$$\sin\theta = \pm\frac{\lambda}{a}$$

If we divide the slit into four equal parts and use similar reasoning, we find that the viewing screen is also dark when

$$\sin\theta = \pm\frac{2\lambda}{a}$$

Likewise, we can divide the slit into six equal parts and show that darkness occurs on the screen when

$$\sin\theta = \pm\frac{3\lambda}{a}$$

Therefore, the general condition for destructive interference is

$$\sin\theta_{\text{dark}} = m\frac{\lambda}{a} \qquad m = \pm 1, \pm 2, \pm 3, \ldots$$

(38.1)....................

Condition for destructive interference for a single slit

This equation gives the values of θ_{dark} for which the diffraction pattern has zero light intensity-that is, when a dark fringe is formed. However, it tells us nothing about the variation in light intensity along the screen. The general features of the intensity distribution are shown in Figure 38.6. A broad central bright fringe is observed; this fringe is flanked by much weaker bright fringes alternating with dark fringes. The various dark fringes occur at the values of θ_{dark} that satisfy Equation 38.1. Each bright-fringe peak lies approximately halfway between its bordering darkfringe minima. Note that the central bright maximum is twice as wide as the secondary maxima.

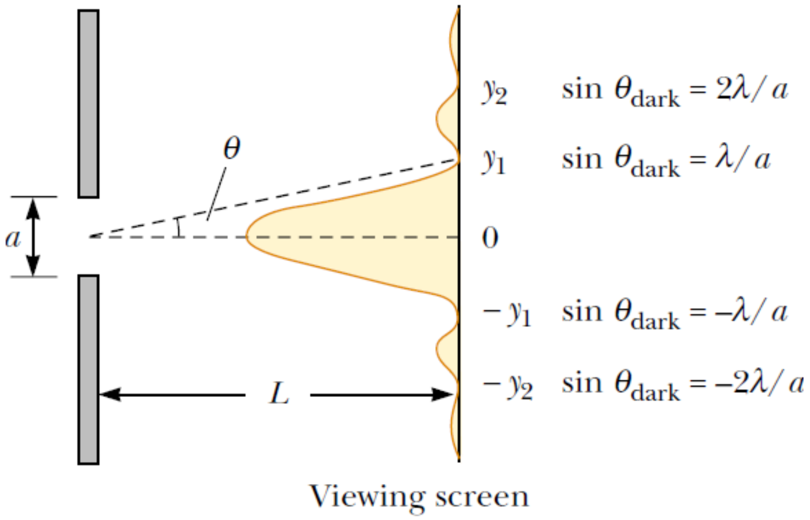

Figure 38.6 Intensity distribution for a Fraunhofer diffraction pattern from a single slit of width *a*. The positions of two minima on each side of the central maximum are labeled.

Example 38.1 Where Are the Dark Fringes?

Light of wavelength 580 nm is incident on a slit having a width of 0.300 mm. The viewing screen is 2.00 m from the slit. Find the positions of the first dark fringes and the width of the central bright fringe.

Solution: The problem statement cues us to conceptualize a single-slit diffraction pattern similar to that in Figure 38.6. We categorize this as a straightforward application of our discussion of single-slit diffraction patterns. To analyze the problem, note that the two dark fringes that flank the central bright fringe correspond to $m = \pm 1$ in Equation 38.1. Hence, we find that

$$\sin\theta_{dark} = \pm\frac{\lambda}{a} = \pm\frac{5.80\times10^{-7}\text{ m}}{0.300\times10^{-3}\text{ m}} = \pm1.933\times10^{-3}$$

From the triangle in Figure 38.6, note that $\tan\theta_{dark} \approx y_1/L$. Because θ_{dark} is very small, we can use the approximation $\sin\theta_{dark} \approx \tan\theta_{dark}$; thus, $\sin\theta_{dark} \approx y_1/L$. Therefore, the positions of the first minima measured from the central axis are given by

$$y_1 \approx L\sin\theta_{dark} = (2.00\text{ m})(\pm1.933\times10^{-3})$$
$$= \pm3.87\times10^{-3}\text{ m}$$

The positive and negative signs correspond to the dark fringes on either side of the central bright fringe. Hence, the width of the central bright fringe is equal to

$$2|y_1| = 7.74\times10^{-3}\text{ m} = 7.74\text{ mm.}$$

Intensity of Single-Slit Diffraction Patterns

We can use phasors to determine the light intensity distribution for a single-slit diffraction pattern. Imagine a slit divided into a large number of small zones, each of width Δy as shown in Figure 38.7. Each zone acts as a source of coherent radiation,and each contributes an incremental electric field of magnitude ΔE at some point on the screen. We obtain the total electric field magnitude E at a point on the screen by summing the contributions from all the zones. The light intensity at this point is proportional to the square of the magnitude of the electric field.

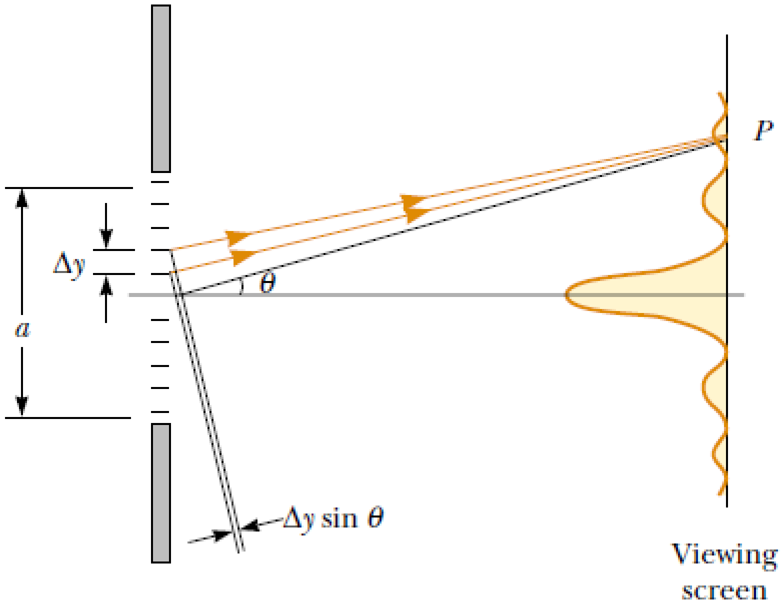

Figure 38.7 Fraunhofer diffraction pattern for a single slit. The light intensity at a distant screen is the resultant of all the incremental electric field magnitudes from zones of width Δy.

The incremental electric field magnitudes between adjacent zones are out of phase with one another by an amount $\Delta\beta$, where the phase difference $\Delta\beta$ is related to the path difference $\Delta y \sin\theta$ between adjacent zones by an expression given by an argument similar to that leading to Equation 37.8:

$$\Delta\beta = \frac{2\pi}{\lambda}\Delta y \sin\theta \qquad \text{..................(38.2)}$$

To find the magnitude of the total electric field on the screen at any angle θ, we sum the incremental magnitudes ΔE due to each zone. For small values of θ, we can assume that all the ΔE values are the same. It is convenient to use phasor diagrams for
various angles, as in Figure 38.8. When $\theta = 0$, all phasors are aligned as in Figure 38.8a because all the waves from the various zones are in phase. In this case, the total

electric field at the center of the screen is $E_0 = N\Delta E$, where N is the number of zones. The resultant magnitude E_R at some small angle θ is shown in Figure 38.8b, where each phasor differs in phase from an adjacent one by an amount $\Delta\beta$. In this case, E_R is the

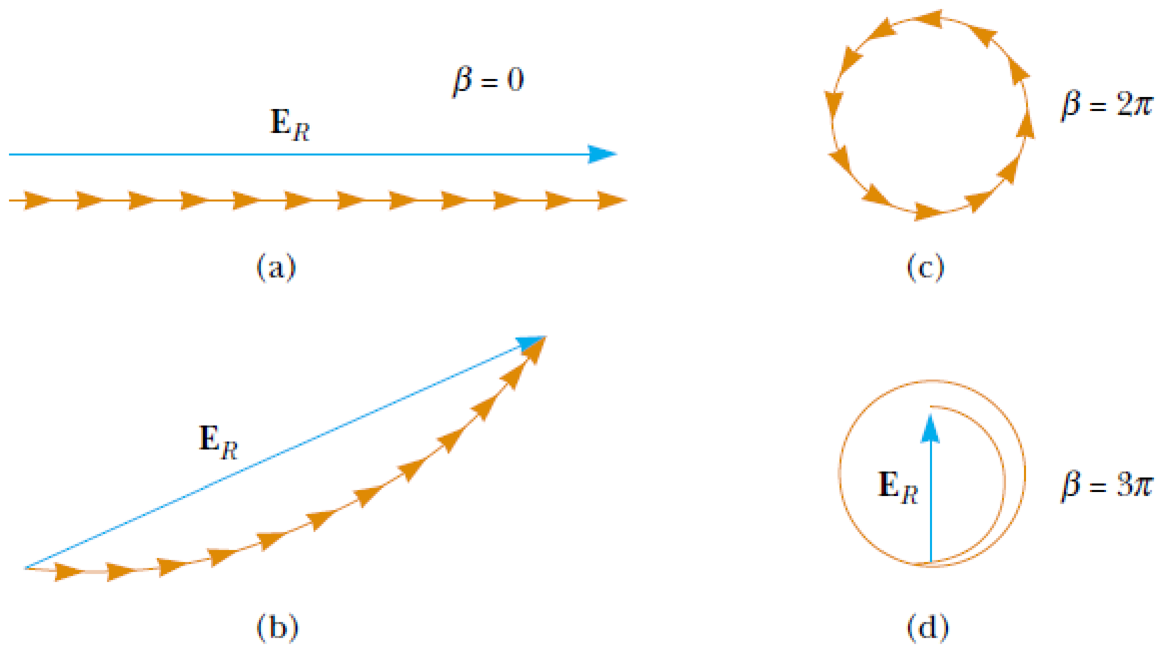

Figure 38.8 Phasor diagrams for obtaining the various maxima and minima of a single-slit diffraction pattern.

vector sum of the incremental magnitudes and hence is given by the length of the chord. Therefore, $E_R < E_0$. The total phase difference β between waves from the top and bottom portions of the slit is

$$\beta = N\Delta\beta = \frac{2\pi}{\lambda} N\Delta y \sin\theta = \frac{2\pi}{\lambda} a\sin\theta \qquad (38.3)$$

where $a = N\Delta y$ is the width of the slit.

As θ increases, the chain of phasors eventually forms the closed path shown in Figure 38.8c. At this point, the vector sum is zero, and so $E_R = 0$, corresponding to the first minimum on the screen. Noting that $\beta = N\Delta\beta = 2\pi$ in this situation, we see from Equation 38.3 that

$$2\pi = \frac{2\pi}{\lambda} a\sin\theta_{\text{dark}}$$

$$\sin\theta_{\text{dark}} = \frac{\lambda}{a}$$

That is, the first minimum in the diffraction pattern occurs where $\sin\theta_{dark} = \lambda / a$; this
is in agreement with Equation 38.1.

At larger values of θ, the spiral chain of phasors tightens. For example, Figure 38.8d represents the situation corresponding to the second maximum, which occurs when β = 360° + 180° = 540° (3π rad). The second minimum (two complete circles, not shown) corresponds to β = 720° (4π rad), which satisfies the condition $\sin\theta_{dark} = 2\lambda / a$.

We can obtain the total electric-field magnitude E_R and light intensity I at any point on the screen in Figure 38.7 by considering the limiting case in which Δy becomes infinitesimal (dy) and N approaches ∞. In this limit, the phasor chains in Figure 38.8 become the curve of Figure 38.9.

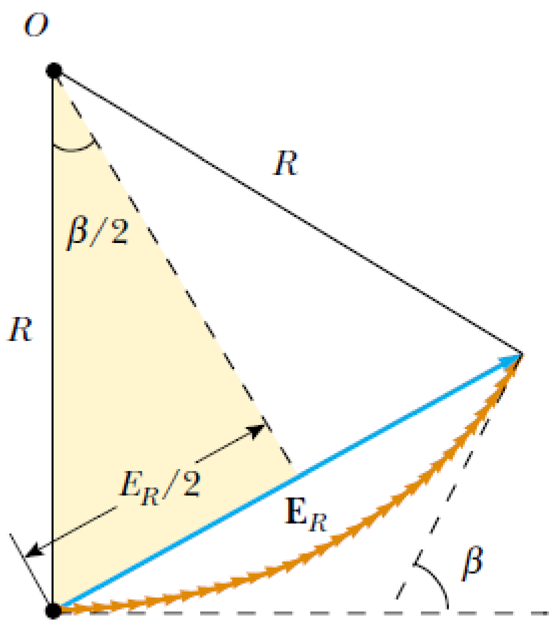

Figure 38.9 Phasor diagram for a large number of coherent sources. All the ends of the phasors lie on the circular arc of radius *R*. The resultant electric field magnitude *ER* equals the length of the chord.

The arc length of the curve is E_0 because it is the sum of the magnitudes of the phasors (which is the total electric field magnitude at
the center of the screen). From this figure, we see that at some angle θ, the resultant electric field magnitude E_R on the screen is equal to the chord length. From the triangle containing the angle β /2, we see that

$$\sin\frac{\beta}{2} = \frac{E_R/2}{R}$$

where R is the radius of curvature. But the arc length E_0 is equal to the product $R\beta$, where β is measured in radians. Combining this information with the previous expression gives

$$E_R = 2R\sin\frac{\beta}{2} = 2\left(\frac{E_0}{\beta}\right)\sin\frac{\beta}{2} = E_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]$$

Because the resultant light intensity I at a point on the screen is proportional to the square of the magnitude E_R, we find that

$$I = I_{\max}\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 \qquad (38.4)$$

Intensity of a single-slit Fraunhofer diffraction patter

where I_{max} is the intensity at $\theta = 0$ (the central maximum). Substituting the expression for β (Eq. 38.3) into Equation 38.4, we have

$$I = I_{\max}\left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right]^2 \qquad (38.5)$$

From this result, we see that *minima* occur when

$$\frac{\pi a\sin\theta_{\text{dark}}}{\lambda} = m\pi$$