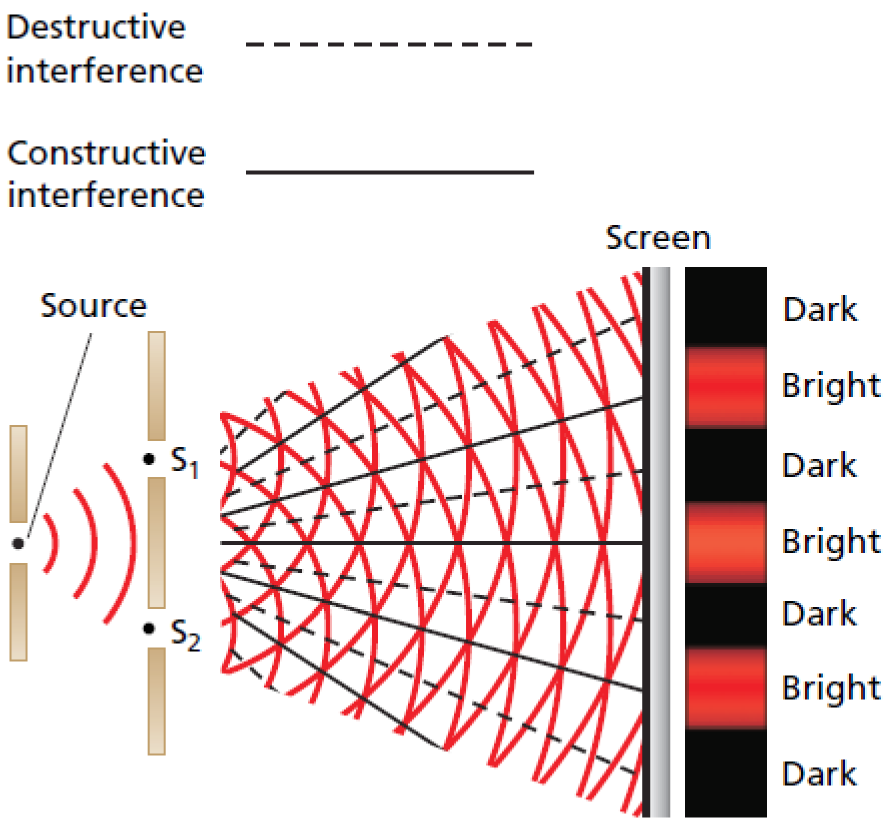

When the light from the two slits combines destructively at any location on the screen, a dark fringe results. Figure 37.3 is a photograph of an interference pattern produced by two coherent vibrating sources in a water tank.

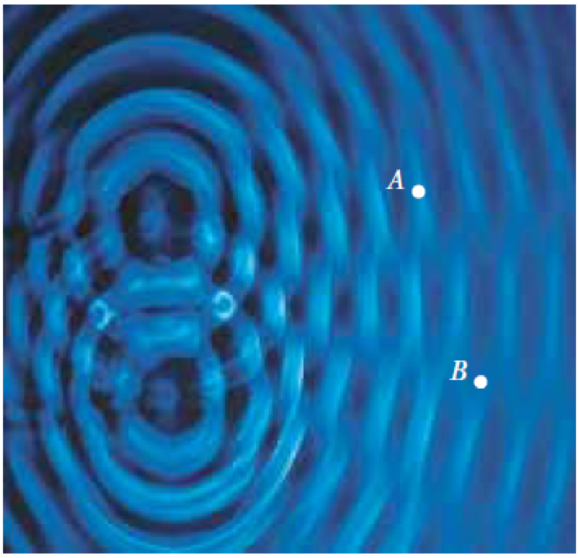

Figure 37.3 An interference pattern involving water waves is produced by two vibrating sources at the water's surface. The pattern is analogous to that observed in Young's double-slit experiment. Note the regions of constructive (*A*) and destructive (*B*) interference.

Figure 37.4 shows some of the ways in which two waves can combine at the screen. In Figure 37.4a, the two waves, which leave the two slits in phase, strike the screen at the central point *P*. Because both waves travel the same distance, they arrive at *P* in phase. As a result, constructive interference occurs at this location, and a bright fringe is observed. In Figure 37.4b, the two waves also start in phase, but in this case the upper wave has to travel one wavelength farther than the lower

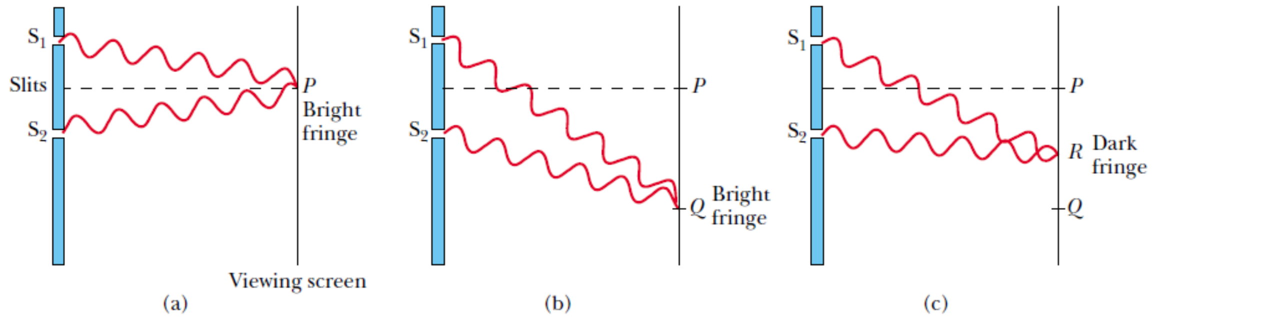

Figure 37.4 (a) Constructive interference occurs at point *P* when the waves combine.
(b) Constructive interference also occurs at point *Q*. (c) Destructive interference occurs at *R* when the two waves combine because the upper wave falls half a wavelength behind the lower wave.

wave to reach point *Q*. Because the upper wave falls behind the lower one by exactly one wavelength, they still arrive in phase at *Q*, and so a second bright fringe appears at this location. At point *R* in Figure 37.4c, however, between points ***P*** and *Q*, the upper wave has fallen half a wavelength behind the lower wave. This means that a trough of the lower wave overlaps a crest of the upper wave; this gives rise to destructive interference at point ***R***. For this reason, a dark fringe is observed at this location.

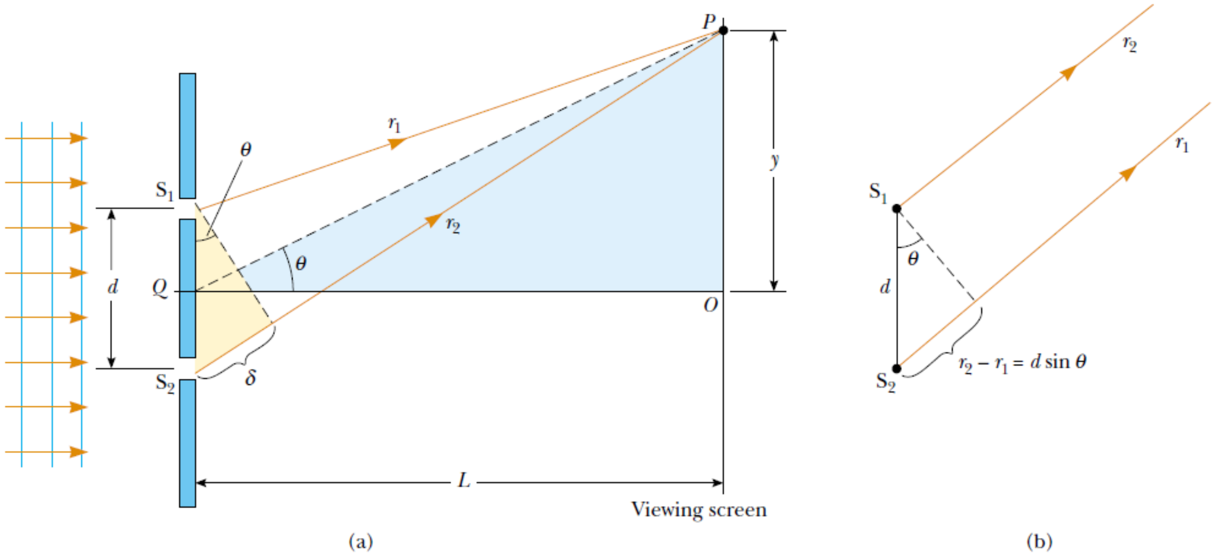

Figure 37.5 (a) Geometric construction for describing Young's double-slit experiment (not to scale). (b) When we assume that r_1 is parallel to r_2, the path difference between the two rays is $r_2 - r_1 = d \sin\theta$. For this approximation to be valid, it is essential that $L >> d$.

We can describe Young's experiment quantitatively with the help of Figure 37.5. The viewing screen is located a perpendicular distance ***L*** from the barrier containing two slits, **S_1** and **S_2**. These slits are separated by a distance ***d***, and the source is monochromatic. To reach any arbitrary point ***P*** in the upper half of the screen, a wave from the lower slit must travel farther than a wave from the upper slit by a distance ***d* sin θ** . This distance is called the path difference δ (lowercase Greek delta). If we assume that **r_1** and **r_2** are parallel, which is approximately true if ***L*** is much greater than **d**, then δ is given by

$$\delta = r_2 - r_1 = d \sin\theta$$

(Path difference) .... (1)

The value of δ determines whether the two waves are in phase when they arrive at point *P*. If δ is either zero or some integer multiple of the wavelength, then the two waves are in phase at point *P* and constructive interference results. Therefore, the condition for bright fringes, or **constructive interference**, at point *P* is

$$\delta = d \sin\theta_{\text{bright}} = m\lambda \qquad (m = 0, \pm1, \pm2, \ . \ . \ .)$$

...... (2)

(Conditions for constructive Interference)

The number *m* is called the order number. For constructive interference, the order number is the same as the number of wavelengths that represents the path difference between the waves from the two slits. The central bright fringe at **θ = 0** is called the *zeroth-order maximum*. The first maximum on either side, where *m*= ± 1, is called the *first-order maximum*, and so forth.

When δ is an odd multiple of **λ/2**, the two waves arriving at point ***P*** are **180°** out of
phase and give rise to destructive interference. Therefore, the condition for dark fringes, or **destructive interference**, at point ***P*** is

$$d \sin\theta_{\text{dark}} = (m + \tfrac{1}{2})\lambda \qquad (m = 0, \pm1, \pm2, \ . \ . \ .)$$

..........(3)

(Conditions for destructive interference)

It is useful to obtain expressions for the positions along the screen of the bright and dark fringes measured vertically from *O* to ***P***. In addition to our assumption that ***L* >> *d***, we assume ***d* >> λ** . These can be valid assumptions because in practice ***L*** is often on the order of 1 m, *d* a fraction of a millimeter, and λ & a fraction of a micrometer for visible light. Under these conditions, θ is small; thus, we can use the
small angle approximation sin θ = tan θ . Then, from triangle *OPQ* in Figure 37.5a, we see that

$$y = L\tan\theta \approx L\sin\theta \qquad \text{..................(4)}$$

Solving Equation 2 for $\sin\theta$ and substituting the result into Equation 4, we see that the positions of the bright fringes measured from *O* are given by the expression

$$y_{\text{bright}} = \frac{\lambda L}{d} m \qquad (m = 0, \pm 1, \pm 2, \ldots) \qquad \text{.............(5)}$$

Using Equations 3 and 4, we find that the dark fringes are located at

$$y_{\text{dark}} = \frac{\lambda L}{d}\left(m + \tfrac{1}{2}\right) \qquad (m = 0, \pm 1, \pm 2, \ldots) \qquad \text{..................(6)}$$

Young's double-slit experiment provides a method for measuring the wavelength of light. In fact, Young used this technique to do just that. Additionally, his experiment gave the wave model of light a great deal of credibility. It was inconceivable that particles of light coming through the slits could cancel each other in a way that would explain the dark fringes.

Example 1 **Measuring the Wavelength of a Light Source:**
A viewing screen is separated from a double-slit source by 1.2 m. The distance between the two slits is 0.03 mm. The second-order bright fringe (*m* = 2) is 4.5 cm from the center line.

(A) Determine the wavelength of the light.

Solution We can use Equation 37.5, with $m = 2$, $y_{\text{bright}} = 4.5 \times 10^{-2}$ m, $L = 1.2$ m, and $d = 3.0 \times 10^{-5}$ m:

$$\lambda = \frac{y_{\text{bright}} d}{mL} = \frac{(4.5 \times 10^{-2}\text{ m})(3.0 \times 10^{-5}\text{ m})}{2(1.2\text{ m})}$$

$$= 5.6 \times 10^{-7}\text{ m} = 560\text{ nm}$$

which is in the green range of visible light.

(B) Calculate the distance between adjacent bright fringes.

Solution From Equation 5 and the results of part (A), we obtain

$$y_{m+1} - y_m = \frac{\lambda L}{d}(m+1) - \frac{\lambda L}{d} m$$

$$= \frac{\lambda L}{d} = \frac{(5.6 \times 10^{-7}\text{ m})(1.2\text{ m})}{3.0 \times 10^{-5}\text{ m}}$$

$$= 2.2 \times 10^{-2}\text{ m} = 2.2\text{ cm}$$

Example 2 : Separating Double-Slit Fringes of Two Wavelengths
A light source emits visible light of two wavelengths: λ = 430 nm and λ' = 510 nm. The source is used in a double-slit interference experiment in which L =1.50 m and d=0.025 mm. Find the separation distance between the third-order bright fringes.

Solution Using Equation 5, with m= 3, we find that the fringe positions corresponding to these two wavelengths are

$$y_{\text{bright}} = \frac{\lambda L}{d} m = 3\frac{\lambda L}{d} = 3\frac{(430 \times 10^{-9}\text{ m})(1.50\text{ m})}{0.0250 \times 10^{-3}\text{ m}}$$
$$= 7.74 \times 10^{-2}\text{ m}$$

$$y'_{\text{bright}} = \frac{\lambda' L}{d} m = 3\frac{\lambda' L}{d} = 3\frac{(510 \times 10^{-9}\text{ m})(1.50\text{ m})}{0.0250 \times 10^{-3}\text{ m}}$$
$$= 9.18 \times 10^{-2}\text{ m}$$

Hence, the separation distance between the two fringes is

$$\Delta y = 9.18 \times 10^{-2}\text{ m} - 7.74 \times 10^{-2}\text{ m}$$
$$= 1.40 \times 10^{-2}\text{ m} = 1.40\text{ cm}$$

:Intensity distribution in the fringe system

To find the intensity on the screen at points between the maxima, we may apply the vector method of compounding amplitudes and illustrated for the present case in Fig.4. For the maxima, the angle δ is zero, and the component amplitudes a_1 and a_2 are parallel, so that if they are equal, the resultant $A=2a$. For the minima, a_1 .and a_2 are in opposite directions, and $A= 0$

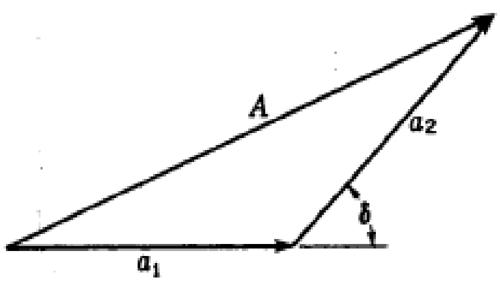

Fig.4.The composition of two waves of the same frequency and amplitude but different phase.

In general, for any value of δ, A is the closing side of the triangle. The value of A^2, which measures the intensity, is then given by Eq. (2). And varies according to $\cos^2(\delta/2)$