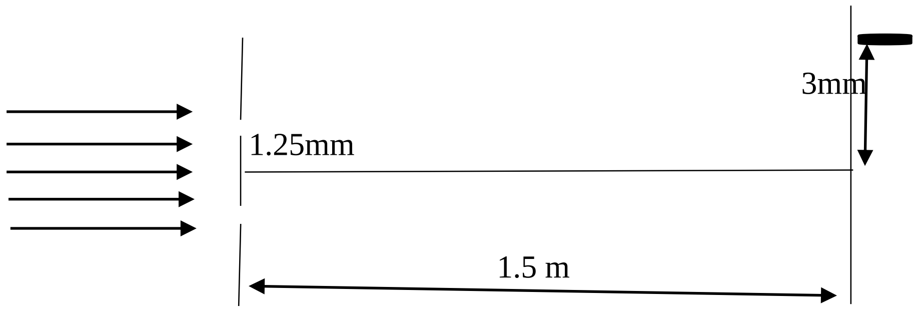
Solved problems

2.1 White light (400 to 700 nm) is used to illuminate a double slit with a spacing of 1.25 mm. An interference pattern falls on a screen 1.5 m away. A pinhole in the screen allows some light to enter a spectrograph of high resolution. If the pinhole in the screen is 3 mm from the central white fringe, where would one expect dark lines to show up in the spectrum of the pinhole source?



$$y_m = \frac{(m + \frac{1}{2})\lambda_o L}{d}$$

$$\lambda = \frac{d y_m}{\left(m+\frac{1}{2}\right)L} = \frac{25000}{(m+\frac{1}{2})} = \begin{cases} 555.5\ nm \\ 454.5\ nm \end{cases}$$

2.2 A Newton's ring apparatus is illuminated by light with two wavelength components. One of the wavelengths is 546 nm. If the eleventh bright ring of the 546-nm fringe system coincides with the tenth ring of the other, what is the second wavelength? What is the radius at which overlap takes place and the thickness of the air film there? The spherical surface has a radius of 1 m.

Ans.

$$r_m^2 = 2t_m R$$

$$r_m = \begin{cases} \left[\left(m - \frac{1}{2}\right)\lambda R\right]^{1/2} \\ [m\lambda R]^{1/2} \end{cases}$$

Since, the eleventh bright ring of the 546-nm fringe system coincides with the tenth ring of the other, i.e,

11th ring of the first wavelength= 10th ring of the second wavelength

$$\left[\left(11 - \frac{1}{2}\right)\lambda_1 R\right]^{1/2} = \left[\left(10 - \frac{1}{2}\right)\lambda_2 R\right]^{1/2}$$

λ_2=603.5 nm

$r_m = \left[\left(11 - \frac{1}{2}\right) 546(nm) 1(m)\right]^{1/2}$ = 2.39 mm

$$t_m = \frac{r_m^2}{2R} = \frac{(2.39mm)^2}{2x1m} = 2.86\text{x}10^{-4}\text{ cm}$$

2.3 A beam of white light (a continuous spectrum from (400 to 700 nm, let us say) is incident at an angle of 45° on two parallel glass plates separated by an air film 0.001 cm thick. The reflected light is admitted into a prism spectroscope. How many dark "lines" are seen across the entire

Ans.

$$2n_f t \cos\theta_t + \frac{\lambda}{2} = \left(m + \frac{1}{2}\right)\lambda$$

From the snell's law

$$n_a \sin\theta_i = n_f \sin\theta_t$$

$$\cos\theta_t = \sqrt{1 - \frac{n_a^2}{n_f^2}\sin^2\theta_i} = \sqrt{1 - \sin^2 45} = 0.707$$

At λ=400 nm, the number of dark lines are

$$2x0.001(cm)x0.707 = mx400x10^{-7}$$

Therefore, m=35

For λ=700 nm, then m=20

Thus the number of the dark lines are 15.

2.4 A thin film of MgF_2 (n = 1.38) is deposited on glass so that it is antireflecting at a wavelength of 580 nm under normal incidence. What wavelength is minimally reflected when the light is incident instead at 45°?

Ans.

The condition of deconstructive interference is

$$2n_f t \cos\theta_t + \frac{\lambda}{2} = \left(m + \frac{1}{2}\right)\lambda$$

At normal incident $2 \times 1.38 \times t = 1 \times 580(nm)$

t=210.145 nm

at θ_i =45

$$\cos\theta_t = \sqrt{1 - \frac{n_a^2}{n_f^2}\sin^2\theta_i} = 0.858$$

θ_t=42.5

$$2 \times 1.38 \times 210.145(nm) \times \cos 42.5 = 1 \times \lambda$$

λ=498 nm

2.5 Light of continuously variable wavelength illuminates normally a thin oil (index of 1.30) film on a glass surface. Extinction of the reflected light is observed to occur at wavelengths of 525 and 675 nm in the visible spectrum. Determine the thickness of the oil film and the orders of the interference.

Ans.

$2n_f t = (m_1 + \frac{1}{2})\lambda_1$, $2n_f t = m_2\lambda_2$

Order, m	1	2	3	4
$t = \frac{(m_1+\frac{1}{2})\lambda_1}{2n_f} =$	302.88	504.8	706.7	908.6x10^{-5} cm
$t = \frac{(m_2+\frac{1}{2})\lambda_2}{2n_f} =$	389.4	649.6	908.6	1168.2x10^{-5}cm

Thickness of the film is 908x10^{-5} cm and the orders are 4 and 3 respectively.

2.6. A ray of green light (λ = 565.69 nm) impinges at 30° on a thin planar film of index 1.500 immersed in air. (a) What is the smallest film thickness for which the point of reflection appears on a maximum fringe? (b) What would that point look like if the film were 1500 nm thick?

Ans. (a) An irradiance maximum will occur when

$$\delta = \frac{4\pi t}{\lambda_o}({n_f}^2 - n^2 \sin^2\theta i)^{1/2} \pm \pi$$

is an integral multiple of 2π. This, in turn, will result for the smallest t when

$$\frac{4\pi d}{\lambda_o}({n_f}^2 - n^2 \sin^2\theta i)^{1/2} = \frac{1}{4}, \text{ or } d = \frac{\lambda_o}{4}(1.5^2 - \sin^2 30)^{-1/2} = 100\ nm$$

(b) If d were 1500 nm, we would have == 15π ± π, which again would correspond to a maximum.

2.7 White light falling on two long narrow slits emerges and is observed on a distant screen. If red light (λ = 780 nm) in the first order fringe overlaps violet in the second-order fringe, what is the latter's wavelength?

Ans.

$$y_m = \frac{m\lambda_o L}{d}, \qquad y_{red} = \frac{1x\lambda_{red} L}{d}, \qquad y_{violet} = \frac{2x\lambda_{violet} L}{d}$$

$\lambda_{violet} = \lambda_{red}/2 = 780/2 = 390$ nm

2.8 Sunlight incident on a screen containing two long narrow slits 0.20 mm apart casts a pattern on a white sheet of paper 2.0 m beyond. What is the distance separating the violet (λ = 400 nm) in the first order band from the red (λ = 600 nm) in the second-order band?

Ans.

$$y_m = \frac{m\lambda_o L}{d}, \; y_{1,red} = \frac{2\times1\times4\times10^{-7}}{2\times10^{-4}} = 4\text{x}10^{-3}\text{m}$$

$$y_{2,vielot} = \frac{2\times2\times6\times10^{-7}}{2\times10^{-4}} = 12\text{x}10^{-3}\text{m}$$

Distance=8x10^{-3}m

2.9 Fringes are observed when a parallel beam of light of wavelength 500 nm is incident perpendicularly onto a wedge-shaped film with an index of refraction of 1.5. What is the angle of the wedge if the fringe separation is1/3cm?

Ans.

$$\Delta x = \frac{\lambda_o}{2n_f\alpha} \; or \; \alpha = \frac{\lambda_o}{2\Delta x \, n_f} = 5\text{x}10^{-5} \text{ rad} = 10.2 \text{ seconds}$$

2.10 Suppose a wedge-shaped air film is made between two sheets of glass, with a piece of paper 7.618 X 10 m thick used as the spacer at their very ends. If light of wavelength 500 nm comes down from directly above, determine the number of bright fringes that will be seen across the wedge

Ans. $\Delta x = \frac{x\lambda_o}{2n_f d}$ for fringe separation where $\alpha = \frac{d}{x}$ is the angle of wedge

Number of fringes =length/separation=x/Δx, so $\frac{x}{\Delta x} = \frac{2d}{2n_f\lambda_o} = \frac{[2(7.618x10^{-5}m)]}{5x10^{-7}m} =$

2.11 A Michelson Interferometer is illuminated with monochromatic light. One of its mirrors is then moved 2.53 x 10^{-5} m, and it is observed that 92 fringe-pairs, bright and dark, pass by in the process. Determine the wavelength of the incident beam.

Ans. A motion of λ/2 causes a single fringe pair to shift past, hence

92λ/2= 2.53 x 10^{-5} m and λ= 550 nm.

2.12 Given that the mirrors of a Fabry-Perot Interferometer have a reflection coefficient of *r* = 0.8944, find
(a) the coefficient of finesse,
(b) the half-width,
(c) the finesse,

Ans.

R=r^2=0.8, $F = \frac{4R}{(1-R)^2} = 80,\ \gamma = 4\sin^{-1}\left(\frac{1}{\sqrt{F}}\right) = 0.448$

Further problems:

2.13 A nonreflecting, single layer of a lens coating is to be deposited on a lens of refractive index n = 1.78. Determine the refractive index of a coating material and the thickness required to produce zero reflection for light of wavelength 550 nm.

2.14 Determine the refractive index and thickness of a film to be deposited on a glass surface (*ng* = 1.54) such that no normally incident light of wavelength 540 nm is reflected.

2.15 White light is incident normally on a thin film which has n = 1.5 and a thickness of 5000 A. For what wavelengths in the visible spectrum {4000 - 7000 A) will the intensity of the reflected light be a maximum?

2.16 A thin glass sheet has a thickness of 1.2 x 10^{-6} m and an index of refraction n = 1.50. Visible light with wavelengths between 400 nm and 700 nm is normally incident on the glass sheet. What wavelengths are most intensified in the light reflected from the sheet?

2.17 In the Fresnel double mirror s = 1 m, λ= 589 nm, and the separation of the fringes was found to be 0.5 mm. What is the angle of inclination of the mirrors, if the perpendicular distance of the actual point source to the intersection of the two mirrors is 1 m?

2.18 The Fresnel biprism is used to obtain fringes from a point source that is placed 2 m from the screen, and the prism is midway between the source and the screen. Let the wavelength of the light be λ = 500 nm and the index of refraction of the glass be n = 1.5. What is the prism angle, if the separation of the fringes is 0.5 mm?

2.19 What is the general expression for the separation of the fringes of a Fresnel biprism of index n immersed in a medium having an index of refraction n'

2.20 prove that

(a) $R.P. = \mathcal{F}m$ (b) $(\Delta\nu)_{min} = \frac{c}{\mathcal{F}2n_f t}$ (c) Compute FSR in terms of frequency.