Where $V = \frac{n}{s}, \quad V'' = \frac{n''}{s''}, \quad P_1 = \frac{n' - n}{r_1}, \quad P_2 = \frac{n'' - n'}{r_2}$ .... (4u)

Equation (4t) can be written $P = P_1 + P_2$ and $P = V + V''$

Thick Lenses

Two Spherical Surfaces:

A simple form of thick lens comprises two spherical surfaces as shown in Fig. 5A. A treatment of the image-forming capabilities of such a system follows directly from procedures outlined in Chaps. 3 and 4. Each surface, acting as an image-forming component, contributes to the final image formed by the system as a whole.

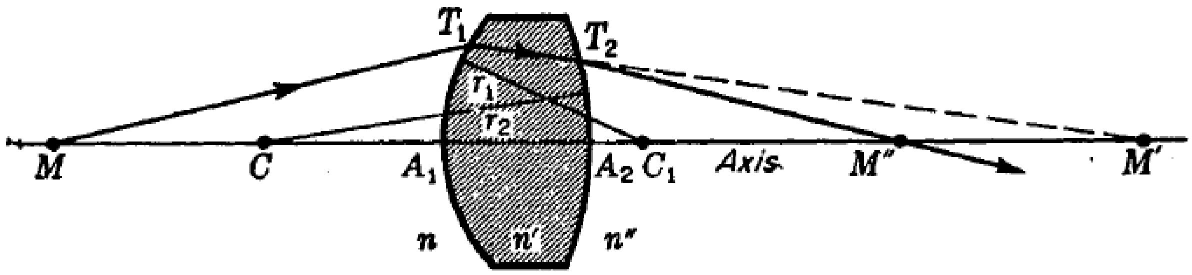


Figure *5A* : Details of the refraction of a ray at both surfaces of a lens.

Let *n, n',* and *n''* represent the refractive indices of three media separated by two spherical surfaces of radius r_1 and r_2 . A light ray from an axial object point *M is* shown refracted by the first surface in a direction T_1M' and then further refracted by the second surface in a direction T_2M''. Since the lens axis may be considered as a second ray of light originating at *M* and passing through the system, *M''* is the final image of the object point *M*. Hence *M* and *M''* are conjugate points for the thick lens as a whole, and all rays from *M* should come to a focus at *M''*.

We shall first consider the parallel-ray method for graphically locating an image formed by a thick lens and then apply the general formulas already given for calculating image distances.

$$\frac{n}{s_1} + \frac{n'}{s_1'} = \frac{n' - n}{r_1} \qquad \frac{n'}{s_2'} + \frac{n''}{s_2''} = \frac{n'' - n'}{r_2} \qquad (5a)$$

For first surface *For second surface*

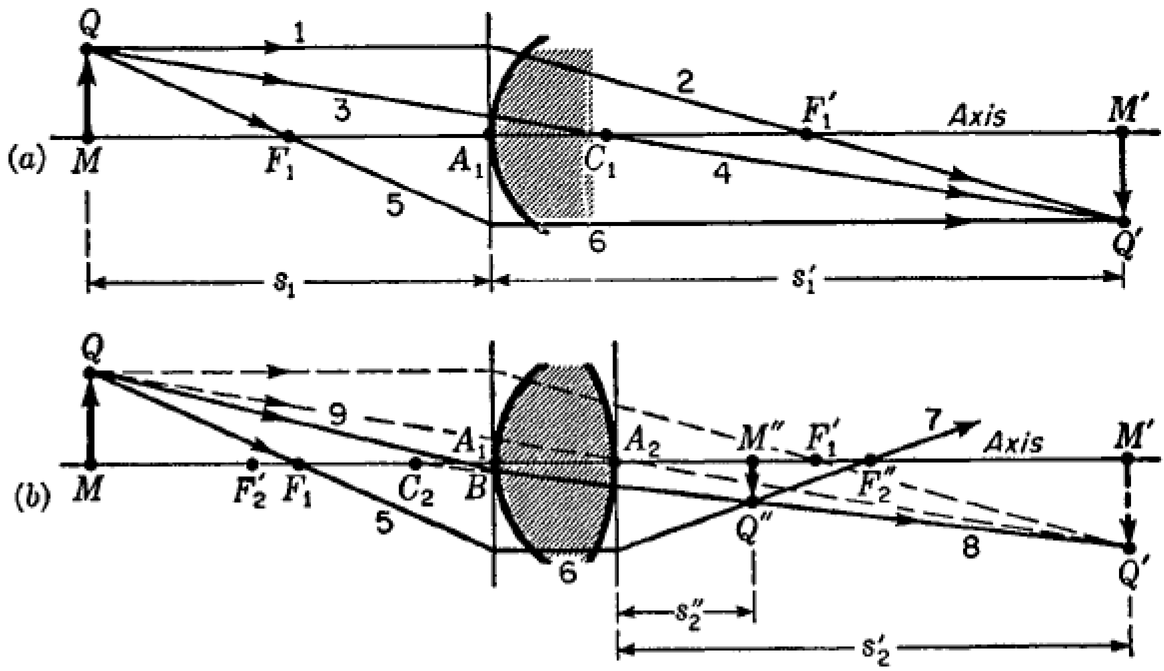


Figure 5B : The parallel-ray method for graphically locating the image formed by a thick lens.

Example 1: An equiconvex lens 2 cm thick and having radii of curvature of 2 cm is mounted in the end of a water tank. An object in air is placed on the axis of the lens 5 cm from its vertex. Find the position of the final image. Assume refractive indices of 1, 1.5, and 1.33 for air, glass, and water, respectively.

Solution The relative dimensions in this problem are approximately those shown in Fig. 5B(b). If we apply Eq.(Sa) to the first surface alone, we find the image distance to be

$$\frac{1}{5} + \frac{1.5}{s_1'} = \frac{1.5 - 1}{2} \qquad \text{or} \qquad s_1' = +30cm$$

When the same equation is applied to the second surface, we note that the object distance is s'_1 minus the lens thickness, or 28 cm, and that since it pertains to a virtual object it has a negative sign. The substitutions to be made are, therefore, $S_2 = -28$ cm, $n' = 1.5$, $n'' = 1.33$, and $r_2 = -2$ cm.

$$\frac{1.5}{-28} + \frac{1.33}{s_2''} = \frac{1.33 - 1.5}{-2} \qquad \text{or} \qquad s_2'' = +9.6cm$$

FOCAL POINTS AND PRINCIPAL POINTS:

Diagrams showing the characteristics of the two focal points of a thick lens are given in Fig. 5C. In the first diagram diverging rays from the primary focal point F emerge parallel to the axis, while in the second diagram parallel incident rays are brought to a focus at the secondary focal point F''. In each case the incident and refracted rays have been extended to their point of intersection between the surfaces.

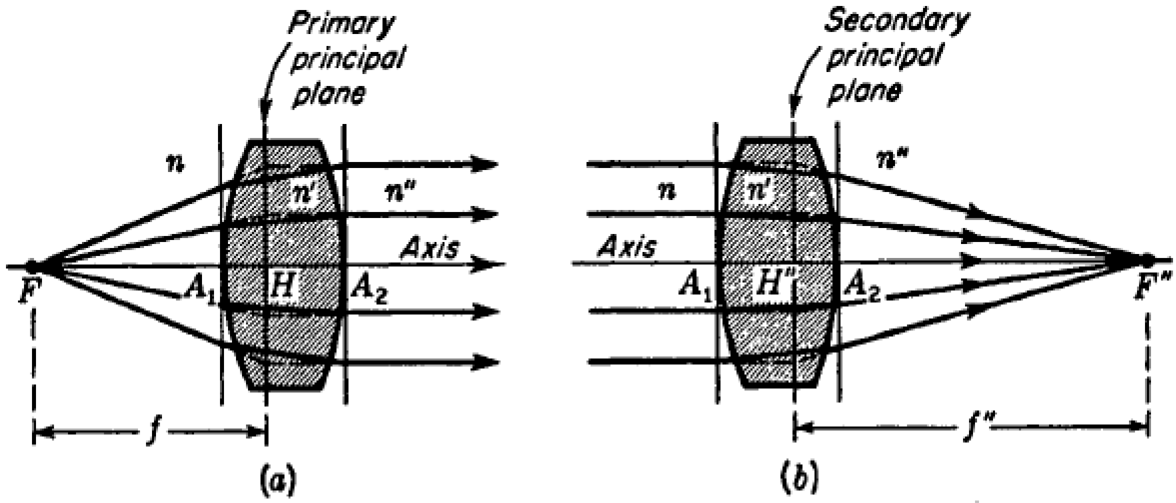


Figure 5C: Ray diagrams showing the primary and secondary principal planes of a thick lens.

Transverse planes through these intersections constitute *primary* and *secondary principal planes*. These planes cross the axis at points Hand *H"*, called the *principal points*.

The focal lengths, as shown in the figure (5C), are measured from the focal points *F* and *F"* to their respective principal points *H* and *H"* and not to their respective vertices A1 and A2. If the medium is the same on both sides of the lens, $n'' = n$, the primary focal length f is exactly equal to the secondary focal length f''.

If the media on the two sides of the lens are different so that n'' is not equal to n, the two focal lengths are different and have the ratio of their corresponding refractive indices:

$$\frac{n''}{n} = \frac{f''}{f} \qquad (5b)$$

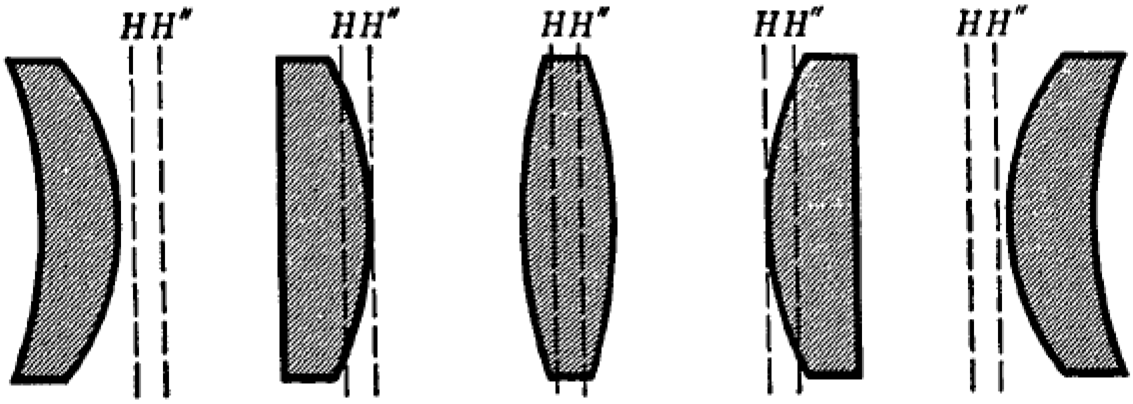


Figure 5D: The variation of the positions of the primary and secondary principal planes as a thick lens

GENERAL THICK-LENS FORMULAS

A set of formulas that can be used for the calculation of important constants generally associated with a thick lens is presented below in the form of two equivalent sets.

Gaussian formulas

$$\frac{n}{f} = \frac{n'}{f_1'} + \frac{n''}{f_2''} - \frac{dn''}{f_1' f_2''} = \frac{n''}{f''}$$

$$A_1F = -f\left(1 - \frac{d}{f_2'}\right)$$

$$A_1H = +f\frac{d}{f_2'}$$

$$A_2F'' = +f''\left(1 - \frac{d}{f_1'}\right)$$

$$A_2H'' = -f''\frac{d}{f_1'}$$

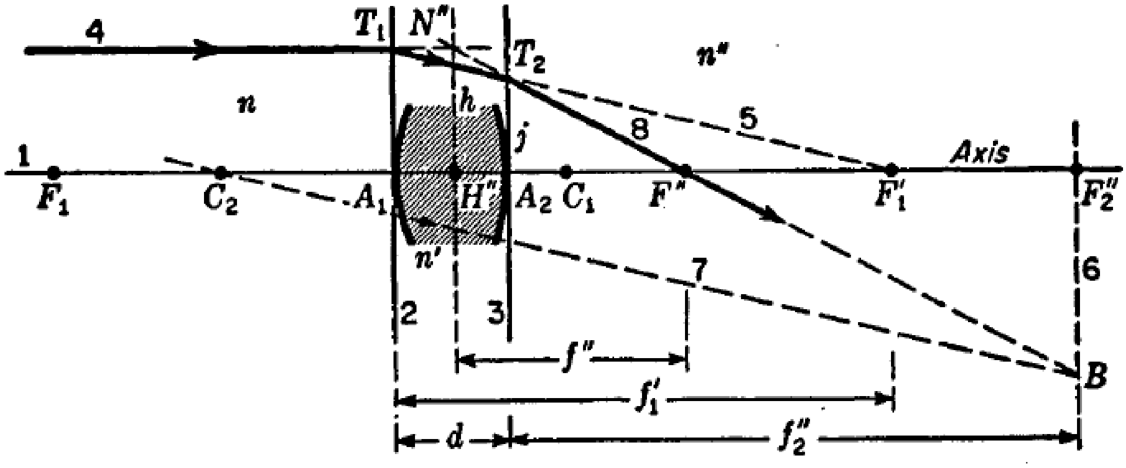


Figure 5G : The oblique-ray method for graphically tracing paraxial rays through a thick lens.

Power formulas

$$P = P_1 + P_2 - \frac{d}{n'} P_1 P_2 \qquad (5g)$$

$$A_1F = -\frac{n}{P}\left(1 - \frac{d}{n'} P_2\right) \qquad (5h)$$

$$A_1H = +\frac{n}{P}\frac{d}{n'} P_2 \qquad (5i)$$

$$A_2F'' = +\frac{n''}{P}\left(1 - \frac{d}{n'} P_1\right) \qquad (5j)$$

$$A_2H'' = -\frac{n''}{P}\frac{d}{n'} P_1 \qquad (5k)$$

Example : An equiconvex lens with radii of 4 cm and index *nl* = 1.50 is located 2.0 cm in front of an equiconcave lens with radii of 6.0 cm and index *n2* = 1.60. The lenses are to be considered as thin. The surrounding media have indices $n = 1.00$, $n' = 1.33$, and $n'' = 1.00$. Find *(a)* the power, *(b)* the focal lengths, *(c)* the focal points, and *(d)* the principal points of the system.

Solution *(a)*: In this instance we shall solve the problem by the use of the power formulas. By Eqs. (5t) the powers of the two lenses in their surrounding media are

$$P_1 = \frac{1.50 - 1.00}{0.04} + \frac{1.33 - 1.50}{-0.04} = 12.50 + 4.17 = +16.67\ \text{D}$$

$$P_2 = \frac{1.60 - 1.33}{-0.06} + \frac{1.00 - 1.60}{0.06} = -4.45 - 10.0 = -14.45\ \text{D}$$

By Eq. (5g), we obtain

$$P = 16.67 - 14.45 + 0.015 \times 16.67 \times 14.45$$

$$P = +5.84\ \text{D}$$

(b) Using Eq. (51),we find

$$f = \frac{n}{P} = \frac{1.00}{5.84} = 0.171\ \text{m} = 17.1\ \text{cm}$$

$$f'' = \frac{n''}{P} = \frac{1.00}{5.84} = 0.171\ \text{m} = 17.1\ \text{cm}$$

(*c*) By Eqs. (5h) to (5k) we obtain

$$A_1F = -\frac{1.00}{5.84}(1 + 0.015 \times 14.45) = -0.208\ \text{m} = -20.8\ \text{cm}$$

$$A_1H = +\frac{1.00}{5.84}\,0.015\,(-14.45) = -0.037\ \text{m} = -3.7\ \text{cm}$$

$$A_2F'' = +\frac{1.00}{5.84}(1 - 0.015 \times 16.67) = +0.128\ \text{m} = +12.8\ \text{cm}$$

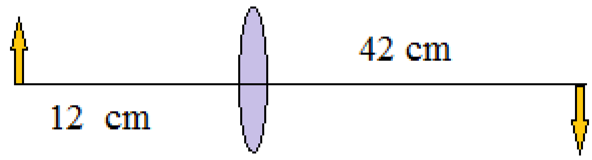
(*d*) The principal points are

$$A_2H'' = -\frac{1.00}{5.84}\,0.015 \times 16.67 = -0.043\ \text{m} = -4.3\ \text{cm}$$

As a check on these results we find that the difference between the first two intervals A_1F and A_1H gives the primary focal length $FH = 17.1$ cm. Similarly the sum of the second two intervals A_2F'' and A_2H'' gives the secondary focal length $H''F'' = 17.1$ cm.

Example: An object located 12 cm in front of a thin lens has its image formed on the opposite side 42 cm from the lens. Calculate *(a)* the focal length of the lens and *(b)* the lens power.

Solution:



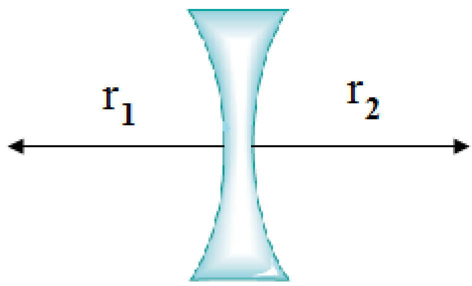
$$\frac{1}{f}=\frac{1}{S_\circ}+\frac{1}{S_i}=\frac{S_\circ S_i}{S_\circ+S_i}=\frac{12\times 42}{12+42}$$

$$\frac{504}{5}=9.333$$

$$P=\frac{1}{f}=\frac{1}{9.333}=10.714D$$

Example: An equiconcave lens is to be made of flint glass of index 1.75. Calculate the radii of curvature if it is to have a power of - 3 D.

Solution:



$$f=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \quad = \quad f=(n-1)\left(\frac{1}{-r}-\frac{1}{r}\right)$$

$$P=-\frac{2}{r}(n-1)$$

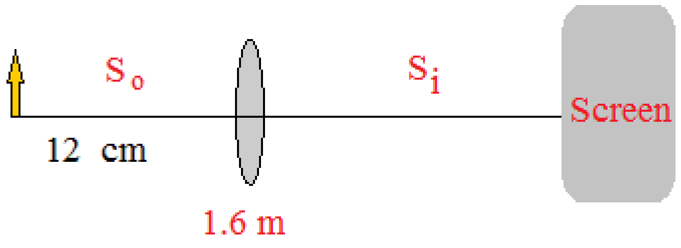
$$\therefore r=-\frac{2}{P}(n-1)$$

$$=-\frac{2}{3}(n-1)=-\frac{2}{3}(1.75-1)=-\frac{1.5}{3}=-0.5=-50cm$$

$$\therefore r=50cm$$

Example : An object is located 1.6 m from a white screen. A lens of what focal length will be required to form a real and inverted image on the screen with a magnification of – 6 ?

Solution :



$$m_T = \frac{-S_i}{S_\circ} = -6 \qquad \therefore S_i = 6S_\circ$$

$$\frac{1}{f} = \frac{1}{S_\circ} + \frac{1}{S_i}$$

$$S_\circ + S_i = 1.6 \qquad \Rightarrow \quad S_\circ + 6S_\circ = 1.6 \qquad \therefore S_\circ = 0.2286 \ m$$

$$S_i = 6S_\circ = 0.2286 \times 6 = 1.3714m$$

$$\frac{1}{f} = \frac{1}{S_\circ} + \frac{1}{S_i} = \frac{S_\circ S_i}{S_\circ + S_i} = \frac{0.228 \times 1.3714}{0.2286 + 1.3714} = 0.1959387 \ m$$

$$= 19.59 \text{ cm}$$

Example: An object 2.5 cm high is placed 12 cm in front of a thin lens of focal length 3 cm. Calculate *(a)* the image distance, *(b)* the magnification, and *(c)* the nature of the image.