الفصل الخامس

Focal Point And Focal Length:

Diagrams showing the reflection of a parallel beam of light by a concave mirror and by a convex one are given in Fig. 6A. A ray striking the mirror at some point such as *T* obeys the law of reflection $\phi'' = \phi$. All rays are shown as brought to a common

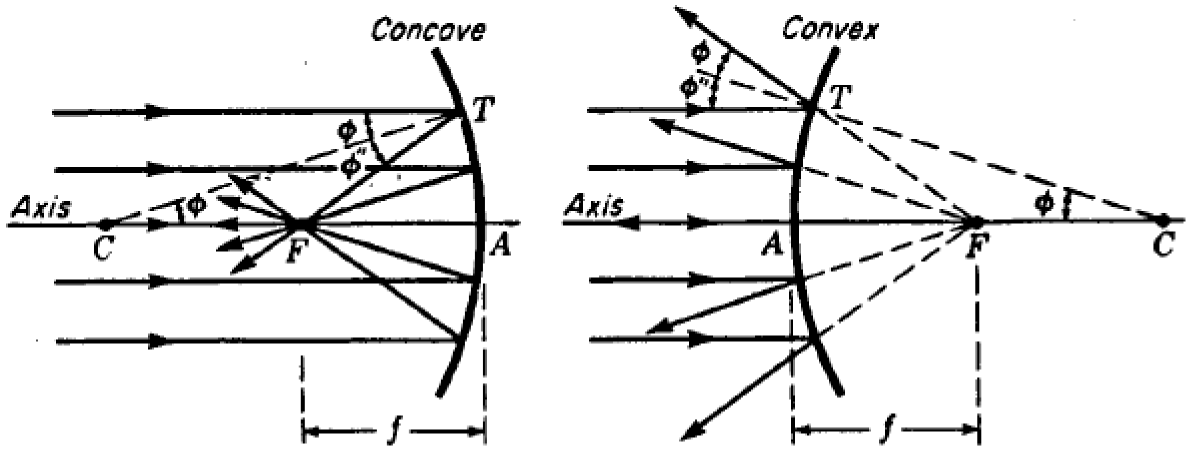

Figure 6A:The primary and secondary focal points of spherical mirrors coincide.

focus at *F,* although this will be strictly true only for paraxial rays. The point ***F*** is called the *focal point* and the distance ***FA*** the *focal length.* In the second diagram the reflected rays diverge as though they came from a common point ***F***. Since the angle ***TCA*** also equals ϕ, the triangle ***TCF*** is isosceles, and in general ***CF* = *FT***. But for very small angles ϕ (paraxial rays), ***FT*** approaches equality with ***FA***. Hence

$$FA = \frac{1}{2}(CA) \quad \text{-------------} \quad f = -\frac{1}{2}r \quad \ldots\ldots\ldots\ldots\ldots\ (6a)$$

The negative sign is introduced in Eq. (6a) so that the focal length of a concave mirror, which behaves like a positive or converging lens, will also be positive.

Graphical Constructions:

The parallel-ray method of construction is given for a concave mirror in Fig. 6E. Where a similar procedure is applied to a convex mirror in Fig. 6F. and the image is virtual. The oblique-ray method can also be used for mirrors, as illustrated in Fig. 6G for a concave mirror.

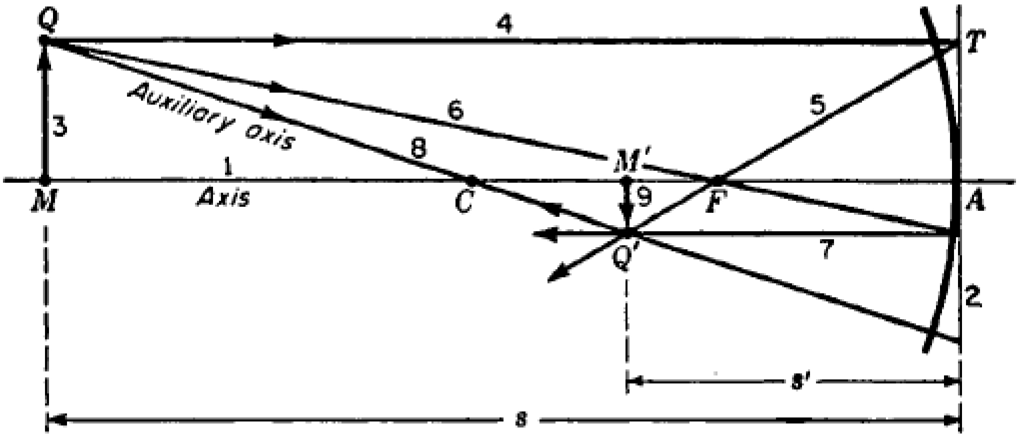

Figure 6E:Parallel-ray method for graphically locating the image formed by a concave mirror.

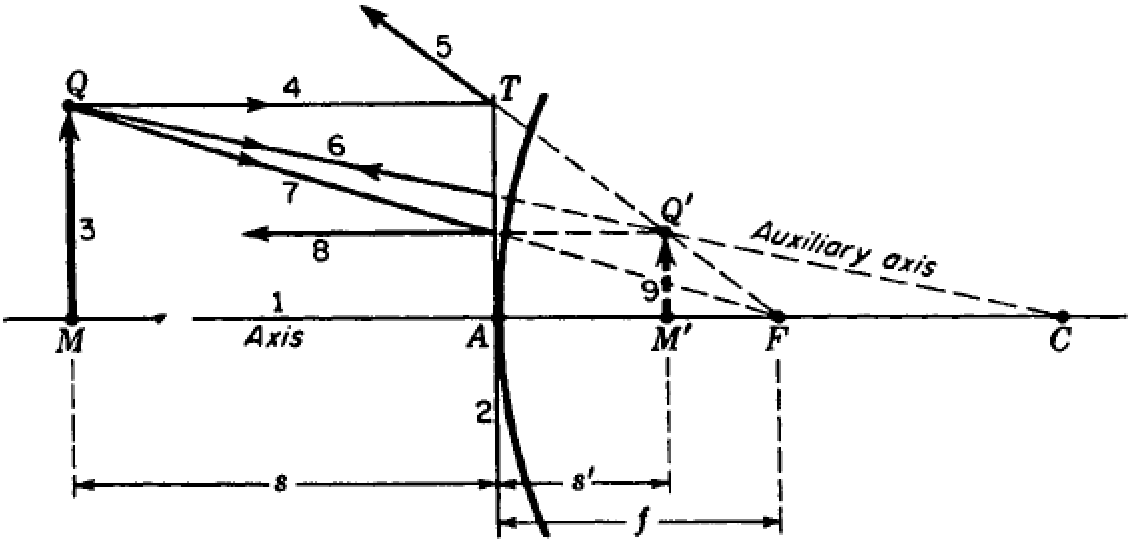

Figure 6F: Parallel-ray method for graphically locating the image formed by a convex mirror.

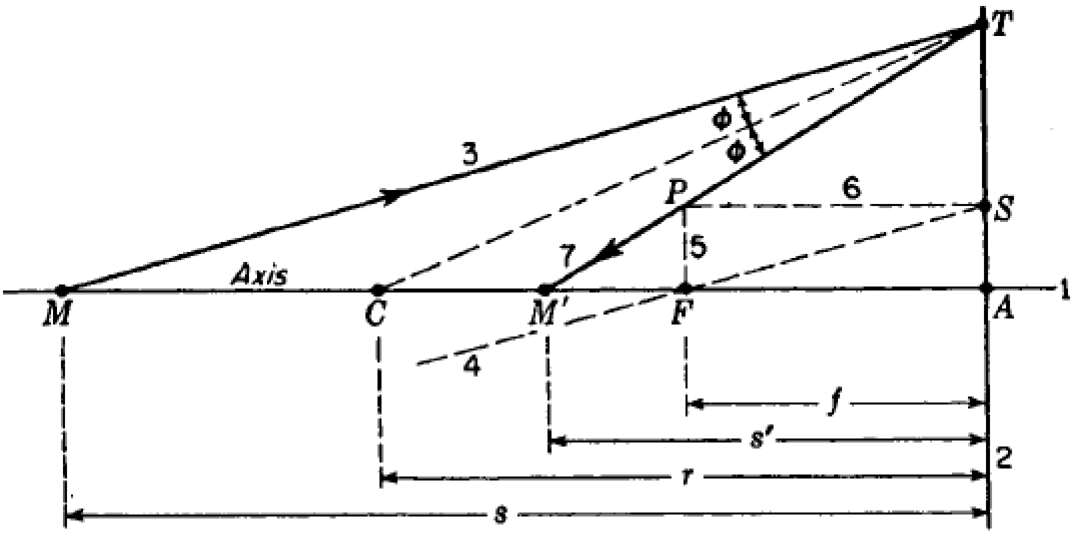

Figure 6G: Oblique-ray method for locating the image formed by a concave mirror.

Mirror Formulas:

In order to be able to apply the standard lens formulas of the preceding chapters to spherical mirrors with as little change as possible, we must adhere to the following sign conventions:

1- *Distances measured from left to right are positive while those measured from right to left are negative.*

2- *Incident rays travel from left to right and reflected rays from right to left.*

3 -*The focal length is measured from the focal point to the vertex.* This gives f a positive sign for concave mirrors and a negative sign for convex mirrors.

4- *The radius is measured from the vertex to the center of curvature.* This makes r negative for concave mirrors and positive for convex mirrors.

5- *Object distances s and image distances s' are measured from the object and from the image respectively to the vertex.* This makes both sand s' positive and the object and image real when they lie to the left of the vertex; they are negative and virtual when they lie to the right.

In Fig. 6G it is observed that by the law of reflection the radius *CT* bisects the angle *MTM'*. Using a well-known geometrical theorem, we can then write the proportion:

$$\frac{MC}{MT} = \frac{CM'}{M'T}$$

Now, for paraxial rays, $MT \approx MA = s$ and $M'T \approx M'A = s'$, where the symbol ≈ means "is approximately equal to." Also, from the diagram,

$$MC = MA - CA = s + r$$

and

$$CM' = CA - M'A = -r - s' = -(s' + r)$$

Substituting in the above proportion gives

$$\frac{S+r}{S} = -\frac{S'+r}{S'}$$

$$S'S + S'r = -SS' - Sr$$

$$S'S + SS' = -S'r - Sr$$

$$S'r + Sr = -2SS' \qquad \div SS'r$$

which can easily be put in the form

$$\frac{1}{S}+\frac{1}{S'}=-\frac{2}{r}$$ *Mirror formula* (6b)

The primary focal point is defined as that axial object point for which the image is formed at infinity, so substituting $s = f$ and $s' = \infty$ in Eq. (6b), we have

$$\frac{1}{f}+\frac{1}{\infty}=-\frac{2}{r}$$

from which $$\frac{1}{f}+0=-\frac{2}{r} \qquad f=-\frac{r}{2} \qquad (6c)$$

The secondary focal point is defined as the image point of an infinitely distant object point. This is $f' = S'$ and $s = \infty$, so that

$$\frac{1}{\infty}+\frac{1}{f'}=-\frac{2}{r}$$

from which $$\frac{1}{f'}=-\frac{2}{r} \qquad f'=-\frac{r}{2} \qquad (6d)$$

When $-21r$ is replaced by $\frac{1}{f}$, Eq. (6b) becomes

$$\frac{1}{S}+\frac{1}{S'}=\frac{1}{f} \qquad (6e)$$

The lateral magnification of the image from a mirror can be evaluated from the geometry of Fig. 6C. From the proportionality of sides in the similar triangles $Q'AM'$ and QAM we find that $-y'/y = s'/s$, giving

$$\frac{Q'M'}{QM}=\frac{M'A}{MA}$$

$$-\frac{y'}{y}=\frac{S'}{S} \qquad m=\frac{y'}{y}=-\frac{S'}{S} \qquad (6f)$$

Example : An object 2 cm high is situated 10cm in front of a concave mirror of radius 16 cm. Find *(a)* the focal length of the mirror, *(b)* the position of the image, and *(c)* the lateral magnification.

Solution:

The given quantities are $y = +2$cm, $s = +10$cm, and $r = -16$cm. The unknown quantities are f, s', and m. *(a)* By Eq. (6c),

$$f = -\frac{-16}{2} = 8cm$$

(b) By Eq. (6e),

$$\frac{1}{10} + \frac{1}{S'} = \frac{1}{8} \qquad or \qquad \frac{1}{S'} = \frac{1}{8} - \frac{1}{10} = \frac{1}{40} \qquad S' = +40cm$$

(c) By Eq. (6f), $m = -\frac{40}{10} = -4$

Power of Mirrors:

As definitions, we let

$$P = \frac{1}{f}, \qquad V = \frac{1}{S}, \qquad V' = \frac{1}{S'}, \qquad K = \frac{1}{r} \qquad (6g)$$

Equations (6b), (6e), (6c), and (6f) then take the forms

$$V + V' = -2K \qquad (6h)$$

$$V + V' = P \qquad (6i)$$

$$P = -2K \qquad (6j)$$

$$m = \frac{y'}{y} = -\frac{V}{V'} \qquad (6k)$$

Example : An object is located 20 cm in front of a convex mirror of radius 50 cm. Calculate *(a)* the power of the mirror, *(b)* the position of the image, and *(c)* its magnification.

Solution: Expressing all distances in meters, we have

(a) By Eq. (6j),

$$P = -2K = -4D$$

(b) By Eq. (6i),

$$5 + V' = -4 \quad \text{or} \quad V' = -9\ D$$

or

$$S' = \frac{1}{V'} = -\frac{1}{9} = -0.111\ m = -11.1\ cm$$

(c) By Eq. (6k),

$$m = -\frac{5}{-9} = +0.555$$