Hence the distance moved equals the number of fringes counted, multiplied by a half wavelength.

Twyman and Green interferometer

If a Michelson interferometer is illuminated with strictly parallel monochromatic light, produced by a point source at the principal focus of a well-corrected lens, it becomes a very powerful instrument for testing the perfection of optical parts such as prisms and lenses. The piece to be tested is placed in one of the light beams, and the mirror behind it is so chosen that the reflected waves, after traversing the test piece a second time, again become plane. These waves are then brought to interference with the plane waves from the other arm of the interferometer by another lens, at the focus of which the eye is placed. If the prism or lens is optically perfect, so that the returning waves are strictly plane, the field will appear uniformly illuminated. Any local variation of the optical path will, however, produce fringes in the corresponding part of the field, which are essentially the contour lines of the distorted wave front. Even though the surfaces of the test piece may be accurately made, the glass may contain regions that are slightly more or less dense. With the Twyman and Green interferometer these can be detected and corrected for by local polishing of the surface.

Index of refraction by interference methods

If a thickness t of a substance having an index of refraction n is introduced into the path of one of the interfering beams in the interferometer, the optical path in this beam is increased because of the fact light travels more slowly in the substance and consequently has a shorter wavelength. The optical path is now nt through the medium, whereas it was practically t through the corresponding thickness of air $(n = 1)$. Thus the increase in optical path due to insertion of the substance is $(n - 1)t$. This will introduce $(n - 1)t/\lambda$ extra waves in the path of one beam; so if we call Δm the number of fringes by which the fringe system is displaced when the substance is placed in the beam, we have

$$(n - 1)t = \Delta m\, \lambda \qquad (11)$$

In principle a measurement of Δm, t, and λ thus gives a determination of n.

Interference Involving Multiple Reflections

Some of the most beautiful effects of interference result from the multiple reflection of light between the two surfaces of a thin film of transparent material. These effects require no special apparatus for their production or observation and are familiar to anyone who has noticed the colors shown by thin films of oil on water, by soap bubbles, or by cracks in a piece of glass. We begin our investigation of this class of interference by considering the somewhat idealized case of reflection and refraction from the boundary separating different optical media. In Fig. 14A(a) a ray of light in air or vacuum incident on a plane surface of a transparent medium like water is indicated by *a*. The reflected and refracted rays are indicated by *ar* and *at*, respectively.

A question of particular interest from the standpoint of physical optics is that of a possible abrupt *change of phase* of waves when they are reflected from a boundary. For a given boundary the result will differ, as we shall now show, according to whether the waves approach from the side of higher velocity or from that of lower velocity. Thus, let the symbol *a* in the left-hand part of Fig. 14A represent the amplitude (not the intensity) of a set of waves striking the surface, let *r* be the fraction of the amplitude reflected, and let *t* be the fraction transmitted. The amplitudes of the two sets of waves will then be *ar* and *at*, as shown. Now, following a treatment given by Stokes., imagine the two sets reversed in direction, as in part *(b)* of the figure. Provided there is no dissipation of energy by absorption, a wave motion is a strictly reversible phenomenon. The two reversed trains, of amplitude *ar* and *at*, should accordingly have as their net effect after striking the surface a wave in air equal in amplitude to the incident wave in part *(a)* but traveling in the opposite direction. The wave of amplitude *ar* gives a reflected wave of amplitude *arr* and a refracted wave of amplitude *art*. If we call *r′* and *t′* the fractions of the amplitude reflected and refracted when the reversed wave *at* strikes the boundary from below, this contributes amplitudes *att′* and *atr′* to the two waves, as indicated. Now, since the resultant effect must consist only of a wave in air of amplitude *a*, we have

$$att' + arr = a \qquad (12a)$$

and

$$art + atr' = 0 \qquad (12b)$$

The second equation states that the two incident waves shall produce no net disturbance on the water side of the boundary. From Eq. (12a) we obtain

$$tt' = 1 - r^2 \qquad (13a)$$

and from Eq. (12b)

$$r' = -r \qquad (13b)$$

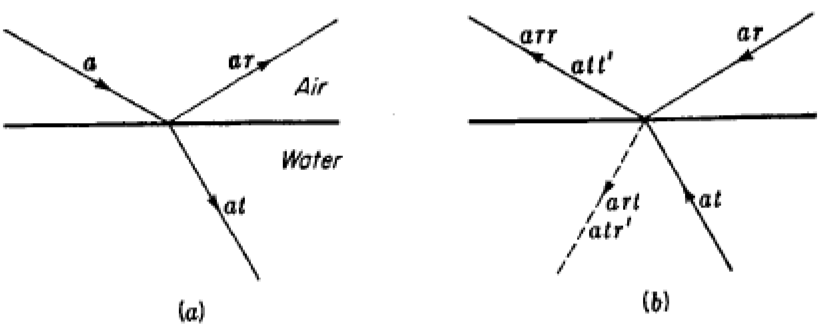


Fig.14A Stokes' treatment of reflection.

Interference in Thin Films:

Interference effects are commonly observed in thin films, such as thin layers of oil on water or the thin surface of a soap bubble. The varied colors observed when white light is incident on such films result from the interference of waves reflected from the two surfaces of the film. Consider a film of uniform thickness t and index of refraction n, as shown in Figure 15. Let us assume that the light rays traveling in air are nearly normal to the two surfaces of the film. To determine whether the reflected rays interfere constructively or destructively, we first note the following facts:

- A wave traveling from a medium of index of refraction n_1 toward a medium of index of refraction n_2 undergoes a 180° phase change upon reflection when $n_2 > n_1$ and undergoes no phase change if $n_2 < n_1$.
- The wavelength of light ✦$_n$ in a medium whose index of refraction is n

$$\lambda_n = \frac{\lambda}{n} \quad \text{.....................................14}$$

where ✦ is the wavelength of the light in free space.

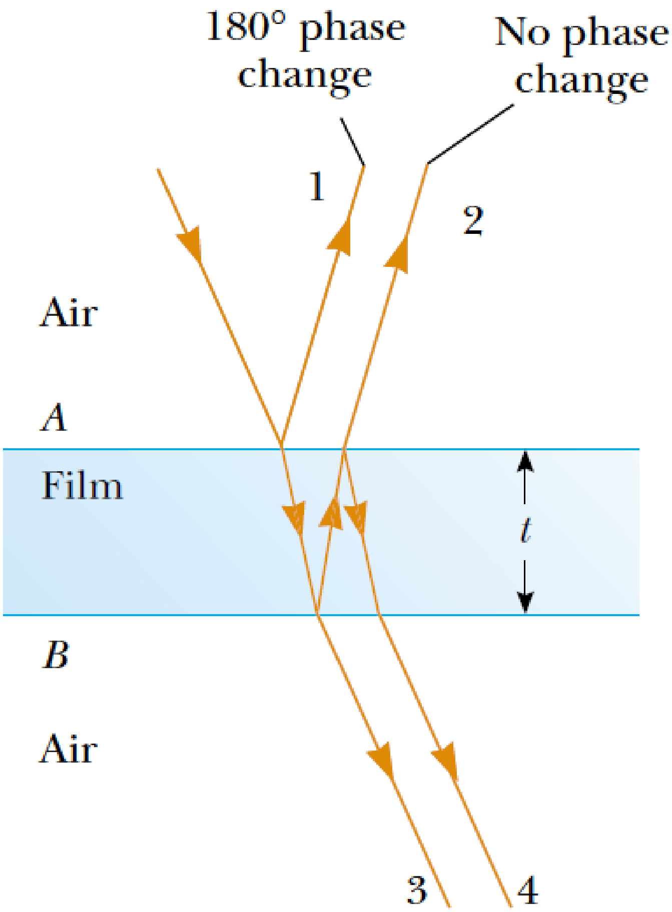


Figure 15 Interference in light reflected from a thin film is due to a combination of rays 1 and 2 reflected from the upper and lower surfaces of the film. Rays 3 and 4 lead to interference effects for light transmitted through the film.

Let us apply these rules to the film of Figure 15, where $n_{film} > n_{air}$. Reflected ray 1, which is reflected from the upper surface (A), undergoes a phase change of 180° with respect to the incident wave. Reflected ray 2, which is reflected from the lower film surface (B), undergoes no phase change because it is reflected from a medium (air) that has a lower index of refraction. Therefore, ray 1 is 180° out of phase with ray 2which is equivalent to a path difference of **$\lambda_n/2$**. However, we must also consider that ray 2 travels an extra distance 2t before the waves recombine in the air above surface A. Remember that we are considering light rays that are close to normal to the surface. If the rays are not close to normal, the path difference is larger than (2t). If **$2t = \lambda_n/2$**, then rays 1 and 2 recombine in phase, and the result is constructive interference. In general, the condition for constructive interference in thin films is.

$$2t = (m + \tfrac{1}{2})\lambda_n \qquad (m = 0, 1, 2, \ . \ . \ .) \qquad ....15$$

This condition takes into account two factors: (1) the difference in path length for the two rays (the term **$m\lambda_n$**) and (2) the 180° phase change upon reflection (the term
$\lambda_n/2$). Because **$\lambda_n = \lambda/n$**, we can write Equation 15 as

$$2nt = (m + \tfrac{1}{2})\lambda \qquad (m = 0, 1, 2, \ . \ . \ .) \qquad ......16$$

Conditions for constructive interference in thin films

If the extra distance **2t** traveled by ray 2 corresponds to a multiple of **λ_n**, then the two waves combine out of phase, and the result is destructive interference. The general equation for *destructive* interference in thin films is

$$2nt = m\lambda \qquad (m = 0, 1, 2, \ . \ . \ .) \qquad ....17$$

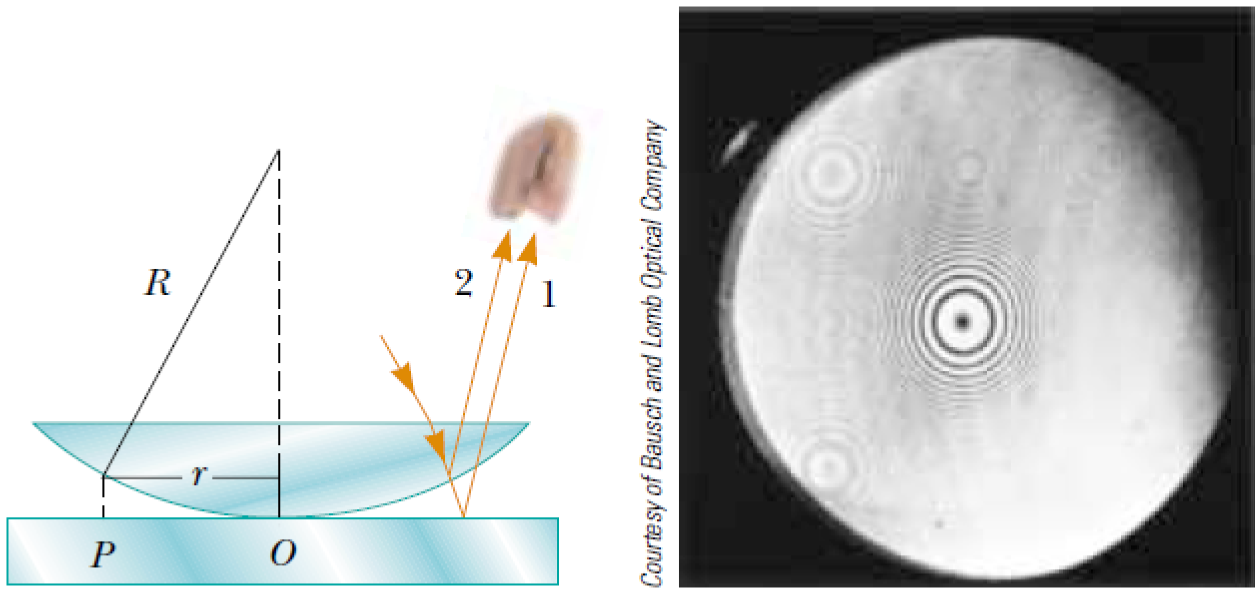
Conditions for destructive interference in thin films

The foregoing conditions for constructive and destructive interference are valid when the medium above the top surface of the film is the same as the medium below the bottom surface or, if there are different media above and below the film, the index of refraction of both is less than n. If the film is placed between two different media, one with **$n<n_{film}$** and the other with **$n>n_{film}$**, then the conditions for constructive and destructive interference are reversed. In this case, either there is a phase change of 180° for both ray 1 reflecting from surface A and ray 2 reflecting from surface B, or there is no phase change for either ray; hence, the net change in relative phase due to the reflections is zero.

Newton's Rings

Another method for observing interference in light waves is to place a plano-convex lens on top of a flat glass surface, as shown in Figure 16-a. With this arrangement, the air film between the glass surfaces varies in thickness from zero at the point of contact to some value *t* at point *P*. If the radius of curvature *R* of the lens is much greater than the distance ***r***, and if the system is viewed from above, a pattern of light and dark rings is observed, as shown in Figure 16-b. These circular fringes, discovered by Newton, are called Newton's rings.

The interference effect is due to the combination of ray 1, reflected from the flat plate, with ray 2, reflected from the curved surface of the lens. Ray 1 undergoes a phase change of **180°** upon reflection (because it is reflected from a medium of higher index of refraction), whereas ray 2 undergoes no phase change (because it is reflected from a medium of lower refractive index). Hence, the conditions for constructive and destructive interference are given by Equations 16 and 17, respectively, with $n = 1$ because the film is air.



(a) (b)

Fig.16 (a) The combination of rays reflected from the flat plate and the curved lens surface gives rise to an interference pattern known as Newton's ring. (b) Photograph of Newton's rings.

The contact point at O is dark, as seen in Figure 16-b, because there is no path difference and the total phase change is due only to the **180°** phase change upon reflection.

Using the geometry shown in Figure 16-a, we can obtain expressions for the radii of the bright and dark bands in terms of the radius of curvature **R** and wavelength λ. For example, the dark rings have radii given by the expression. $r \approx \sqrt{m\lambda R/n}$ The details are eft as a problem for you to solve . We can obtain the wavelength of the light causing the interference pattern by measuring the radii of the rings, provided **R** is known. Conversely, we can use a known wavelength to obtain **R**.

One important use of Newton's rings is in the testing of optical lenses. A circular pattern like that pictured in Figure 16-b is obtained only when the lens is ground to a perfectly symmetric curvature.

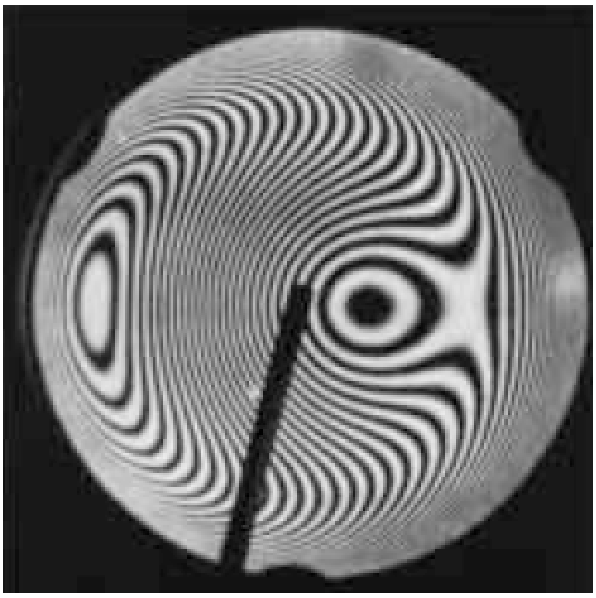


Figure .17 This asymmetrical interference pattern indicates imperfections in the lens of a Newton's-rings apparatus.

Variations from such symmetry might produce a pattern like that shown in Figure .17. These variations indicate how the lens must be reground and repolished to remove imperfections.

Example(1) **interference in a Soap Film**

Calculate the minimum thickness of a soap-bubble film that results in constructive interference in the reflected light if he film is illuminated with light whose wavelength in free pace is λ =600 nm.

Solution : The minimum film thickness for constructive interference in the reflected light corresponds to *m* =0 in Equation 37.16. This gives $2nt = \lambda/2$, or

$$t = \frac{\lambda}{4n} = \frac{600 \text{ nm}}{4(1.33)} = 113 \text{ nm}$$

Example(2) **:**

A Young's interference experiment is performed with monochromatic light. The separation between the slits is 0.5 mm, and the interference pattern on a screen 3.3 m away shows the first side maximum 3.4 mm from the center of the pattern. What is the wavelength?

Solution:

$$y_{\text{bright}} = \frac{\lambda L}{d} m$$

For $m = 1$,
$$\lambda = \frac{yd}{L} = \frac{(3.4 \times 10^{-3} \text{ m})(5 \times 10^{-4} \text{ m})}{3.3 \text{ m}} = \boxed{515 \text{ nm}}$$

Example(3) :

Young's double-slit experiment is performed with 589nm light and a distance of 2 m between the slits and the screen. The tenth interference minimum is observed 7.26 mm from the central maximum. Determine the spacing of the slits.

Solution:

In the equation $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$.

The first minimum is described by $m = 0$

and the tenth by $m = 9$: $\sin\theta = \frac{\lambda}{d}\left(9 + \frac{1}{2}\right)$.

Also, $\tan\theta = \frac{y}{L}$

but for small θ, $\sin\theta \approx \tan\theta$.

Thus, $d = \frac{9.5\lambda}{\sin\theta} = \frac{9.5\lambda L}{y}$

$$d = \frac{9.5(5\,89 \times 10^{-10}\ \text{m})(2\ \text{m})}{7.26 \times 10^{-3}\ \text{m}} = 1.54 \times 10^{-3}\ \text{m} = \boxed{1.54\ \text{mm}} .$$

Example(4)

Light with wavelength 442 nm passes through a double-slit system that has a slit separation $d = 0.4$ mm. Determine how far away a screen must be placed in order that a dark fringe appear directly opposite both slits, with just one bright fringe between them.

Solution:

Taking $m = 0$ and $y = 0.2$ mm in Equation 37.6 gives

$$L \approx \frac{2dy}{\lambda} = \frac{2(0.4\ \times 10^{-3}\ \text{m})(0.2 \times 10^{-3}\ \text{m})}{442 \times 10^{-9}\ \text{m}} = 0.362\ \text{m}$$

$$L \approx \boxed{36.2\ \text{cm}}$$

Example (5)
Mirror M1 in Figure 37.22 is displaced a distance ΔL. During this displacement, 250 fringe reversals (formation of successive dark or bright bands) are counted. The light
being used has a wavelength of 632.8 nm. Calculate the displacement ΔL.

Solution:
When the mirror on one arm is displaced by $\Delta \ell$, the path difference changes by $2\Delta \ell$. A shift resulting in the reversal between dark and bright fringes requires a path length change of one-half
wavelength. Therefore, $2\Delta\ell = \frac{m\lambda}{2}$, where in this case, $m = 250$.

$$\Delta\ell = m\frac{\lambda}{4} = \frac{(250)(6.328\times10^{-7}\ \text{m})}{4} = \boxed{39.6\ \mu\text{m}}$$

Example(6)
Monochromatic light is beamed into a Michelson interferometer. The movable mirror is displaced 0.382 mm, causing the interferometer pattern to reproduce itself 1 700 times. Determine the wavelength of the light. What color is it?

Solution:

$$\text{Distance} = 2(3.82\times10^{-4}\ \text{m}) = 1\,700\lambda$$

$$\lambda = 4.49\times10^{-7}\ \text{m} = \boxed{449\ \text{nm}}$$

The light is $\boxed{\text{blue}}$.