Focal Points And Focal Lengths

From Fig. 4-A(a,c) the primary focal point F is an axial point having the property that any ray coming from it or proceeding toward it travels parallel to the axis after refraction.
While from Fig 4-A(b,d) The secondary focal point F' is an axial point having the property that any incident ray traveling parallel to the axis will, after refraction, proceed toward, or appear to come from, F'.
The distance between the center of a lens and either of its focal points is its focal length. These distances, designated *f* and *f'*, usually measured in centimeters or inches, have a positive sign for converging lenses and a negative sign for diverging lenses.

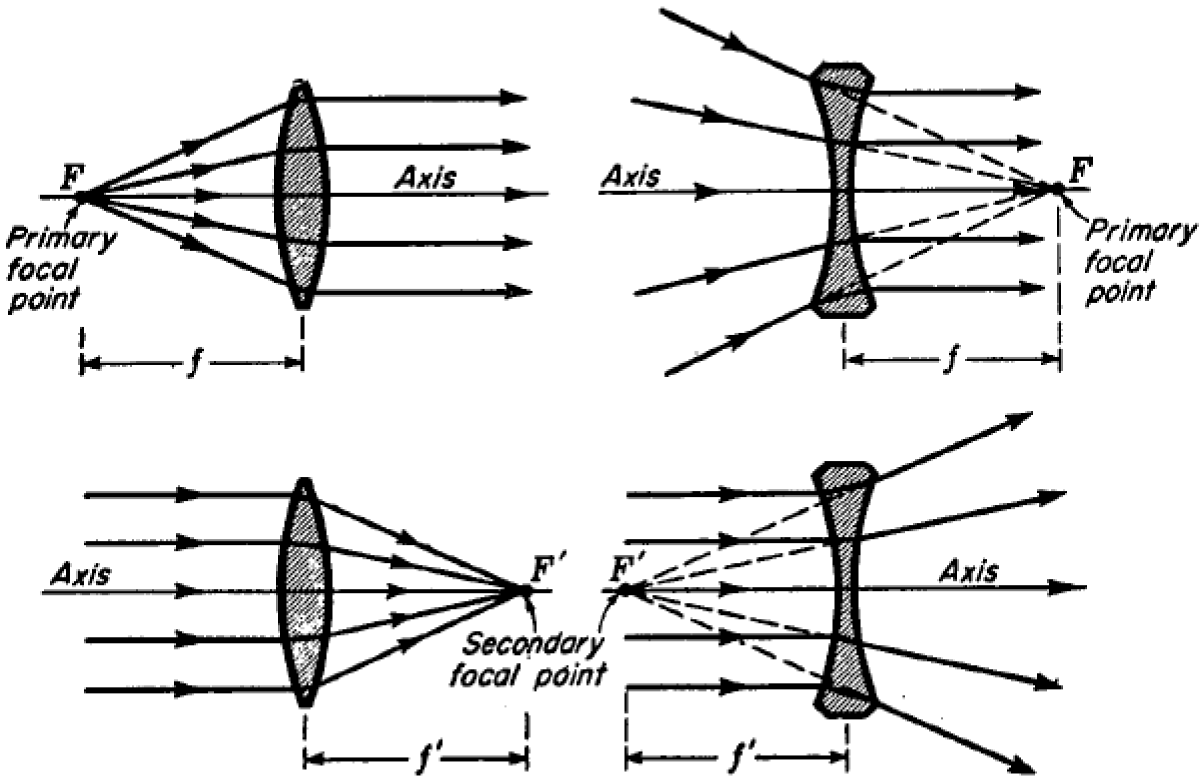

Figure 4A : Ray diagrams illustrating the primary and secondary focal points *F* and *F'* and the corresponding focal lengths *f* and *f'* of thin lenses.

For a lens with the same medium on both sides, we have, by the reversibility of light rays,

$$f = f'$$

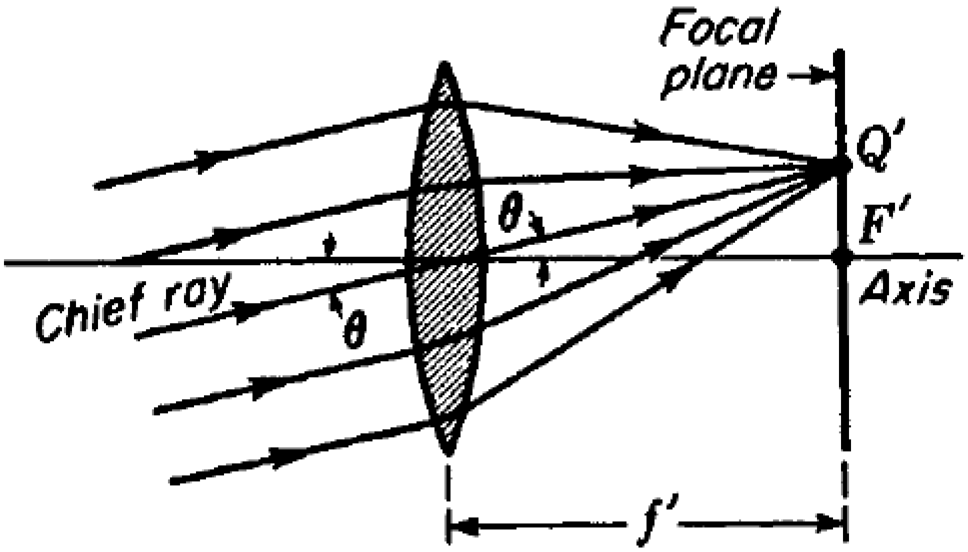

Figure 4B: How parallel incident rays are brought to a focus at the secondary focal plane of a thin lens.

Image Formation:

When an object is placed on one side or the other of a converging lens and beyond the focal plane, an image is formed on the opposite side (see Fig. 4C). If the object is moved closer to the primary focal plane, the image will be formed farther away from the secondary focal plane and will be larger, If the object is moved farther away from *F,* the image will be formed closer to *F'* and will be smaller.
From the Fig. 4c conditions only for paraxial rays, i.e., rays close to lens axis and making small angles with it.

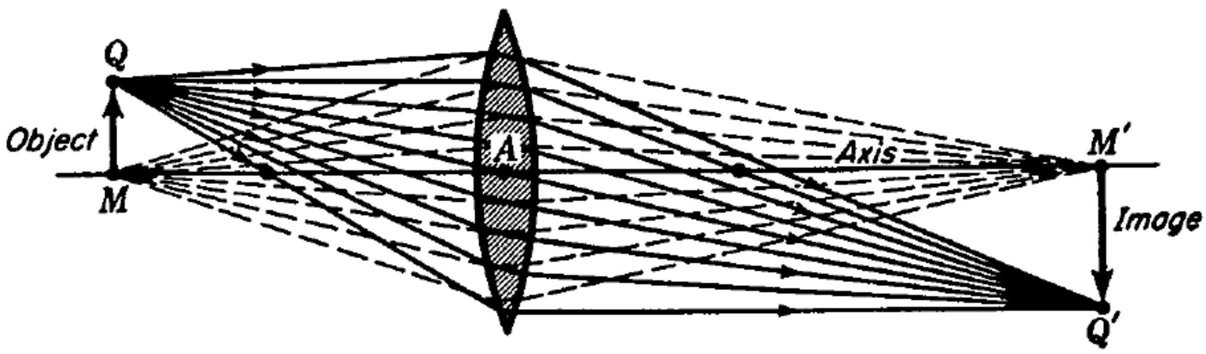

Figure 4c Image formation by an ideal thin lens. All rays from an object point *Q* which pass through the lens are refracted to pass through the image point *Q'*.

CONJUGATE POINTS AND PLANES:

If the principle of the reversibility of light rays is applied to Fig. 4C, we observe that *Q'M'* becomes the object and *QM* becomes its image. The object and image are therefore *conjugate,* just as they are for a single spherical surface (see Sec. 3.4). Any pair of object and image points such as *M* and *M'* in Fig. 4C are called *conjugate* *points,* and planes through these points perpendicular to the axis are called *conjugate planes.*

If we know the focal length of a thin lens and the position of an object, there are three. methods of determining the position of the image: (1) graphical construction, (2) experiment, and (3) use of the lens formula

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \quad \ldots\ldots\ldots\ldots\ldots (4a)$$

Here s is the object distance, s' is the image distance, and f is the focal length, all measured to or from the center of the lens.

The Parallel-Ray Method:

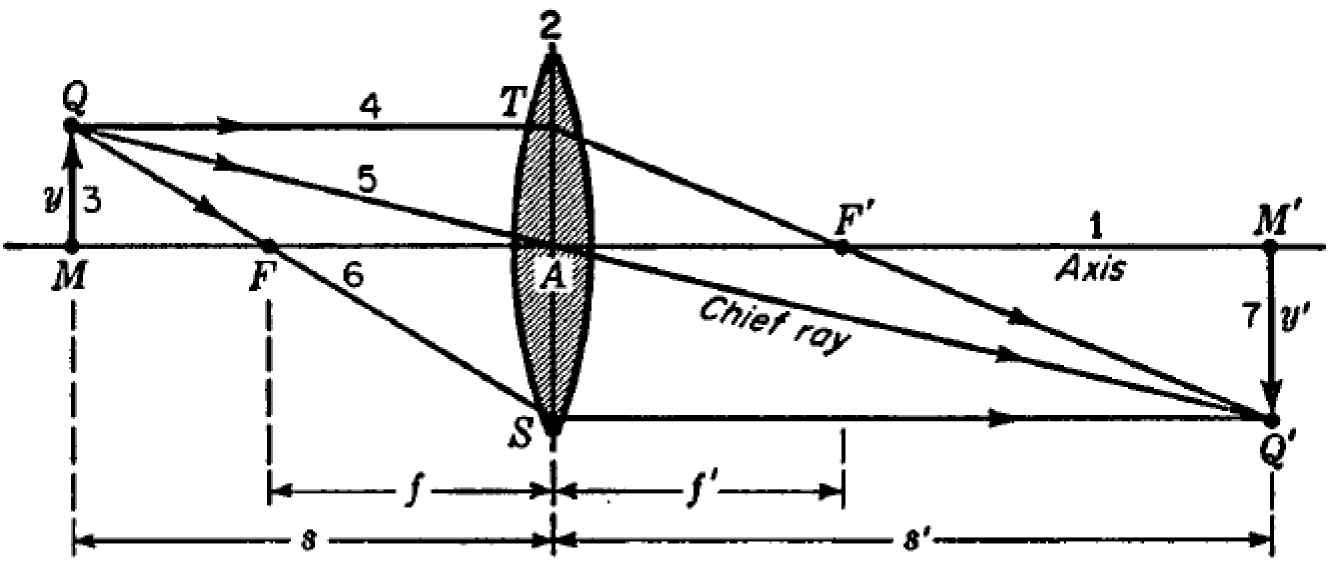

Figure 4D: The parallel-ray method for graphically locating the image formed by a thin lens.

The parallel-ray method is illustrated in Fig. 4D. Consider the light emitted from the extreme point Q on the object. Of the rays emanating from this point in different directions the one *(QT)* traveling parallel to the axis will by definition of the focal point be refracted to pass through F'. The ray QA, which goes through the lens center where the faces are parallel, is undeviated and meets the other ray at some point Q'. These two rays are sufficient to locate the tip of the image at Q', and the rest of the image lies in the conjugate plane through this point. All other rays from Q will also be brought to a focus at Q'. As a check, we note that the ray QF which passes through the primary focal point will by definition of F be refracted parallel to the axis and will cross the others at Q' as shown in the figure.

The Oblique-Ray Method:

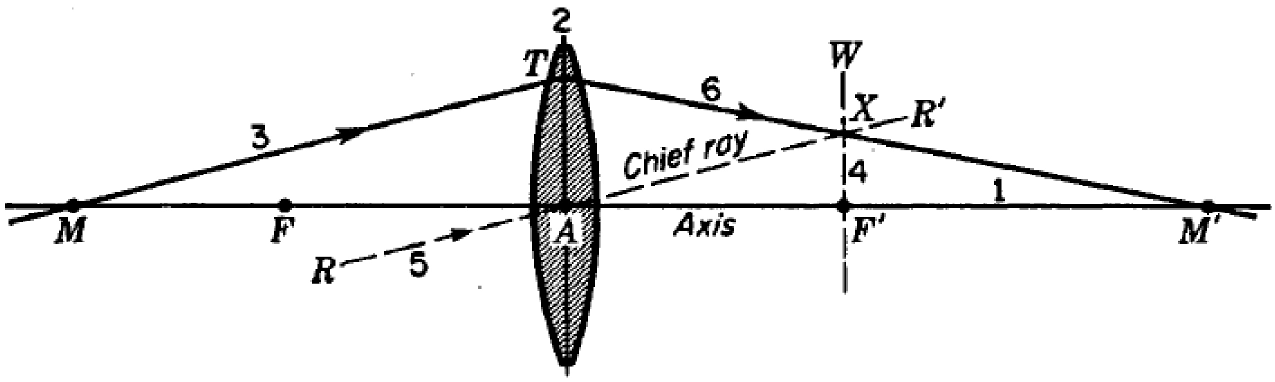

Figure 4E :The oblique-ray method for graphically locating the image formed by a thin lens.

Let *MT* in Fig. 4E represent any ray incident on the lens from the left. It is refracted in the direction *TX* and crosses the axis at *M′*. The point *X* is located at the intersection between the secondary focal plane *F′* Wand the dashed line *RR′* drawn through the center of the lens parallel to *MT*.
The order in which each step of the construction is made is again indicated by the numbers 1,2,3, ... The principle involved in this method may be understood by reference to Fig. 4B. Note that *RR′* is not an actual ray in this case.
Parallel rays incident on the lens are always brought to a focus at the focal plane, the ray through the center being the only one undeviated. Therefore, if we actually have rays diverging from *M,* as in Fig. 4E, we can find the direction of anyone of them after it passes through the lens by making it intersect the parallel line *RR′* through *A* in the focal plane.

Use of the lens Formula:

To illustrate the application of Equation (4a) to find the image position,

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \quad \rightarrow \frac{1}{s'}=\frac{1}{f}-\frac{1}{s} \quad \therefore \quad s'=\frac{fs}{s-f}$$

Let an object be located 6cm in front of a positive lens of focal length + 4 cm. The given quantities are s = +6cm and f = +4cm , and the unknown is s'. As a first step we transpose Equation (4a) by solving for s':

$$s'=\frac{fs}{s-f} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(4b)}$$

From direct substitution of the given quantities in this equation we have

s' = (+6) x (+4) / (+6)-(+4) =+ 12 cm

The image is formed 12 cm from the lens and is *real*.

Lateral Magnification:

From Fig. 4D it is seen that the right triangles *QM A* and *Q′ M′ A* are similar.

where AM' is the image distance s' and AM is the object distance s.

$$\frac{M'Q'}{MQ} = \frac{AM'}{AM}$$

Taking upward directions as positive, $y = MQ$, and $y' = -M'Q'$; so we have by direct substitution $y'/y = -s'/s$. The lateral magnification is therefore

$$m = \frac{y'}{y} = -\frac{s'}{s} \qquad \text{............ (4c)}$$

When s and s' are both positive, as in Fig. 4D, the negative sign of the magnification signifies an inverted image.

Virtual Images:

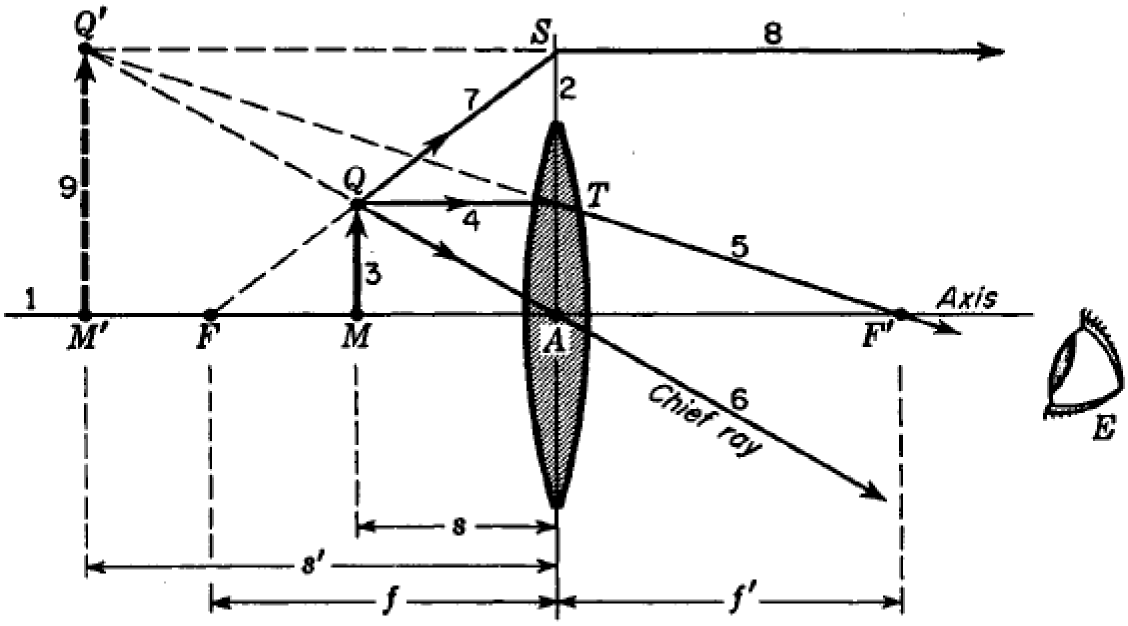

Figure 4F

The parallel-ray method for graphically locating the virtual image formed by a positive lens when the object is between the primary focal point and the lens.

The images formed by the converging lenses in Figs. 4C and 4D are real in that they can be made visible on a screen. They are characterized by the fact that rays of light are actually brought to a focus in the plane of the image. A virtual image cannot be formed on a screen. The rays from a given point on the object do not actually come together at the corresponding point in the image; instead they must be projected backward to find this point. Virtual images are produced with converging lenses when the object is placed between the focal point and the lens and with diverging lenses when the object is in any position. Examples are shown in Figs. 4F and 4G.

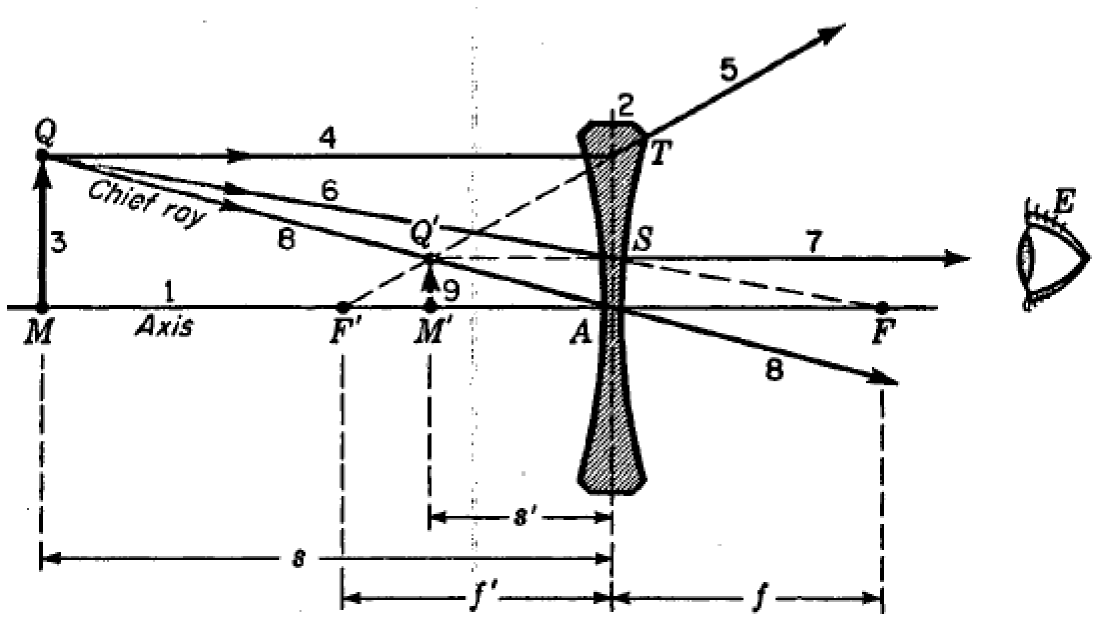

Figure 4G

The parallel-ray method for graphically locating the virtual image formed by anegative lens.

Example :If an object is located 6cm in front of a lens of focal length 10cm, where will the image be formed

Solution : The given quantities are $s = +6$cm, and $f = +10$cm, while the unknown quantities are s' and m. By making direct substitutions in Eq. (4b) we obtain

$$s' = \frac{sf}{s-f} \rightarrow = \frac{(+6)\times(+10)}{(+6)-(+10)} = \frac{+60}{-4} = -15cm$$

The magnification is obtained by

$$m = -\frac{s'}{s} = -\frac{-15}{6} = +2.5$$

The positive sign means that the image is erect.

Example 2: An object is placed 12cm in front of a diverging lens of focal length 6cm. Find the image.

Solution The given quantities are $s = +12$cm and $f = -6.0$ cm, while the unknown quantities are s' and m. We substitute directly in Equation (4b), to obtain

$$s' = \frac{(+12)\times(-6)}{(+12)-(-6)} = \frac{-72}{+18} = -4cm$$

from which $s' = -4$cm. For the image size Equation (4c) gives

$$m = -\frac{s'}{s} = -\frac{-4}{12} = +1/3$$