fringe.

Example 38.2 **Relative Intensities of the Maxima**

Find the ratio of the intensities of the secondary maxima to the intensity of the central maximum for the single-slit Fraunhofer diffraction pattern.

Solution To a good approximation, the secondary maxima lie midway between the zero points. From Figure 38.10a, nwe see that this corresponds to β/2 values of 3π/2, 5π/2, 7π/2, ... Substituting these values into Equation 38.4 gives for the first two ratios

$$\frac{I_1}{I_{\max}} = \left[\frac{\sin(3\pi/2)}{(3\pi/2)}\right]^2 = \frac{1}{9\pi^2/4} = 0.045$$

$$\frac{I_2}{I_{\max}} = \left[\frac{\sin(5\pi/2)}{5\pi/2}\right]^2 = \frac{1}{25\pi^2/4} = \boxed{0.016}$$

Intensity of Two-Slit Diffraction Patterns:

When more than one slit is present, we must consider not only diffraction patterns due to the individual slits but also the interference patterns due to the waves coming from different slits. Notice the curved dashed lines in Figure 37.14, which indicate a decrease in intensity of the interference maxima as θ increases. This decrease is due to a diffraction pattern. To determine the effects of both two-slit interference and a single-slit diffraction pattern from each slit, we combine Equations 37.12 and 38.5:

$$I = I_{\max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)\left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2 \qquad (38.6)$$

Although this expression looks complicated, it merely represents the single-slit diffraction pattern (the factor in square brackets) acting as an "envelope" for a two-slit

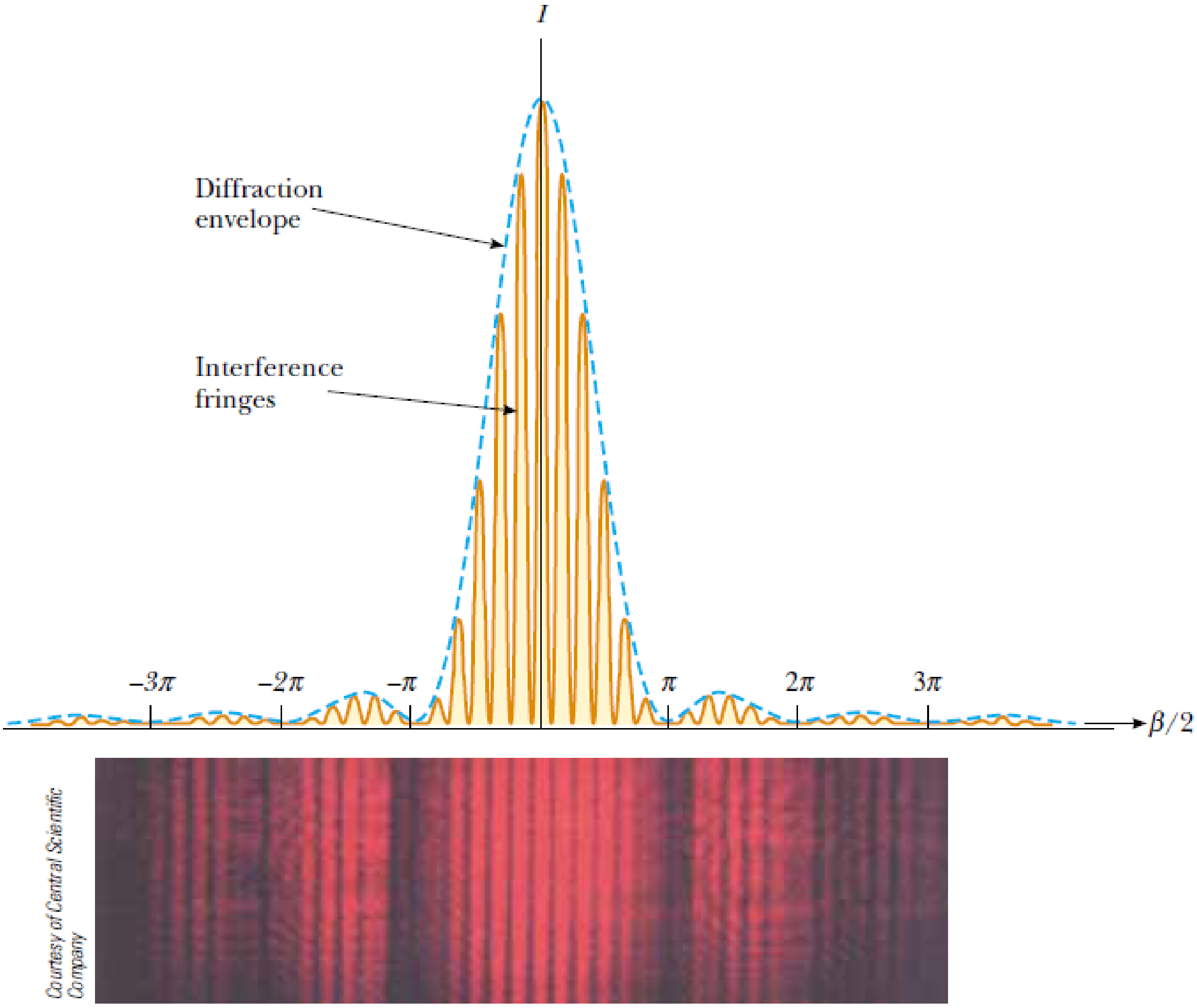


Figure 38.11 The combined effects of two-slit and single-slit interference. This is the pattern produced when 650 nm light waves pass through two 3 μm slits that are 18 μm apart. Notice how the diffraction pattern acts as an "envelope" and controls the intensity of the regularly spaced interference maxima.

interference pattern (the cosine-squared factor), as shown in Figure 38.11. The broken blue curve in Figure 38.11 represents the factor in square brackets in Equation 38.6. The cosine-squared factor by itself would give a series of peaks all with the same height as the highest peak of the red-brown curve in Figure 38.11. Because of the effect of the square-bracket factor, however, these peaks vary in height as shown.

Equation 37.2 indicates the conditions for interference maxima as $d \sin\theta = m\lambda$ where d is the distance between the two slits. Equation 38.1 specifies that the first diffraction minimum occurs when $a \sin\theta = m\lambda$ where a is the slit width. Dividing Equation 37.2 by Equation 38.1(with m =1) allows us to determine which interference maximum coincides with the first diffraction minimum:

$$\frac{d\sin\theta}{a\sin\theta} = \frac{m\lambda}{\lambda}$$

$$\frac{d}{a} = m \qquad (38.7)$$

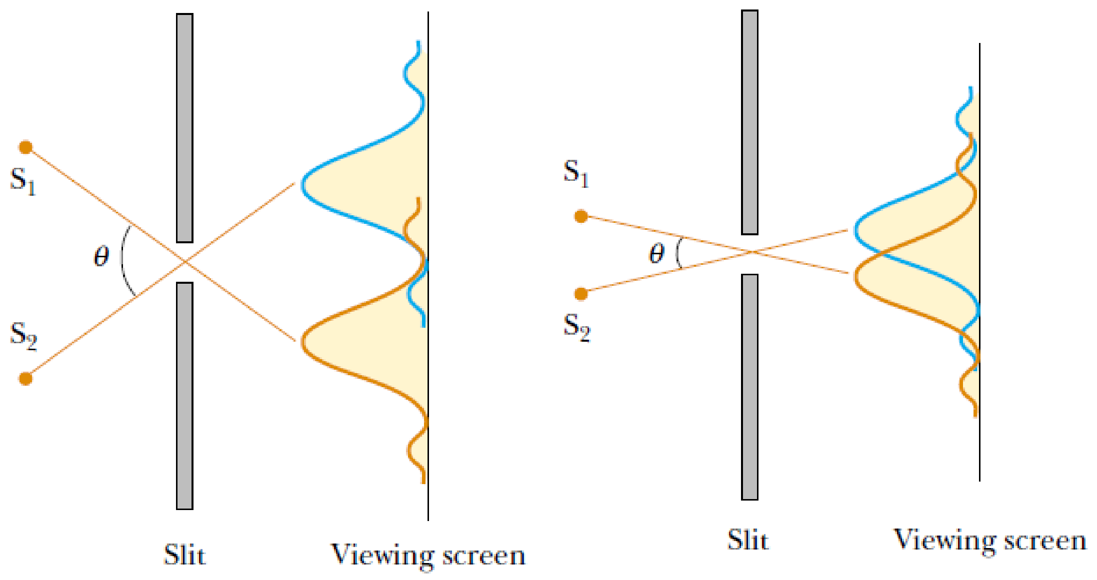
In Figure 38.11, d/a =18 μm/3 μm =6. Therefore, the sixth interference maximum (if we count the central maximum as m =0) is aligned with the first diffraction minimum and cannot be seen.

Resolution of Single-Slit and Circular Apertures:

The ability of optical systems to distinguish between closely spaced objects is limited because of the wave nature of light. To understand this difficulty, consider Figure 38.12, which shows two light sources far from a narrow slit of width *a*. The sources can be two noncoherent point sources S_1 and S_2 - for example, they could be two distant stars. If no interference occurred between light passing through different parts of the slit, two distinct bright spots (or images) would be observed on the viewing screen. However, because of such interference, each source is imaged as a bright central region flanked by weaker bright and dark fringes—a diffraction pattern. What is observed on the screen is the sum of two diffraction patterns: one from S_1, and the other from S_2.

If the two sources are far enough apart to keep their central maxima from overlapping as in Figure 38.12a, their images can be distinguished and are said to be *resolved*. If the sources are close together, however, as in Figure 38.12b, the two central maxima overlap, and the images are not resolved. To determine whether two images are resolved, the following condition is often used:

When the central maximum of one image falls on the first minimum of another image, the images are said to be just resolved. This limiting condition of resolution is known as Rayleigh's criterion.



(a) (b)

Figure 38.12 Two point sources far from a narrow slit each produce a diffraction pattern. (a) The angle subtended by the sources at the slit is large enough for the diffraction patterns to be distinguishable. (b) The angle subtended by the sources is so small that their diffraction patterns overlap, and the images are not well resolved. (Note that the angles are greatly exaggerated.)

From Rayleigh's criterion, we can determine the minimum angular separation θ_{min} subtended by the sources at the slit in Figure 38.12 for which the images are just resolved. Equation 38.1 indicates that the first minimum in a single-slit diffraction pattern occurs at the angle for which

$$\sin\theta = \frac{\lambda}{a}$$

where a is the width of the slit. According to Rayleigh's criterion, this expression gives the smallest angular separation for which the two images are resolved. Because $\lambda \ll a$ in most situations, $\sin\theta$ is small, and we can use the approximation $\sin\theta \approx \theta$.

Therefore, the limiting angle of resolution for a slit of width a is

$$\theta_{min} = \frac{\lambda}{a} \qquad (38.8)$$

where θ_{min} is expressed in radians. Hence, the angle subtended by the two sources at the slit must be greater than λ/a if the images are to be resolved.