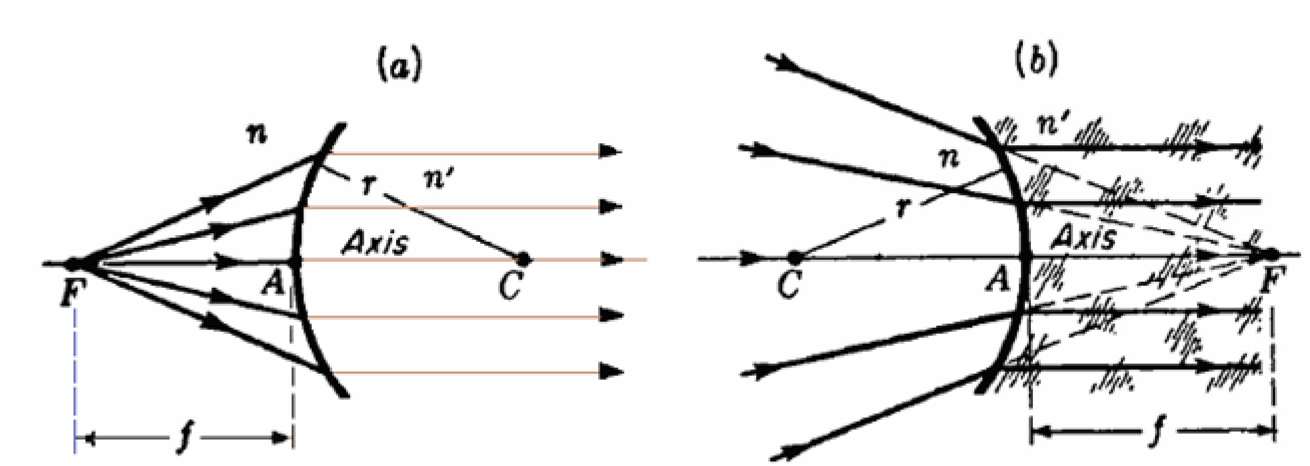
1- FOCAL POINTS AND FOCAL LENGTHS

Characteristic diagrams showing the refraction of light by convex and concave spherical surfaces are given in Fig. 3B. Each ray in being refracted obeys Snell's law as given by

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

The principal axis in each diagram is a straight line through the center of curvature C. The point *A* where the axis crosses the surface is called the *vertex.* In diagram *(a)* rays are shown diverging from a point source *F* on the axis in the first medium and refracted into a beam everywhere parallel to the axis in the second medium. Diagram *(b)* shows a beam converging in the first medium toward the point *F* and then refracted into a parallel beam in the second medium. *F* in each of these two cases is called the *primary focal point,* and the distance *f* is called the *primary focallength.*

In diagram *(c)* a parallel incident beam is refracted and brought to a focus at the point F',and in diagram *(d)* a parallel incident beam is refracted to diverge as if it came from the point *P*. *F'* in each case is called the *secondary focal point,* and the distance *f'* is called the *secondary focal length.* Returning to diagrams *(a)* and *(b)* for reference, we now state that *the primary focal point F is an axial point having the property that any ray coming from it or proceeding toward it travels parallel to the axis after refraction.* Referring to diagrams *(c)* and *(d),* we make the similar statement that *the secondary focal point F' is an axial point having the property that any incident ray traveling parallel to the axis will, after refraction, proceed toward, or appear to come from, F'.*



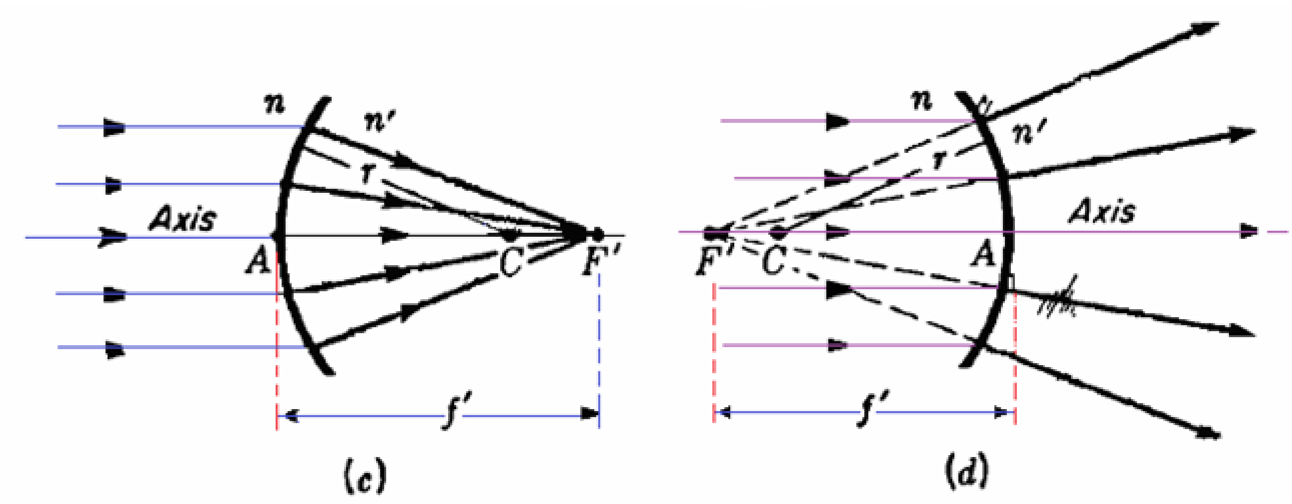


Figure 3B

The focal points **F** and **F'** and focal lengths ***f*** and ***f'*** associated with a single spherical refracting surface of radius ***r*** separating two media of index ***n*** and ***n'*** .

2- VIRTUAL IMAGES

The image *M'Q'* in Fig. 3D is a real image in the sense that if a flat screen is located there, a sharply defined image of the object *MQ* will be formed on the screen.

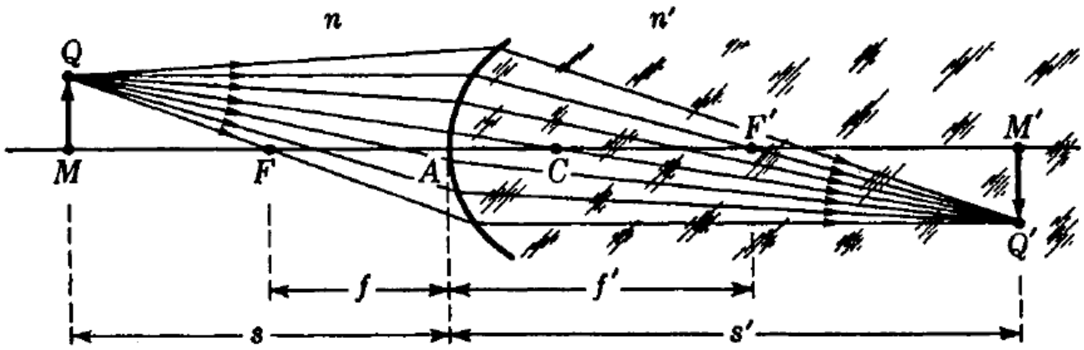


Fig. 3D

Not all images, however, can be formed on a screen, as is illustrated in Fig. 3E.

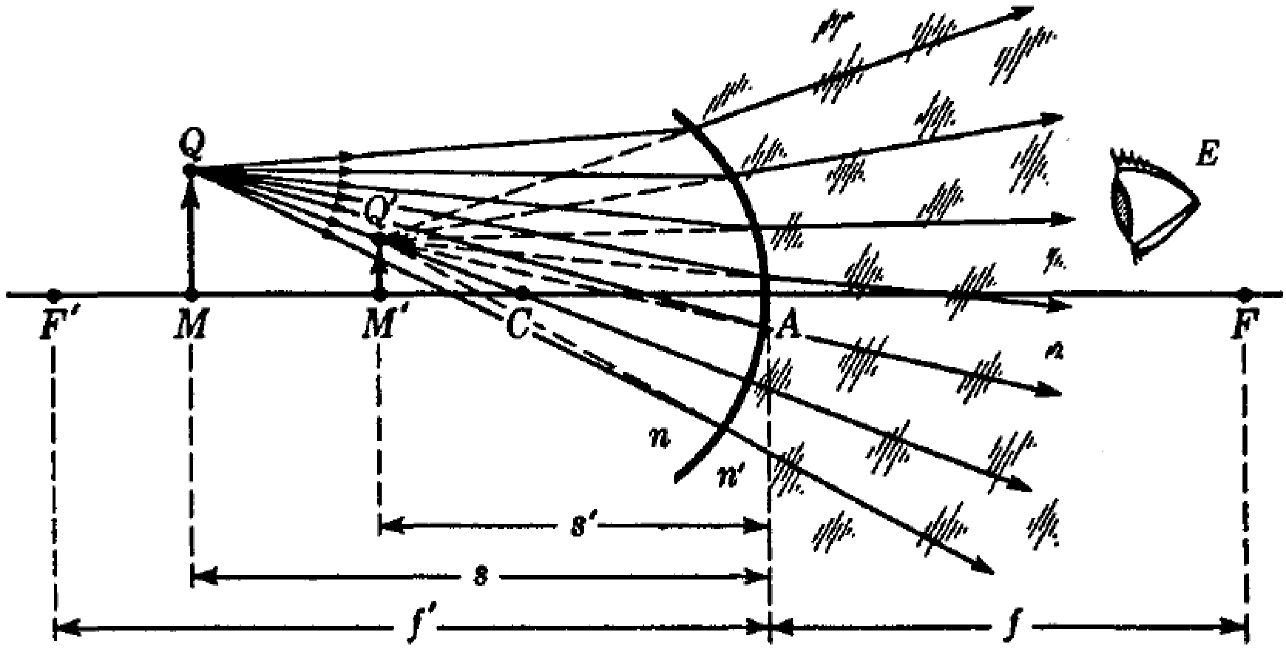


Fig 3E

Light rays from an object point Q are shown refracted by a concave spherical surface separating the two media of index $n = 1$ and $n' = 1.5$, respectively. The focal lengths have the ratio 1: 1.5.

Since the refracted rays are diverging, they will not come to a focus at any point.

To an observer's eye located at the right, however, such rays will appear to be coming from the common point Q'. In other words, Q' is the image point corresponding to the object point Q. Similarly M' is the image point corresponding to the object point M. Since the refracted rays do not come from Q' but only appear to do so, no image can be formed on a screen placed at M'. For this reason such an image is said to be *virtual.*

3- Convension of Signs

1. *Draw all figures with light incident on the refracting or reflecting surface from the left.*
2. *Consider object distances (s) positive when the object lies at the left of the vertex of the refracting or reflecting surface.*
3. *Consider image distances (s') positive when the image lies at the right of the vertex.*
4. *Consider radii of curvature (R) positive when the center of curvature lies at the right of the vertex.*
5. *Consider angles positive when the slope of the ray with respect to the axis (or with respect to a radius of curvature) is positive.*
6. *Consider transverse dimensions positive when measured upward from the axis.*

4- Conjugate Points And Planes:

The principle of the reversibility of light rays has the consequence that if *Q'M'* in Fig. 3D were an object, an image would be formed at *QM*. Hence, if any object is placed at the position previously occupied by its image, it will be imaged at the position previously occupied by the object. The object and image are thus interchangeable, or conjugate. Any pair of object and image points such as *M* and *M'* in Fig. 3D are called *conjugate points,* and planes through these points perpendicular to the axis are called *conjugate planes.* If one is given the radius of curvature *r* of a spherical surface separating two media of index n and *n',* respectively, as well as the position of an object, there are three general methods that may be employed to determine the position and size of the image: (1) graphical methods, (2) experiment, and (3) calculation using the formula

$$\frac{n}{s}+\frac{n'}{s'}=\frac{n'-n}{r} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

In this equation *s* is the object distance and *s'* is the image distance. This equation, called the *gaussian formula for a single spherical surface,* As an object *M* is brought closer to the primary focal point, Eq. (1) shows that the distance *AM'* of the image from the vertex becomes steadily greater and that in the limit when the object reaches *F* the refracted rays are parallel and the image is formed at infinity. Then we have $s' = \infty$, and Eq. (1) becomes

$$\frac{n}{s}+\frac{n'}{\infty}=\frac{n'-n}{r} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Since this particular object distance is called the primary focal length *f,* we may write

$$\frac{n}{f}=\frac{n'-n}{r} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

if the object distance is made larger and eventually approaches infinity, the image distance diminishes and becomes equal to f' in the limit, $s=\infty$. Then

$$\frac{n}{\infty}+\frac{n'}{s'}=\frac{n'-n}{r} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

or, since this value of *s'* represents the secondary focal length *f ',*

$$\frac{n'}{f'}=\frac{n'-n}{r} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Equating the left-hand members of Eqs. (3) and (5), we obtain

$$\frac{n}{f}=\frac{n'}{f'} \quad \text{or} \quad \frac{n'}{n}=\frac{f'}{f} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

When $(n'-n)/r$ in Eq. (1) is replaced by n/f or by n'/f' according to Eqs. (3) and (5), there results

$$\frac{n}{s}+\frac{n'}{s'}=\frac{n}{f} \quad \text{or} \quad \frac{n}{s}+\frac{n'}{s'}=\frac{n'}{f'} \qquad \ldots\ldots\ldots\ldots\ldots (7)$$

Both these equations give the conjugate distances for a single spherical surface.

EXAMPLE : A concave surface with a radius of 4 cm separates two media of refractive index $n = 1$ and $n' = 1.5$. An object is located in the first medium at a distance of 10 cm from the vertex. Find *(a)* the primary focal length, *(b)* the secondary focal length, and *(c)* the image distance.

Solution: The given quantities are $n = 1$, $n' = 1.5$, $r = -4$ cm, and $s = +10$cm. The unknown quantities are f , f' , and s'. *(a)* We use Eq. (3) directly to obtain

$$\frac{1}{f} = \frac{1.5-1}{-4} \quad \rightarrow \quad f = \frac{-4}{0.5} = -8cm$$

(b) We use Eq. (5) directly and obtain

$$\frac{1.5}{f'} = \frac{1.5-1}{-4} \quad \rightarrow\Rightarrow \quad f' = \frac{-6}{0.5} = -12cm$$

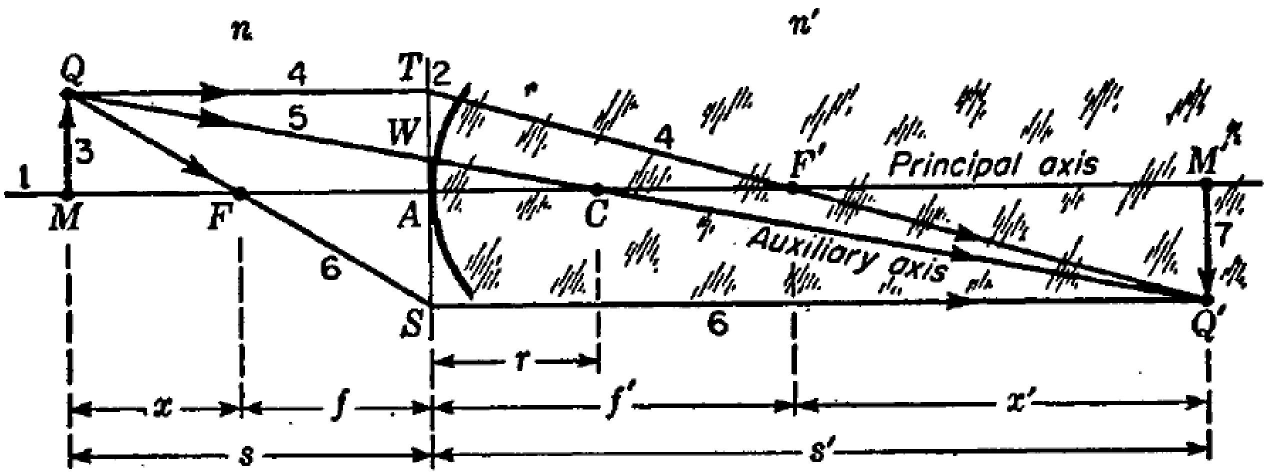
Note that in this problem both focal lengths are negative and that the ratio f/f' is *1/1.5* as required by Eq. (6).

(c) We use Eq. (7) and obtain, by direct substitution,

$$\frac{1}{10} + \frac{1.5}{s'} = \frac{1}{-8} \quad \text{giving} \quad s' = -6.66cm$$

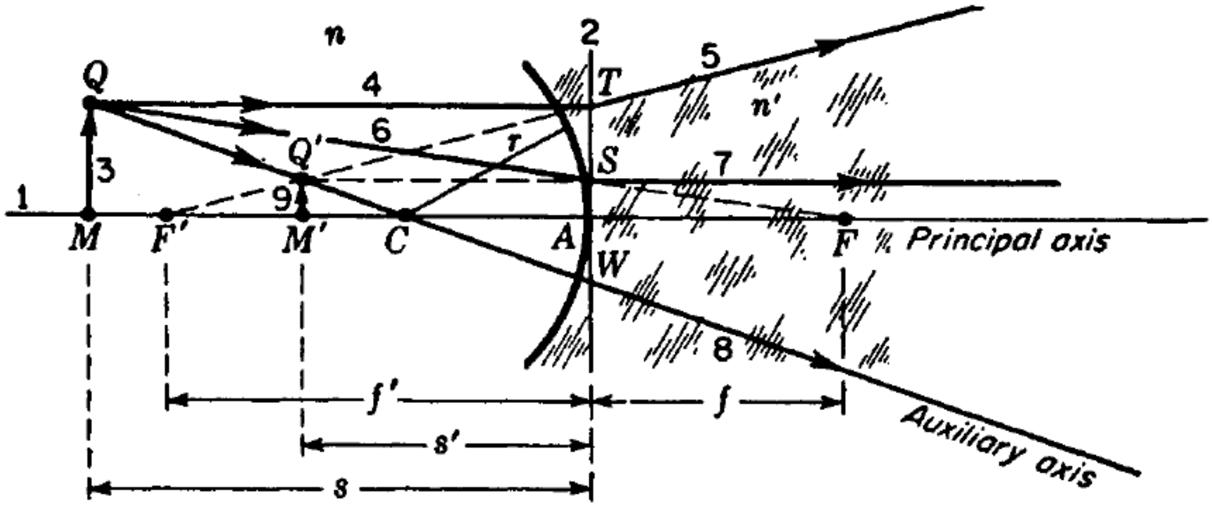
5- Graphical constructions.
5-1 The parallel-ray method:

The parallel-ray method of construction is illustrated in Figs. 3F and 3G for convex and concave surfaces, respectively. Consider the light emitted from the highest point Q of the object in Fig. 3F.



Figs. 3F

Of the rays emanating from this point in different directions the one *(QT)* traveling parallel to the axis will by definition of the focal point be refracted to pass through *P'*. The ray *QC* passing through the center of curvature is undeviated because it crosses the boundary perpendicular to the surface.



Figs. 3G

As a check we note that the ray *QS,* which passes through the point *P,* will (by definition of the primary focal point) be refracted parallel to the axis and will cross the others at *Q'* as shown in the figure.
This method is called the *parallel-ray method.* The numbers 1, 2, 3, ... indicate the order in which the lines are customarily drawn.

5-2 Oblique-Ray Methods:

Method 1

Let *MT* in Fig. 3H represent any ray incident on the surface from the left. Through the center of curvature C a dashed line *RC* is drawn, parallel to *MT,* and extended to the point where it crosses the secondary focal plane. The line *TX* is then drawn as the refracted ray and extended to the point where it crosses the axis at *M'*. Since the axis may here be considered as a second ray of light, *M* represents an axial object point and *M'* its conjugate image point. The principle involved in this construction is the following. If *MT* and *RA* were parallel incident rays of light, they would (after refraction and by the definition of focal planes) intersect the secondary focal plane *WF'* at *X*. Since *RA* is directed toward C, the refracted ray *ACX* remains undeviated from its original direction.