Example 2 : Separating Double-Slit Fringes of Two Wavelengths
A light source emits visible light of two wavelengths: λ = 430 nm and λ'= 510 nm. The source is used in a double-slit interference experiment in which L =1.50 m and d=0.025 mm. Find the separation distance between the third-order bright fringes.

Solution Using Equation 5, with m= 3, we find that the fringe positions corresponding to these two wavelengths are

$$y_{\text{bright}} = \frac{\lambda L}{d} m = 3\frac{\lambda L}{d} = 3\frac{(430 \times 10^{-9}\ \text{m})(1.50\ \text{m})}{0.0250 \times 10^{-3}\ \text{m}}$$
$$= 7.74 \times 10^{-2}\ \text{m}$$

$$y'_{\text{bright}} = \frac{\lambda' L}{d} m = 3\frac{\lambda' L}{d} = 3\frac{(510 \times 10^{-9}\ \text{m})(1.50\ \text{m})}{0.0250 \times 10^{-3}\ \text{m}}$$
$$= 9.18 \times 10^{-2}\ \text{m}$$

Hence, the separation distance between the two fringes is

$$\Delta y = 9.18 \times 10^{-2}\ \text{m} - 7.74 \times 10^{-2}\ \text{m}$$
$$= 1.40 \times 10^{-2}\ \text{m} = \boxed{1.40\ \text{cm}}$$

:Intensity distribution in the fringe system

To find the intensity on the screen at points between the maxima, we may apply the vector method of compounding amplitudes and illustrated for the present case in Fig.4. For the maxima, the angle δ is zero, and the component amplitudes a_1 and a_2 are parallel, so that if they are equal, the resultant $A=2a$. For the minima, a_1 .and a_2 are in opposite directions, and $A= 0$

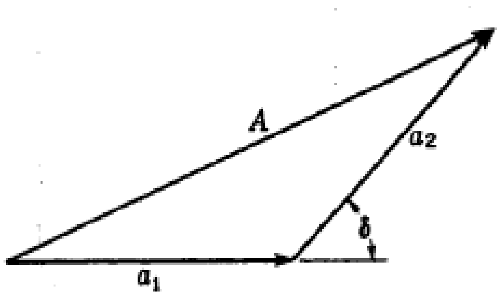


Fig.4.The composition of two waves of the same frequency and amplitude but different phase.

In general, for any value of δ, A is the closing side of the triangle. The value of A^2, which measures the intensity, is then given by Eq. (2). And varies according to $\cos^2(\delta/2)$

$$I = A^2 = 4a^2 \cos^2 (\delta/2)$$

In Fig.5. the solid curve represents a plot of the intensity against the phase .difference

In concluding our discussion of these fringes, one question of fundamental importance should be considered. If the two beams of light arrive at a point on the screen exactly out of phase, they interfere destructively and the resultant intensity is zero. One may well ask what becomes of the energy of the two beams, since the law of conservation of energy tells us that energy cannot be destroyed. The answer to this question is that the energy which apparently disappears at the minima actually is still present at the maxima, where the intensity is greater than would be produced by the two beams acting separately. In other words, the energy is not destroyed but merely redistributed in the interference pattern. The average intensity on the screen is exactly that which would exist in the absence of interference. Thus, as shown in Fig.5, the intensity in the interference pattern varies between $4a^2$ and zero. Now each beam acting separately would contribute $2a^2$, and so without interference we would have a uniform intensity of a^2, as indicated by the broken line.

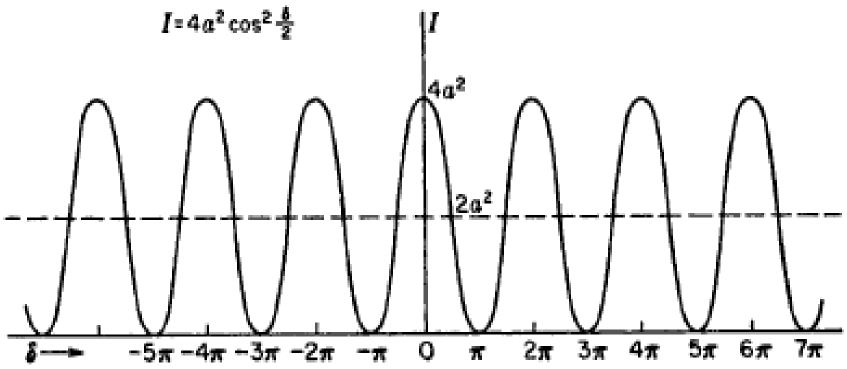


Figure 5 Intensity distribution for the interference fringes from two waves .of the same frequency

To obtain the average intensity on the screen for n fringes, we note that the average value of the square of the cosine is 1/2. This gives, by Eq.2, $I= 2a^2$, justifying the statement made above, and it shows that no violation of the law of conservation of .energy is involved in the interference phenomenon

Intensity Distribution of the Double-Slit Interference Pattern:

Note that the edges of the bright fringes in Figure 37.2b are not sharp-there is a gradual change from bright to dark. So far we have discussed the locations of only the centers of the bright and dark fringes on a distant screen. Let us now direct our attention to the intensity of the light at other points between the positions of maximum constructive and destructive interference. In other words, we now calculate the distribution of light intensity associated with the double-slit interference pattern.

Again, suppose that the two slits represent coherent sources of sinusoidal waves such that the two waves from the slits have the same angular frequency ω and a constant phase difference ϕ . The total magnitude of the electric field at point *P* on the screen in Figure 37.6 is the superposition of the two waves.

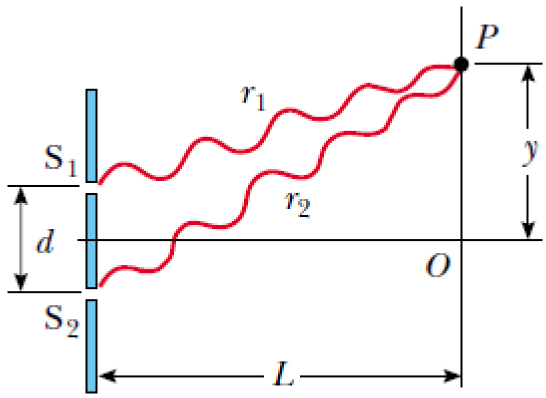


Figure 37.6 Construction for analyzing the double-slit interference pattern. A bright fringe, or intensity maximum, is observed at *O*.

Assuming that the two waves have the same amplitude E_0, we can write the magnitude of the electric field at point *P* due to each wave separately as

$$E_1 = E_0 \sin \omega t \qquad \text{and} \qquad E_2 = E_0 \sin(\omega t + \phi) \qquad \text{....(7)}$$

Although the waves are in phase at the slits, *their phase difference* ϕ *at P depends on the path difference* $\delta = r_2 = r_1 = d \sin\theta$. A path difference of λ (for constructive interference) corresponds to a phase difference of 2π rad. A path difference of δ is the same fraction of λ as the phase difference ϕ is of 2π. We can describe this mathematically with the ratio

$$\frac{\delta}{\lambda} = \frac{\phi}{2\pi}$$

which gives us

$$\phi = \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda} d \sin\theta$$

..................(8) {phase difference}

This equation tells us precisely how the phase difference ϕ depends on the angle θ in Figure 37-5.

Using the superposition principle and Equation 7, we can obtain the magnitude of the resultant electric field at point *P* :

$$E_P = E_1 + E_2 = E_0[\sin \omega t + \sin(\omega t + \phi)] \qquad \text{.......(9)}$$

To simplify this expression, we use the trigonometric identity

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

Taking $A = \omega t + \phi$ and $B = \omega t$, we can write Equation 9 in the form

$$E_P = 2E_0 \cos\left(\frac{\phi}{2}\right) \sin\left(\omega t + \frac{\phi}{2}\right) \qquad \text{......(10)}$$

This result indicates that the electric field at point P has the same frequency - as the light at the slits, but that the amplitude of the field is multiplied by the factor $2\cos(\phi/2)$. To check the consistency of this result, note that if $\phi = 0, 2\pi, 4\pi, \ldots$ then
the magnitude of the electric field at point P is $2E_0$, corresponding to the condition for maximum constructive interference. These values of ϕ are consistent with Equation 2 for constructive interference. Likewise, if $\phi = \pi, 3\pi, 5\pi, \ldots$, then the magnitude of the electric field at point P is zero; this is consistent with Equation 3 for total destructive interference.
Finally, to obtain an expression for the light intensity at point P, recall from Section 34.3 that *the intensity of a wave is proportional to the square of the resultant electric field magnitude at that point* . Using Equation 10, we can therefore express the light intensity at point P as

$$I \propto E_P^2 = 4E_0^2 \cos^2\left(\frac{\phi}{2}\right) \sin^2\left(\omega t + \frac{\phi}{2}\right)$$

Most light-detecting instruments measure time-averaged light intensity, and the timeaveraged value of $\sin^2(\omega t + \phi/2)$ over one cycle is ½. .Therefore, we can write the average light intensity at point P as

$$I = I_{\max} \cos^2\left(\frac{\phi}{2}\right) \qquad \text{............. (11)}$$

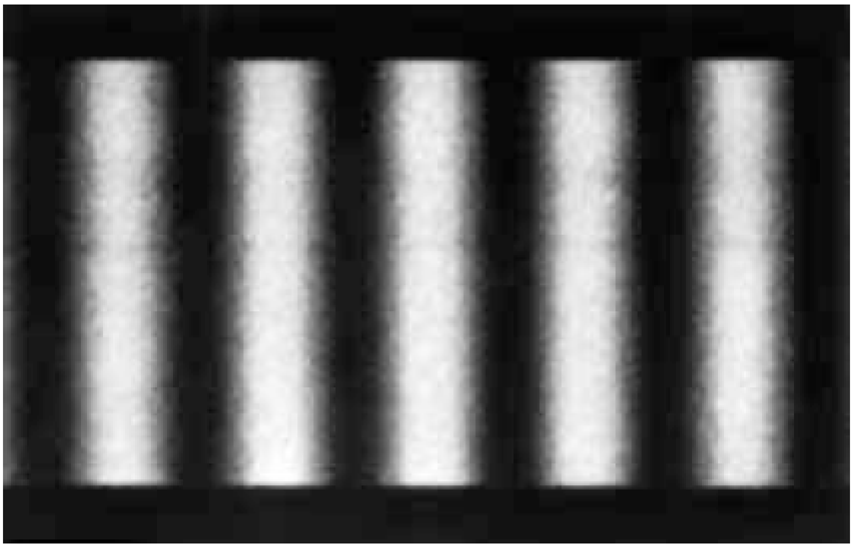
where $I_{\max}$ is the maximum intensity on the screen and the expression represents the time average. Substituting the value for ϕ given by Equation 8 into this expression, we find that

$$\boxed{I = I_{\max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)} \qquad \text{.............(12)}$$

Alternatively, because $\sin\theta \approx y/L$ for small values of θ in Figure 37.5, we can write
Equation 12 in the form

$$I \approx I_{max} \cos^2\left(\frac{\pi d}{\lambda L}\, y\right) \qquad \text{.................(13)}$$

Constructive interference, which produces light intensity maxima, occurs when the quantity $\pi dy/\lambda L$ is an integral multiple of π, corresponding to $y = (\lambda L/d)m$. This is
consistent with Equation 5.



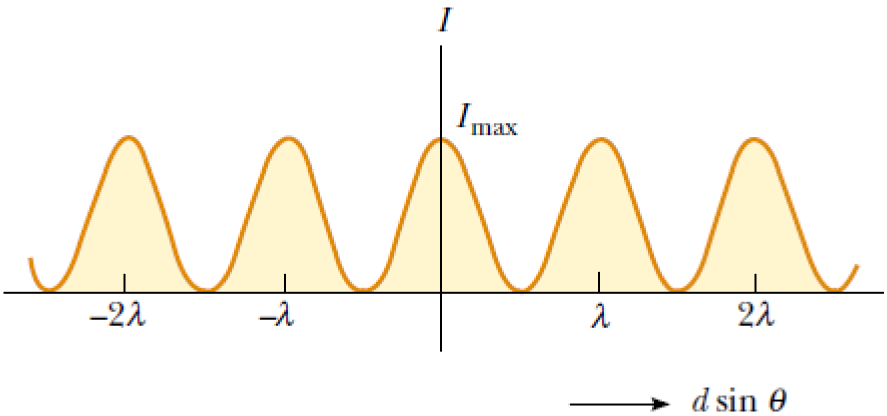


Figure 37.7 Light intensity versus $d \sin\theta$ for a double-slit interference pattern when the screen is far from the two slits ($L >> d$).

A plot of light intensity versus $d \sin\theta$ is given in Figure 37.7. The interference pattern consists of equally spaced fringes of equal intensity. Remember, however, that this result is valid only if the slit-to-screen distance L is much greater than the slit separation, and only for small values of θ.