The power P = -4 D, and the image is virtual and erect. It is located 11.1 cm to the right of the mirror and has a magnification of 0.555 x .

Thick Mirrors:

The term *thick mirror* is applied to a lens system in which one of the spherical surfaces is a reflector. Under these circumstances the light passing through the system is reflected by the mirror back through the lens system, from which it emerges finally into the space from which it entered the lens. Three common forms of optical systems that may be classified as thick mirrors are shown in Fig. 6H. In each case the surface farthest to the right has been drawn with a heavier line than the others, designating the reflecting surface. A parallel incident ray is also traced through each system to where it crosses the axis, thus locating the focal point.

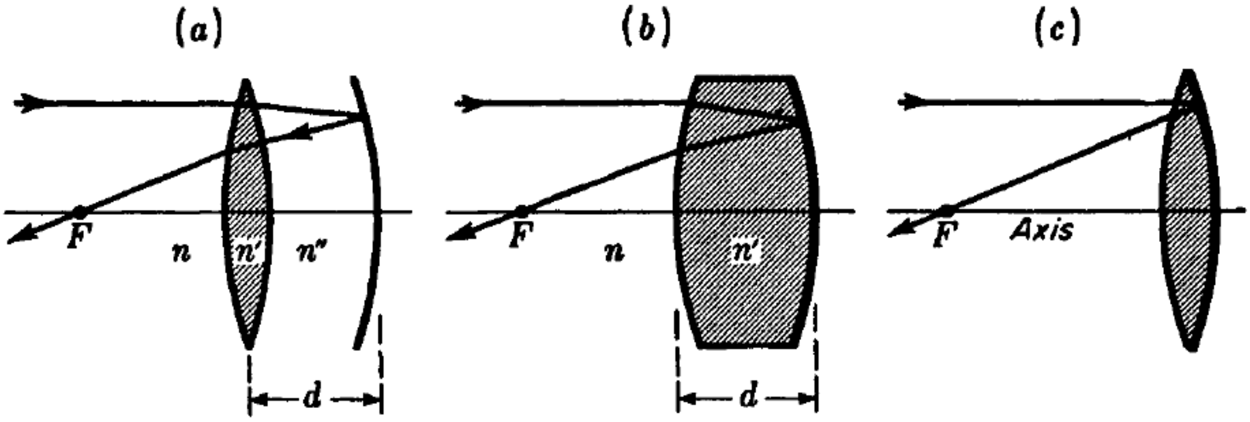


Figure 6H :Diagrams of several types of thick mirrors, showing the location of their respective focal points.

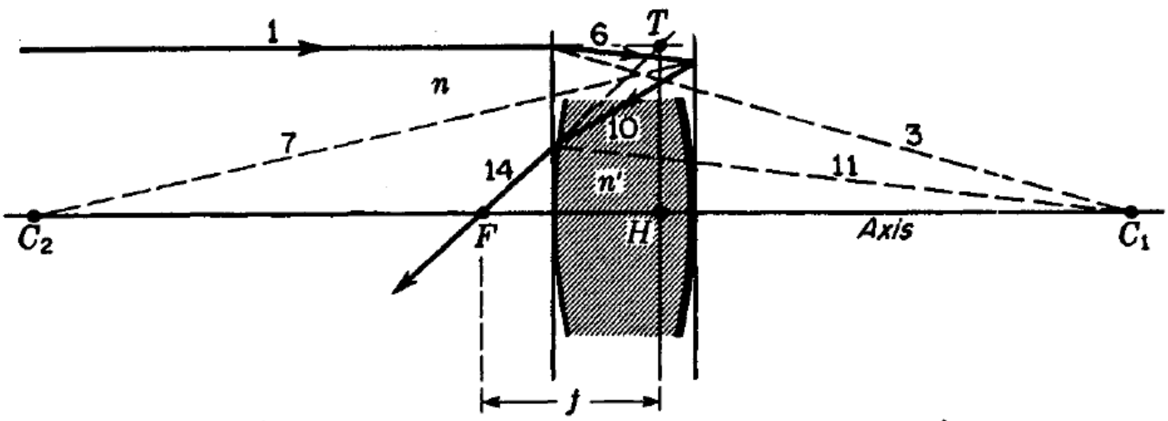


Figure 6J : Auxiliary-diagram method of graphically locating the focal point and principal point of a thick mirror.

Thick-Mirror Formulas:

These formulas will be given in the power notation for case *(a)* shown in Fig. 6H. When r_1, r_2, and r_3 are the radii of the three surfaces consecutively from left to right, the power of the combination can be shown* to be given by

$$P=(1-cP_1)(2P_1+P_2-cP_1P_2) \qquad (6I)$$

where, for the case in diagram *(a)* only and $n'' = n$,

$$P_1 = (n' - n)(K_1 - K_2) \qquad (6m)$$

$$P_2 = -2nK_3 \qquad (6n)$$

and $K_1 = \frac{1}{r_1}$, $K_2 = \frac{1}{r_2}$, $K_3 = \frac{1}{r_3}$

The distance from the lens to the principal point of the combination is given by

$$HH_1 = \frac{c}{1 - cP_1} \qquad (6O)$$

where H_1 is located at the center of the lens and

$$c = \frac{d}{n} \qquad (6P)$$

It is important to note from Eq. (60) that the position of H is independent of the power P_2 of the mirror and therefore of its curvature K_3 .

Example: A thick mirror like that shown in Fig. 6H(a) has as one component a thin lens of index $n' = 1.5$, radii $r_1 = +50$ cm, and $r_2 = -50$ cm. This lens is situated 10 cm in front of a mirror of radius - 50 cm. Assuming that air surrounds both components, find *(a)* the power of the combination, *(b)* the focal length, and *(c)* the principal point.

Solution:

SOLUTION (*a*) By Eq. (6m), the power of the lens is

$$P_1 = (1.50 - 1)\left(\frac{1}{0.50} - \frac{1}{-0.50}\right) = +2\ \text{D}$$

Equation (6n) gives for the power of the mirror

$$P_2 = -2\frac{1}{-0.50} = +4\ \text{D}$$

From Eq. (6p),

$$c = \frac{d}{n} = \frac{0.10}{1} = 0.10\ \text{m}$$

Finally the power of the combination is given by Eq. (6l) as

$$P = (1 - 0.10 \times 2)(2 \times 2 + 4 - 0.10 \times 2 \times 4)$$
$$= 0.8(4 + 4 - 0.8) = +5.76\ \text{D}$$

(*b*) A power of +5.76 D corresponds to a focal length

$$f = \frac{1}{P} = \frac{1}{5.76} = 0.173\ \text{m} = +17.3\ \text{cm}$$

(*c*) The position of the principal point *H* is determined from Eq. (6o) through the distance

$$H_1H = \frac{0.10}{1 - 0.10 \times 2} = \frac{0.10}{0.80} = 0.125\ \text{m} = +12.5\ \text{cm}$$

Other Thick Mirrors

As a second illustration of a thick mirror, consider the thick lens silvered on the back, as shown in Fig. *6H(b).* A comparison of this system with the one in diagram *(a)* shows that Eqs. (6I) to (6p) will apply if the powers P_1 and P_2 are properly defined. For diagram *(b),* P_1 refers to the power of the first surface alone, and P_2 refers to the power of the second surface as a mirror of radius r_2 in a medium of index n'. In other words,

$$P_1 = \frac{n' - n}{r_1}, \quad P_2 = -\frac{2n'}{r_2} \quad \text{and} \quad c = \frac{d}{n'} \qquad (6q)$$

With these definitions the power of thick mirror *(b)* is given by Eq. (6I) and the principal point by Eq. (60).

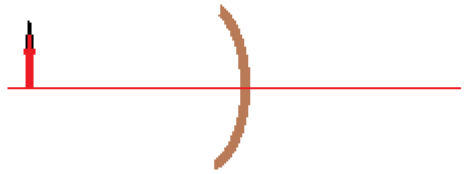
The third illustration of a thick mirror consists of a thin lens silvered on the back surface as shown in Fig. *6H(c).* This system may be looked upon (1) as a special case of diagram *(a),* where the mirror has the same radius as the back surface of the thin lens and the spacing *d* is reduced to zero, or (2) as a special case of diagram *(b),* where the thickness is reduced to practically zero. In either case Eq. (6I) reduces to

$$P = 2P_1 + P_2 \qquad (6r)$$

and the principal point H coincides with H_1 at the common center of the lens and mirror. P_1 represents the power of the thin lens in air and P_2 the power of the mirror in air, or P_1 represents the power of the first surface of radius r_1 and P_2 represents the power of the second surface as a mirror of radius r_2 in a medium of index n' [see Eq. (6q)].

Example (6.1): A spherical mirror has a radius of – 24 cm; An object 3cm high is located in front of the mirror at a distance of *(a)* 48 cm, *(b)* 36 cm, *(c)* 24 cm, *(d)* 12 cm, and *(e)* 6 cm. Find the image distance for each of these object distances.

Solution:



(a) : $\frac{1}{S} + \frac{1}{S'} = -\frac{2}{r}$, $\frac{1}{48} + \frac{1}{S_1'} = -\frac{2}{24} = +\frac{1}{12}$, $S_1' = 16 \ cm$

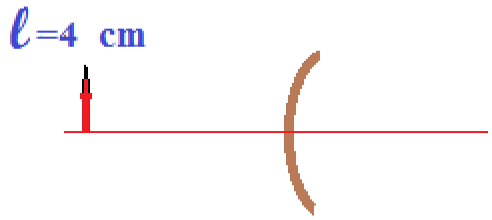
(b): $\frac{1}{36} + \frac{1}{S_2'} = +\frac{1}{12}$, $S_2' = 18 \ cm$

(c) : $\frac{1}{24} + \frac{1}{S_3'} = +\frac{1}{12}$, $S_3' = 24 \ cm$

(d) $\frac{1}{12} + \frac{1}{S_4'} = +\frac{1}{12}$, $S_4' = \infty$

(e) $\frac{1}{6} + \frac{1}{S_5'} = +\frac{1}{12}$, $S_5' = -12 \ cm$

Example (6.5): The radius of a spherical mirror is + 18 cm. An object 4 cm high is located in front of the mirror at a distance of *(a)* 36 cm, *(b)* 24 cm, and *(c)* 12 cm. Find the image distance and image size for each of these object distances.



Solution: $\frac{1}{S}+\frac{1}{S'}=-\frac{2}{r}$, $\frac{1}{S}+\frac{1}{S'}=-\frac{2}{18}$

(a) $\frac{1}{36}+\frac{1}{S'}=-\frac{1}{9}$, $\rightarrow S'=-7.2cm$

$m_1=-\frac{S_1'}{S_1}=-\frac{-7.2}{36}=+0.2$, $\ell_1'=m_1\ell=0.2\times4=0.8cm$

(b) $\frac{1}{24}+\frac{1}{S_2'}=-\frac{1}{9}$, $\rightarrow S_2'=-6.55cm$

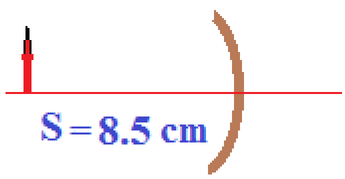
$m_2=-\frac{-6.55}{24}=+0.273$ $\ell_2'=m_2\ell=0.273\times4=1.08cm$

(c) $\frac{1}{12}+\frac{1}{S_3'}=-\frac{1}{9}$, $\rightarrow S_3'=-5.14cm$

$m3=-\frac{-5.14}{12}=+0.428$ $\ell_3'=m_3\ell=0.428\times4=1.712cm$

Example (6.9): A concave mirror is to be used to focus the image of a tree on a photographic film 8.5 m away from the tree. If a lateral magnification of (-1/20) is desired, what should be the radius of curvature of the mirror?

Solution:

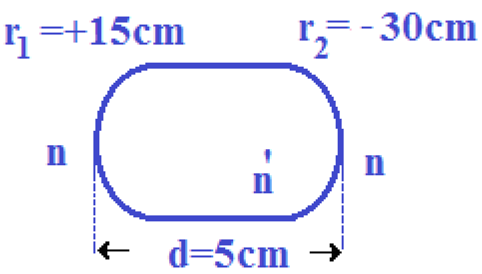


$m_T=-\frac{1}{20}=-\frac{S'}{S}=-\frac{S'}{8.5}$ $\rightarrow$ $S'=+0.425m$

$\frac{1}{S}+\frac{1}{S'}=-\frac{2}{r}$ $\frac{1}{8.5}+\frac{1}{0.425}=-\frac{2}{r}$ $\rightarrow r=0.80952m$ $\rightarrow=80.952cm$

Example (6.17): A thick lens of index 1.56 has radii $r_1=+15$ cm and $r_2=-30$ cm. If the Second surface is silvered and the lens is 5 cm thick, find *(a)* the power, *(b)* the focal length, *(c)* the principal point, and *(d)* the focal point.

Solution:



$$P_1 = \frac{n' - n}{r_1} = \frac{1.56 - 1}{+0.15} = 3.733D$$

$$P_2 = -2nk_2 = -\frac{2n'}{r_2} = -\frac{2\times 1.56}{-0.3} = +10.4D$$

$$C = \frac{d}{n'} = \frac{0.05}{1.56} = 0.0321m$$

(a) $P = (1 - 0.0321\times 3.733)(2\times 3.733 + 10.4 - 0.0321\times 3.733\times 10.4) \rightarrow P = 14.63D$

(b) $$f = \frac{1}{P} = \frac{1}{14.63} = 6.83cm$$

(c) $$H_1H = \frac{C}{1 - CP_1} = \frac{0.0321}{1 - 0.0321\times 3.733} = 0.0365m \rightarrow 3.65cm$$

(d) $$H_1F = H_1H - f = -3.65 + 6.83 = 3.18cm$$