The image is therefore to the left of the lens, virtual, erect, and one-third the size of the object.

Lens Makers' Formula

If a lens is to be ground to some specified focal length, the refractive index of the glass must be known. It is customary for manufacturers of optical glass to specify the refractive index for yellow sodium light, the D line. Supposing the index to be known, the radii of curvature must be so chosen as to satisfy the equation

$$f = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad \text{..................(4d)}$$

As the rays travel from left to right through a lens, all convex surfaces are taken as having a positive radius and all concave surfaces a negative radius. For an equiconvex lens like the one in Fig. 3A(a), r_1 for the first surface is positive and r_2 for the second surface negative. Substituting the value of $1/f$ from Eq. (4a), we write

$$f = \frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad \text{................(4e)}$$

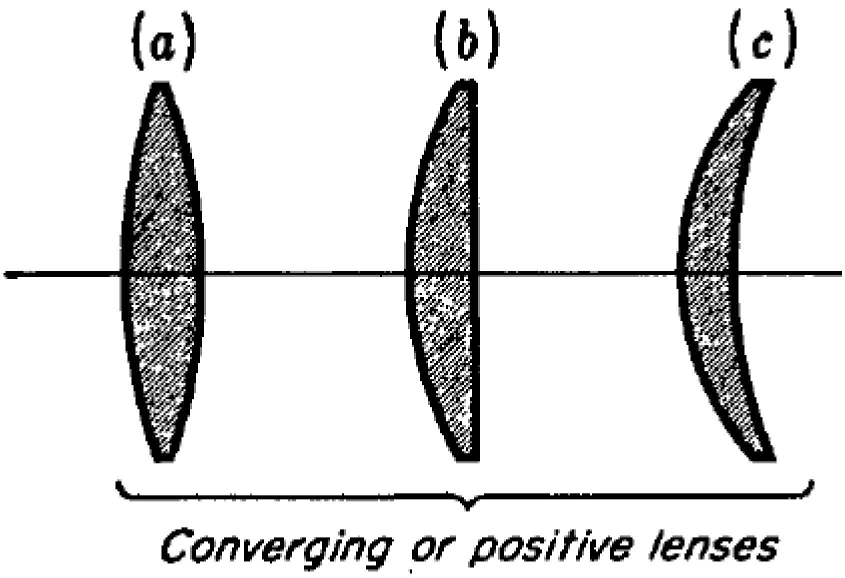

Figure 3A : Cross sections of common types of thin lenses.

Example 3: A plano-convex lens having a' focal length of 25 cm [Fig. *3A(b)*] is to be made of glass of refractive index $n = 1.52$. Calculate the radius of curvature of the grinding and polishing tools that must be used to make this lens.

Solution : Since a plano-convex lens has one flat surface, the radius for that surface is infinite, and r1 in Eq. (4d) is replaced by ∞ . The radius r_2 of the second surface is the unknown. Substitution of the known quantities in Eq. (4d) gives

$$\frac{1}{25}=(1.520-1)(\frac{1}{\infty}-\frac{1}{r_2})$$

Transposing and solving for r_2, we have

$$\frac{1}{25}=0.520(0-\frac{1}{r_2})=-\frac{0.520}{r_2}$$

$$r_2=-(25\times 0.520)=-13cm$$

If this lens is turned around, as in the figure, we shall have $r_1 = +13cm$ and $r_2 = \infty$.

Thin-Lens Combinations:

Consider, two converging lenses spaced some distance apart as shown in Fig. 4H. We first apply the graphical methods to find this image distance and then show how to calculate it by the use of the thin-lens formula. The second step is to imagine the second lens in place and to make the following changes.

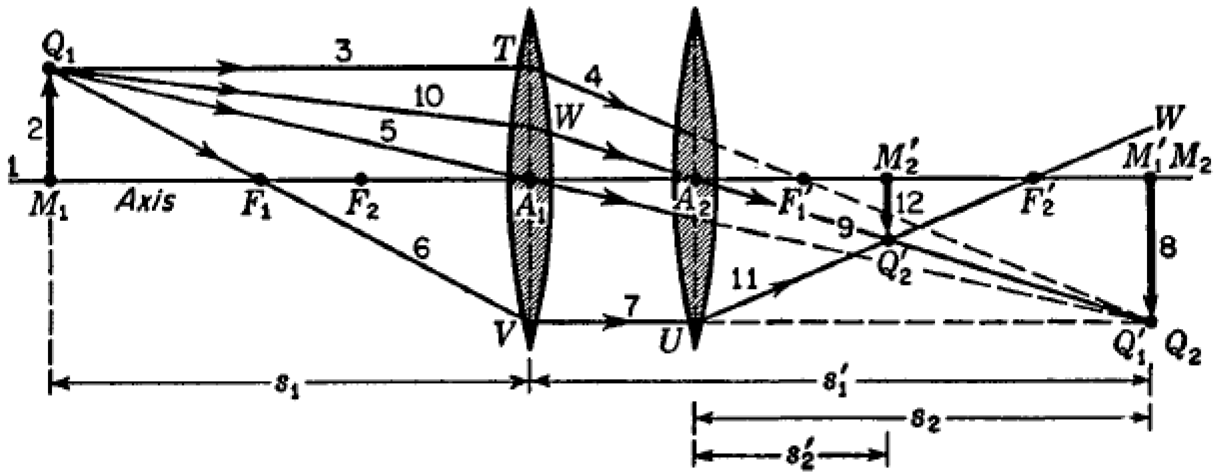

Figure 4H :The parallel-ray method for graphically locating the final image formed by two thin lenses.

The oblique-ray method given in Fig. 4E is applied to the same two lenses in Fig. 4I.

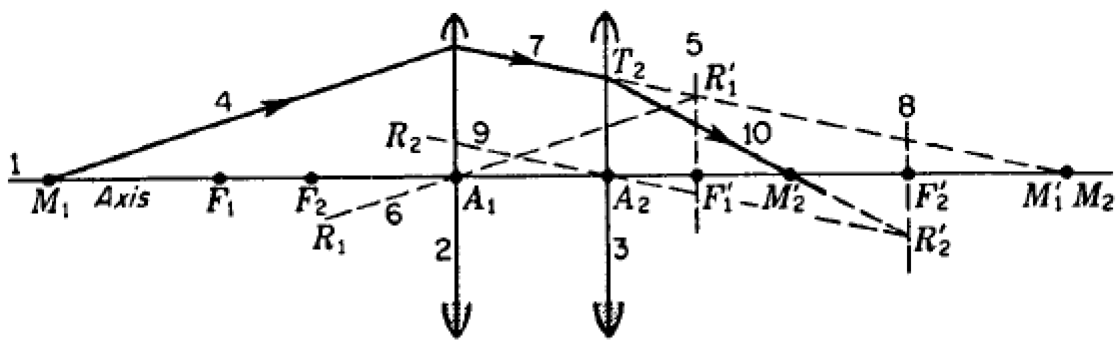

Figure 4I :The oblique-ray method for graphically locating the final image formed by two thin lenses.

The Power of aThin Lens

The concept and measurement of lens power correspond to those used in the treatment of reduced vergence and the power of a single surface as given in (Sec. 3.9). The power of a thin lens in diopters is given by the reciprocal of the focal length in meters:

$$P = \frac{1}{f} \quad diopters \quad = \frac{1}{focal \quad length, \quad m} \quad \ldots\ldots\ldots\ldots\ldots\ldots(4f)$$

$$P = (n-1)\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots(4g)$$

Example 4 : The radii of both surfaces of an equiconvex lens of index 1.6 are equal to 8 cm. Find its power.

Solution The given quantities to be used in Eq. (4g) are n = 1.6, r_1 = 0.08 m, and r_2 = -0.08 m (see Fig. 3A for the shape of an equiconvex lens) .

$$P = (n-1)\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \qquad \rightarrow = (1.6-1)\left(\frac{1}{0.08} - \frac{1}{-0.08}\right) = 0.6 \times \frac{2}{0.08} = +15D$$

Thin Lenses In Contact

Derivation Of The Lens Formula:

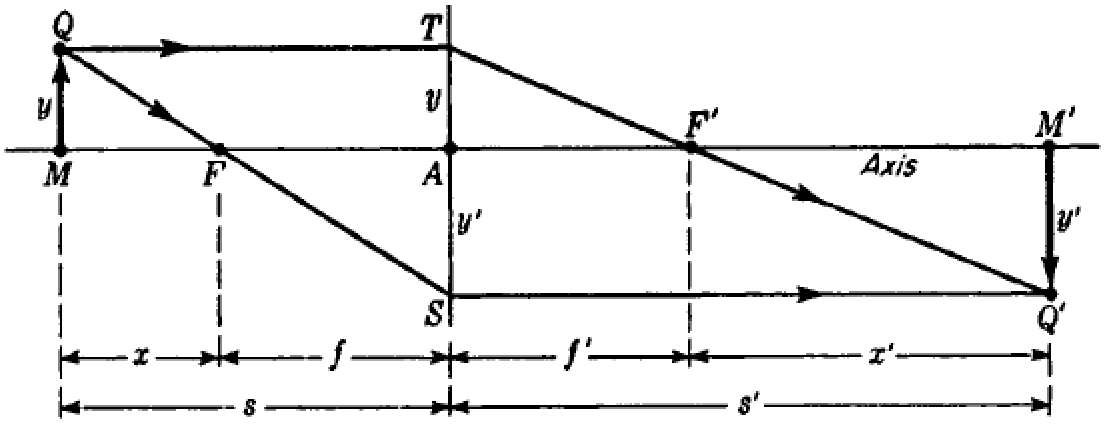

Figure 4K :The geometry used for the derivation of thin-lens formulas.

From Fig. 4K, From similar triangles Q'TS and F' TA the proportionality between corresponding sides gives

$$SQ' = s', \quad TS = y - y', \quad TA = y \quad and \quad AF' = f'$$

$$\therefore \frac{TS}{SQ'} = \frac{TA}{AF'} \rightarrow \frac{y - y'}{s'} = \frac{y}{f'} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

From triangles QTS and FAS

$$\frac{Ts}{QT} = \frac{sA}{FA} \rightarrow \frac{y - y'}{s} = \frac{-y'}{f} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

From Equation 1 and 2 we have

$$\frac{y - y'}{s} + \frac{y - y'}{s'} = \frac{y}{f'} - \frac{y'}{f}$$

When $f = f'$

$$\frac{y-y'}{s}+\frac{y-y'}{s'}=\frac{y-y'}{f}$$

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

Newton Law:

For similar triangles FAS and QMF we have

$$\frac{QM}{MF}=\frac{As}{FA}\rightarrow\frac{y}{x}=\frac{-y'}{f}\quad\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

And from F\`A\`M\` and TAF\`

$$\frac{Q'M'}{M'F'}=\frac{TA}{A'F'}\rightarrow\frac{-y'}{x'}=\frac{y}{f'}\quad\ldots\ldots\ldots\ldots\ldots\ (2)$$

Multiplication of one equation by the other gives

$$\frac{yy'}{xx'}=\frac{yy'}{ff'}$$

$$\therefore xx'=ff'=f^2\quad\ldots\ldots\ldots\ldots(3)$$

When $f=f'$ equation 3 is Newton law

When distances are measured from focal points, one should use the Newton form, which can be obtained directly from Eq. (1, 2):

$$m=\frac{y'}{y}=-\frac{f}{x}=-\frac{x'}{f'}$$

Derivation Of The Lens Makers' Formula:

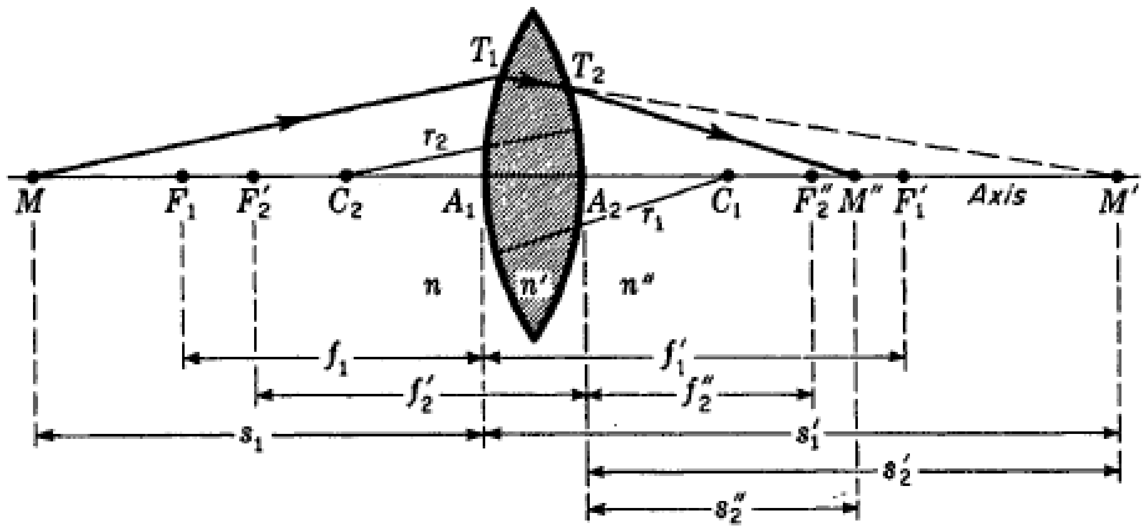

Figure 4L Each surface of a thin lens has its own focal points and focal lengths, as well as separate object and image distances.

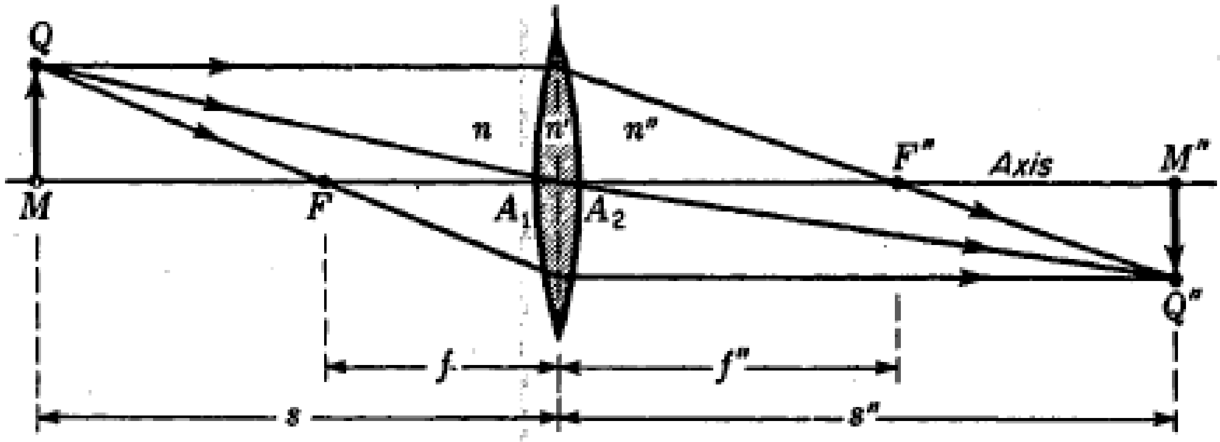

Figure 4M When the media on the two sides of a thin lens have different indices, the primary and secondary focal lengths are not equal and the ray through the lens center is deviated.

The geometry required for this derivation is shown in Fig. 4L. Let n, n', and n'' represent the refractive indices of the three media as shown, f_1 and f_1' the focal lengths for the first surface alone, and f_2' and f_2'' the focal lengths for the second surface alone.
From first surface

$$\frac{n}{s_1}+\frac{n'}{s_1'}=\frac{n'-n}{r_1} \qquad \text{........(4l)}$$

From second surface

$$\frac{n'}{s_2'}+\frac{n''}{s_2''}=\frac{n''-n'}{r_2} \qquad \text{..............(4m)}$$

If we now assume the lens thickness to be negligibly small compared with the object and image distances, we note the image distance s_1' for the first surface becomes equal in magnitude to the object distance s_2' for the second surface. Since M' is a virtual object for the second surface, the sign of the object distance for this surface is negative. As a consequence we can set $s_1' = -s_2'$ and write

$$\frac{n'}{s_1'} = -\frac{n'}{s_2'}$$

If we now add Eqs. (41) and (4m) and substitute this equality, we obtain

$$\frac{n}{s_1} + \frac{n'}{s_1'} + \frac{n'}{s_2'} + \frac{n''}{s_2''} = \frac{n' - n}{r_1} + \frac{n'' - n'}{r_2}$$

$$\frac{n}{s_1} + \frac{n''}{s_2''} = \frac{n' - n}{r_1} + \frac{n'' - n'}{r_2} \quad \text{.....................} \ (4n)$$

If we now call s_1 the object distance and designate it s as in Fig. 4M and call s_2'' the image distance and designate it s'', we can write Eq. (4n) as

$$\frac{n}{s} + \frac{n''}{s''} = \frac{n' - n}{r_1} + \frac{n'' - n'}{r_2} \quad \text{............}(4o)$$

This is the general formula for a thin lens having different media on the two sides. For such cases we can follow the procedure given in Sec. 3.4 and define primary and secondary focal points F and F'', and the corresponding focal lengths f and f'', by setting s or s'' equal to infinity. When this is done, we obtain

$$\frac{n}{f} = \frac{n' - n}{r_1} + \frac{n'' - n'}{r_2} = \frac{n''}{f''}$$

In words, the focal lengths have the ratio of the refractive indices of the two media n and n'' (see Fig. 4M)

$$\frac{f}{f''} = \frac{n}{n''}$$

If the medium on both sides of the lens is the same, $n = n''$, Eq. (40) reduces to

$$\frac{n}{s} + \frac{n''}{s''} = \frac{n' - n}{r_1} + \frac{n - n'}{r_2} = \frac{n' - n}{r_1} - \frac{n' - n}{r_2}$$

$$\frac{n}{s} + \frac{n''}{s''} = (n' - n)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad \text{.............} \ (4r)$$

Finally, if the surrounding medium is air $(n = 1)$, we obtain the lens makers' formula

$$\frac{1}{s} + \frac{1}{s''} = (n' - n)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad \text{............} \ (4s)$$

In the power notation of Eq. (3i), the general formula (Eq. (40)] can be written

$$V + V'' = P_1 + P_2 \quad \text{..................}(4t)$$