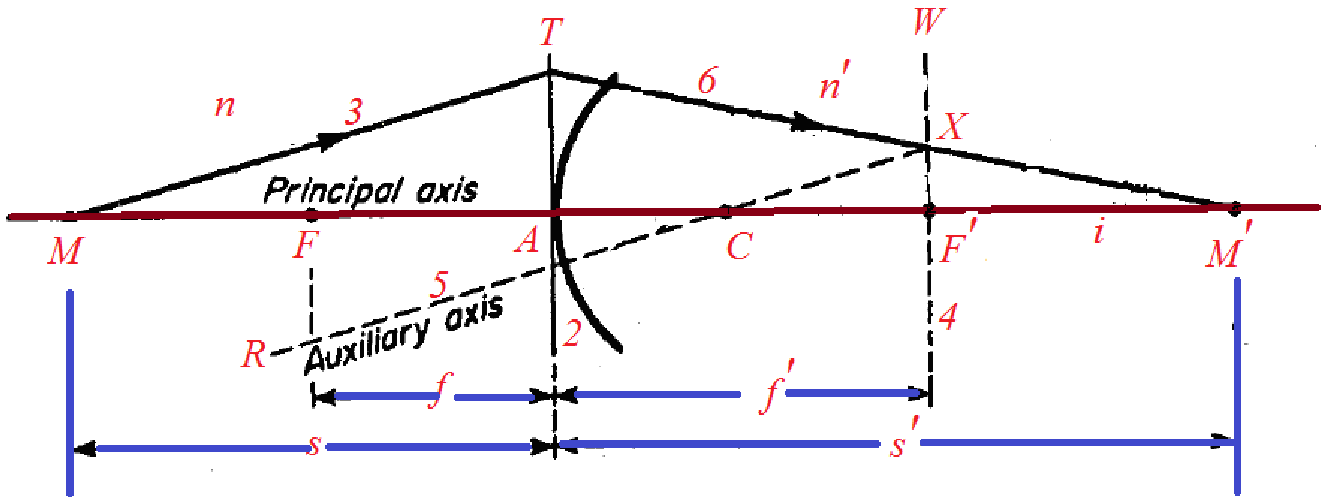

Fig. 3H

Method 2:

This method is shown in Fig. 3I. After drawing the axis *MM'* and the arc representing the spherical surface with a center C, any line such as 1 is drawn to represent any oblique ray of light. Next, an auxiliary diagram is started by drawing *XZ* parallel to the axis. With an origin at *O,* line intervals *OK* and *OL* are laid off proportional to *n* and *n',* respectively, and perpendiculars are drawn through *K, L,* and *A*. From here the construction proceeds in the order of the numbers 1, 2, 3, 4, 5, and 6. Line 2 is drawn through *O* parallel to line 1, line 4 is drawn through *J* parallel to line 3, and line 6 is drawn through *T* parallel to line 5.

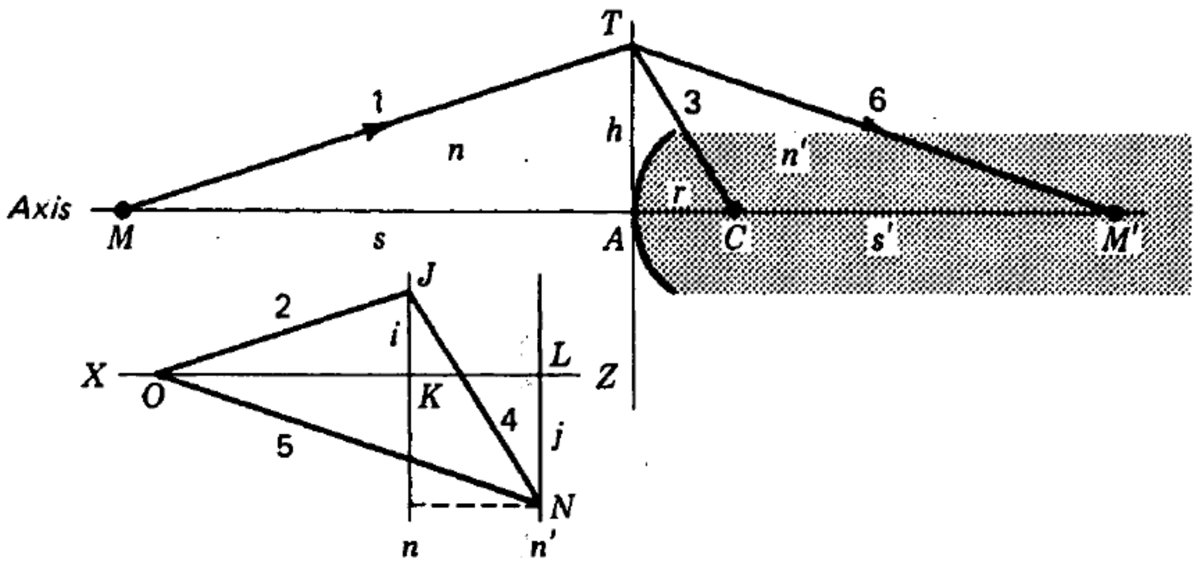

Fig. 3I

Magnification:

In any optical system the ratio between the transverse dimension of the final image and the corresponding dimension of the original object is called the *lateral magnification.* From figure 3F

The theorem of the proportionality of corresponding sides requires that

$$\frac{M'Q'}{MQ}=\frac{CM'}{CM} \qquad \text{or} \qquad \frac{-y'}{y}=\frac{S'-r}{S+r}$$

We now define y'/y as the lateral magnification m and obtain

$$m=\frac{-y'}{y}=-\frac{S'-r}{S+r}$$

If m is positive, the image will be virtual and erect, while if it is negative, the image is real and inverted.

REDUCED VERGENCE:

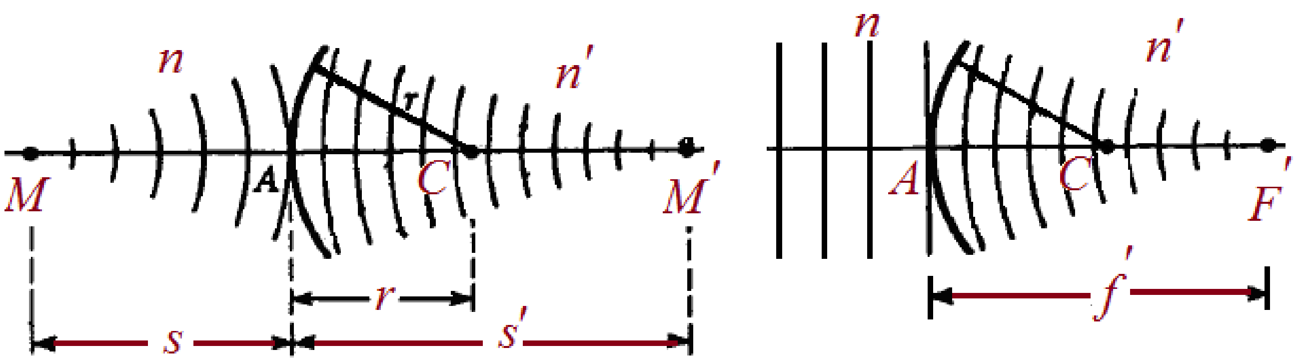

FIGURE 3J : The refraction of light waves at a single spherical surface.

As the waves from M strike the vertex A, they have a radius s and a curvature l/s, and as they leave A, converging toward M', they have a radius s' and a curvature l/s'. Similarly the incident waves arriving at A in the second diagram have an infinite radius ∞ and a curvature of 1/∞, or zero. At the vertex where they leave the surface, the radius of the refracted waves is equal to f' and their curvature is equal to f/f' .The gaussian formulas may therefore be considered as involving the addition and subtraction of quantities proportional to the curvatures of spherical surfaces. When these curvatures rather than radii are used, the formulas become simpler in form and for some purposes more convenient. We therefore introduce at this point the quantities

$$V=\frac{n}{s}, \qquad V'=\frac{n'}{s'}, \qquad K=\frac{1}{r}, \qquad P=\frac{n}{f}, \qquad P=\frac{n'}{f'} \qquad \text{…………..(3h)}$$

The <u>first two</u>, Vand V', are called reduced vergences because they are direct measures of the convergence and divergence of the object and image wave fronts; respectively. For a divergent wave from the object s is positive, and so is the vergence V. For a convergent wave, on the other hand, s is negative, and so is its vergence. For a converging wave front toward the image, V' is positive, and for a diverging wave front, V' is negative.

The third quantity *K* is the curvature of the refracting surface (reciprocal of its radius), while the fourth and 'fifth quantities are, according to Equation 6 , equal and define the refracting power. When all distances are measured in meters, .the reduced vergences V and V', the curvature K, and the power P are in units called diopters.

$$V + V' = P \quad \ldots\ldots\ldots\ldots\ldots\ldots (3i)$$

$$P = \frac{n' - n}{r} \quad \text{or} \quad P = (n' - n)K \quad \ldots\ldots\ldots\ldots\ldots\ldots (3j)$$

Example 3: One end of a glass rod of refractive index 1.5 is ground and polished with a convex spherical surface of radius 10 cm. An object is placed in the air on the axis 40 cm to the left of the vertex. Find (a) the power of the surface and (b) the position of the image.

Solution: The given quantities are n = 1, n' = 1.5, r = + 10cm, and s = +40cm. The unknown quantities are P and s'. To find the solution to (a), we make use of Eq. (3j), substitute the given distance in meters, and obtain

$$P = \frac{1.5 - 1}{0.1} = +5D$$

For the answer to part *(b),* we first use Eq. (3h) to find the vergence *V.*

$$V = \frac{1}{0.4} = +2.5D$$

Direct substitution in Eq. (3i) gives

$$2.5 + V' = 5 \qquad \textit{from which} \qquad V' = +2.5D$$

To find the image distance, we have *V'* = *n'ls',* so that

$$s' = \frac{n'}{V'} = \frac{1.5}{2.5} = +0.6m = +60cm$$

DERIVATION OF THE GAUSSIAN FORMULA:

In Fig. 3K an oblique ray from an axial object point *M* is shown incident on the surface at an angle ϕ and refracted at an angle ϕ'. The refracted ray crosses the axis at the image point *M'.* If the incident and refracted rays *MT* and *TM'* are paraxial, the angles ϕ and ϕ' will be small enough to permit putting the sines of the two angles equal to the angles themselves; for Snell's law we write

$$\frac{\phi}{\phi'} = \frac{n'}{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3k)$$

Since ϕ is an exterior angle of the triangle *MTC* and equals the sum of the opposite interior angles,

$$\phi = \alpha + \beta \quad \ldots\ldots\ldots\ldots\ldots\ldots (3l)$$

Similarly β is an exterior angle of the triangle *TCM'*, so that $\beta = \phi' + \gamma$ and

$$\phi' = \beta - \gamma \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3m)}$$

Substituting these values of ϕ and ϕ' in Eq. (3k) and multiplying out, we obtain

$$n'\beta - n'\gamma = n\alpha + n\beta \qquad \text{or} \qquad n\alpha + n'\gamma = (n' - n)\beta$$

For paraxial rays α , β and γ are very small angles, and we may set $\alpha = h/s$, $\beta = h/\gamma$
and $\gamma = h/s'$. Substituting these values in the last equation, we obtain

$$n\frac{h}{s} + n'\frac{h}{s'} = (n' - n)\frac{h}{r}$$

By canceling *h* throughout we obtain the desired equation

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{r} \quad \ldots\ldots\ldots\ldots\ldots \text{(3n)}$$

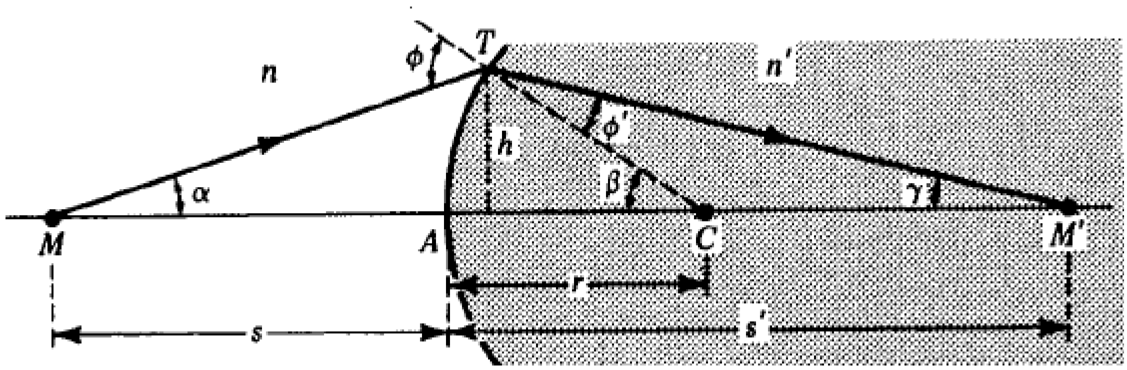

FIGURE 3K Geometry for the derivation of the paraxial formula used in locating images.

Example: The left end of a long glass rod of index 1.635 is ground and polished to a convex spherical surface of radius 2.5 cm. A small object is located in the air and on the axis 9cm from the vertex. Find *(a)* the primary and secondary focal lengths, *(b)* the power of the surface, *(c)* the image distance, and *(d)* the lateral magnification.

Solution (a):

$$f = \frac{n}{n' - n} r = \frac{1}{1.635 - 1} \times 2.5 = 3.937 cm$$

$$f' = \frac{n'}{n' - n} r = \frac{1.635}{1.635 - 1} \times 2.5 = 6.437 cm$$

Solution (b)

$$P = \frac{n'}{f'} = \frac{n}{f} = \frac{1}{0.03937} = 25.54 D$$

Solution (c)

$$\frac{n}{s}+\frac{n'}{s'}=\frac{n'-n}{r} \Rightarrow\Rightarrow\Rightarrow \frac{1}{9}+\frac{1.635}{s'}=\frac{1.635-1}{+2.5} \rightarrow\rightarrow\rightarrow s'=11.434$$

$$m=-\frac{s'-r}{s+r}=-\frac{11.434-2.5}{9+2.5}=-0.777$$

Example: The left end of a water trough has a transparent surface of radius - 2cm. A small object 2.5 cm high is located in the air and on the axis 10cm from the vertex. Find *(a)* the primary and secondary focal lengths, *(b)* the power of the surface, *(c)* the image distance, and *(d)* the size of the image. Assume water to have an index 1.333.

Ans. (a) - 6.01 and - 8.01 em, *(b)* -16.65 0, *(c)* - 5.0 cm, *(d)* +0.938

Solution(a):

$$f=\frac{n}{n'-n}r=\frac{1}{1.333-1}\times(-2)=-6.01cm$$

$$f'=\frac{n'}{n'-n}r=\frac{1.333}{1.333-1}\times(-2)=-8.01cm$$

Solution(d):

$$P=\frac{n}{f}=\frac{n'}{f'}=\frac{1}{-0.0601}=-16.65D$$

Solution(c):

$$\frac{n}{s}+\frac{n'}{s'}=\frac{n'-n}{r} \Rightarrow\Rightarrow \frac{1}{10}+\frac{1.333}{s'}=\frac{1.333-1}{-2} \rightarrow\rightarrow\rightarrow\leftarrow\rightarrow s'=-5cm$$

Solution(d):

$$m=-\frac{s'-r}{s+r}=-\frac{(-5)-(-2)}{10+(-2)}=\frac{+3}{8}=0.375$$

$$\ell'=m\ell=0.375\times2.5 \quad \therefore \ell'=0.9375cm$$

Example: The left end of a long glass rod of index 1.62 is polished to a convex surface of radius + 1.2 cm and then submerged in water of index 1.333. A small object 2.5 cm high is located in the water 10cm in front of the vertex. Calculate *(a)* the primary and secondary focal lengths, *(b)* the power of the surface, *(c)* the image distance, and *(d)* the size of the image.

Ans. (a) + 5.57 and + 6.77 cm, *(b)* 23.91 D, *(c)* + 15.31 cm, *(d)* - 3.150 cm

Solution: