الفصل الثاني

Reflection and refraction at plane surfaces.

We consider the general problem of a train of plane electromagnetic waves, traveling in one medium and incident on a plane surface bounding a second medium in which the velocity of propagation differs from that in the first. It might be expected that if both media are transparent the incident wave train will merely continue on into the second medium.

Fig. A illustrates what actually happens to the train of incident waves. A reflected wave train and a transmitted or reflected wave train, originate at the boundary surface. That is, except in certain special cases, only a part of the incident light passes into the second medium, the remainder being reflected. Furthermore, the directions of travel of the reflected and transmitted waves (again except in special cases) ore different from that of the incident wave.

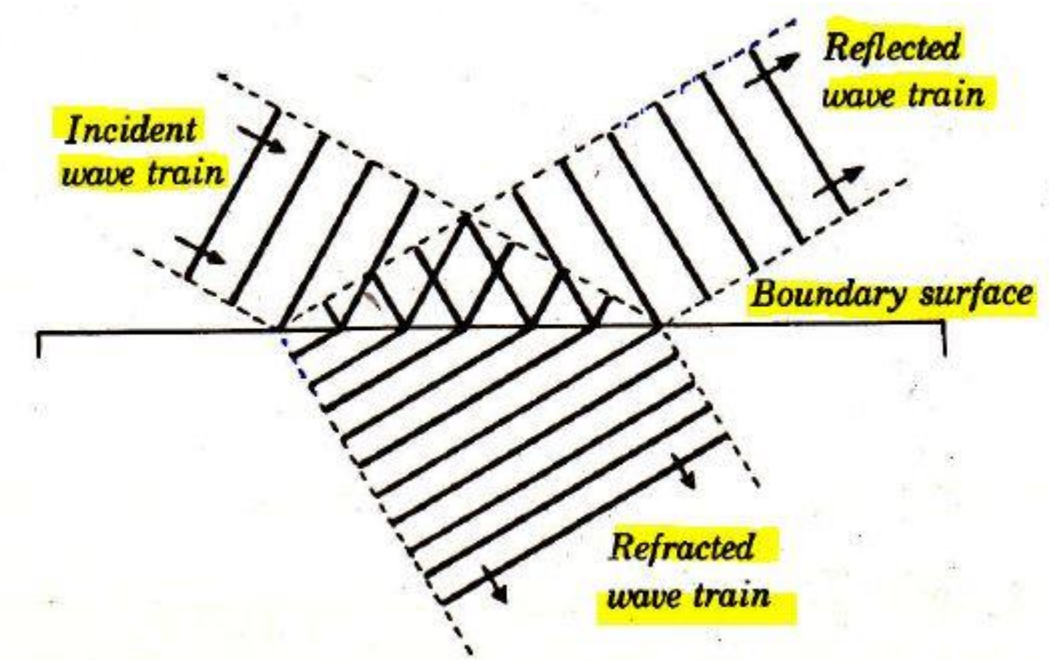


Fig. A . Reflection and refraction of a train of plane waves

The Laws of Reflection and Refraction

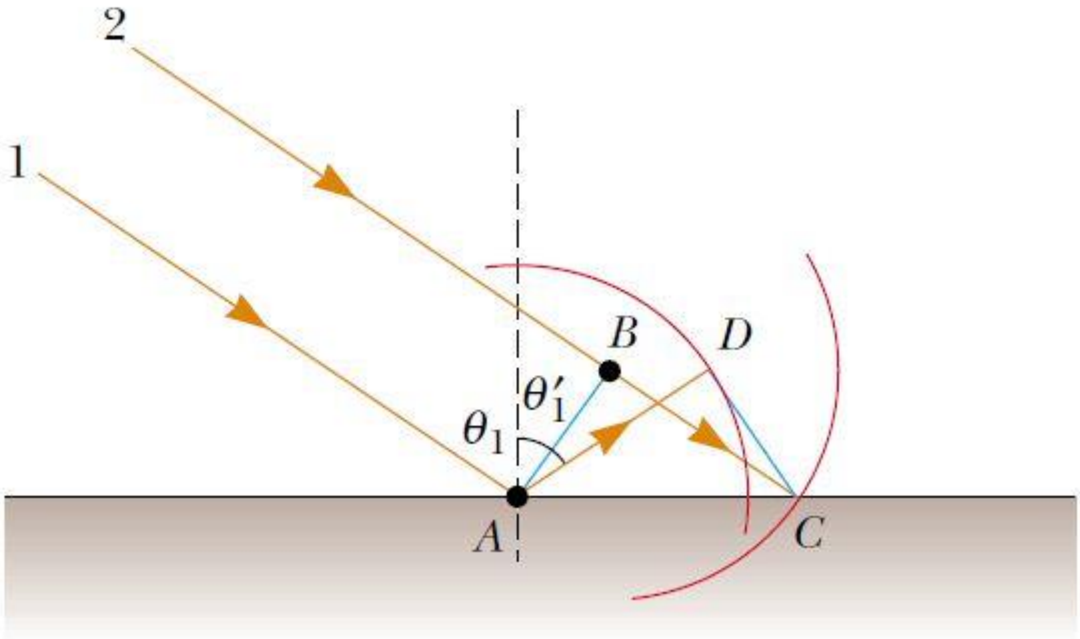
For the law of reflection, refer to Figure 1.a. The line *AB* represents a wave front of the incident light just as ray 1 strikes the surface. At this instant, the wave at *A* sends out a Huygens wavelet (the circular arc centered on *A*) toward *D*. At the same time, the wave at *B* emits a Huygens wavelet (the circular arc centered on *B*) toward *C*. Figure 1.a shows these wavelets after a time interval Δt, after which ray 2 strikes the surface. Because both rays 1 and 2 move with the same speed, we must have $AD = BC = c\,\Delta t$.

The remainder of our analysis depends on geometry, as summarized in Figure 1.b, in which we isolate the triangles *ABC* and *ADC*. Note that these two triangles are congruent because they have the same hypotenuse *AC* and because *AD* = *BC*.
From Figure 1.b, we have

$$\cos\gamma = \frac{BC}{AC} \qquad \text{and} \qquad \cos\gamma' = \frac{AD}{AC}$$

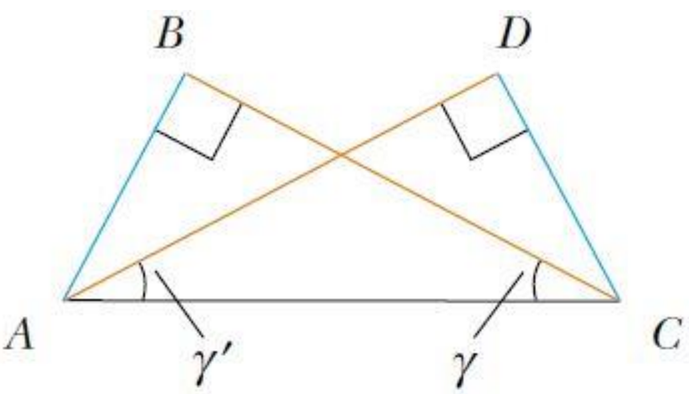
where, comparing Figures 1- a and 1-b, we see that $\gamma = 90° - \theta$ and $\gamma' = 90° - \theta'$.
Because *AD* = *BC*, we have

$$\cos\gamma = \cos\gamma'$$



(a)

Fig.1



(b)

Fig. 1

Therefore,

$$\gamma = \gamma'$$

$$90^\circ - \theta_1 = 90^\circ - \theta_1'$$

$$\theta_1 = \theta_1'$$

which is the law of reflection

Now let us use Huygens's principle and Figure 2 to derive Snell's law of refraction. We focus our attention on the instant ray 1 strikes the surface and the subsequent time interval until ray 2 strikes the surface. During this time interval, the wave at *A* sends out a Huygens wavelet (the arc centered on *A*) toward *D*. In the same time interval, the wave at *B* sends out a Huygens wavelet (the arc centered on *B*) toward *C*.

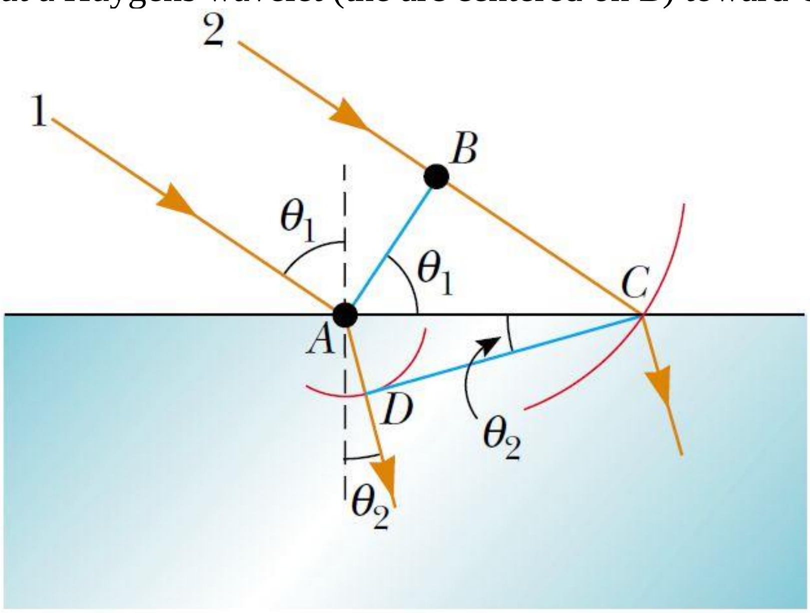


Fig.2

Because these two wavelets travel through different media, the radii of the wavelets are different. The radius of the wavelet from *A* is $AD = v_2 \Delta t$, where v_2 is the wave speed in the second medium. The radius of the wavelet from *B* is $BC = v_1 \Delta t$, where v_1 is the wave speed in the original medium.
From triangles *ABC* and *ADC*, we find that

$$\sin\theta_1 = \frac{BC}{AC} = \frac{v_1\,\Delta t}{AC} \quad \text{and} \quad \sin\theta_2 = \frac{AD}{AC} = \frac{v_2\,\Delta t}{AC}$$

If we divide the first equation by the second, we obtain

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

But from Equation 35.4 we know that $v_1 = c/n_1$ and $v_2 = c/n_2$. Therefore,

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

which is Snell's law of refraction.

Ray treatment of reflection and refraction

It is possible to analyze the passage of tight through any optical system by successive applications of Huygens' principle to the wave fronts as was done in the preceding section at a single surface However, it is often much simpler to trace a few rays through the system. The wave front, if desired, may then be constructed perpendicular to the rays.

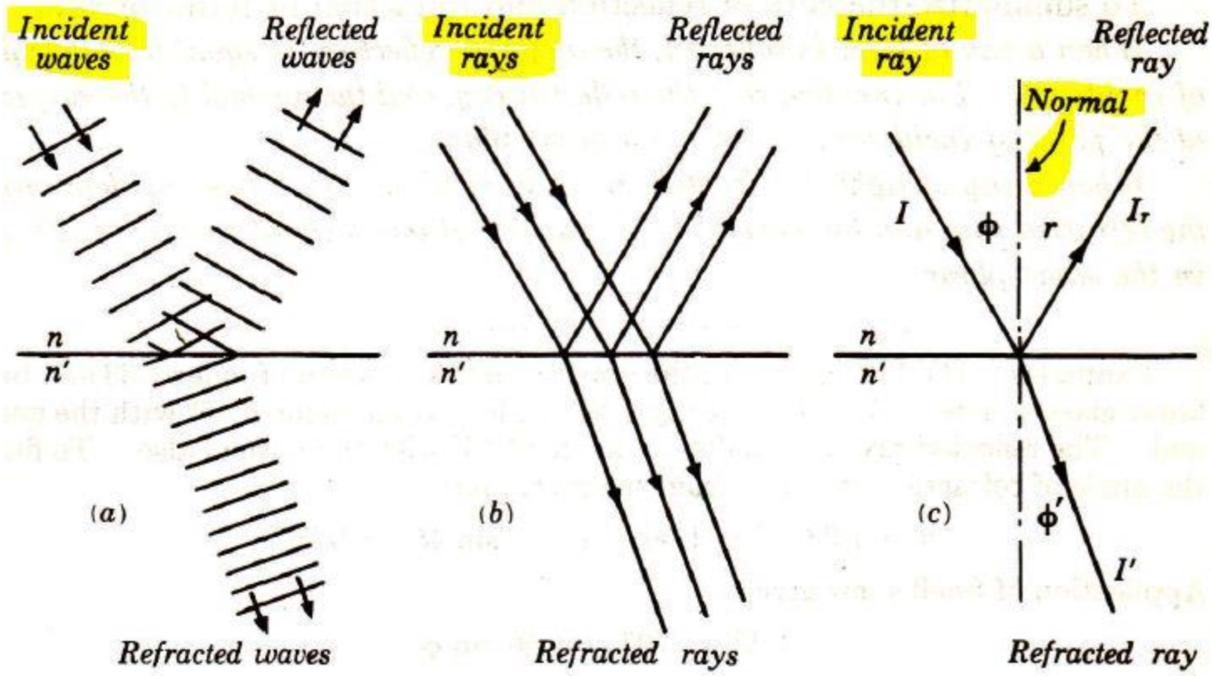


Fig.B : A train of plane waves is in part reflected and in part refracted at the boundary between two media. (b) The waves in (a) are represented by rays. (c) For simplicity, only a single incident, reflected, and refracted ray are drawn.

To summarize the laws of reflection and refraction in terms of rays: when a ray of light Is reflected the angle of reflection is equal to the angle of incidence. The incident ray and the normal of surface at the point of incidence, all lie in the same plane.

When a ray of light is reflected $n \sin\phi = n' \sin\phi'$. The incident ray, the refracted ray and the normal to the surface at the point of incidence all lie in the same plane.

Fermat's Principle

Pierre de Fermat (1601–1665) developed a general principle that can be used to determine the path that light follows as it travels from one point to another. Fermat's principle states that when a light ray travels between any two points, its path is the one that requires the smallest time interval

Let us illustrate how Fermat's principle can be used to derive Snell's law of refraction. Suppose that a light ray is to travel from point *P* in medium 1 to point *Q* in medium 2 (Fig. 3),

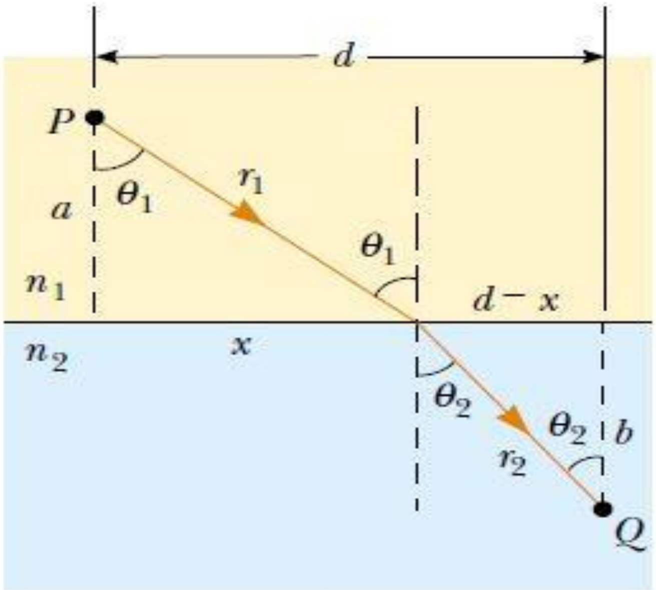


Fig.3

where *P* and *Q* are at perpendicular distances *a* and *b*, respectively, from the interface. The speed of light is c/n_1 in medium 1 and c/n_2 in medium 2. Using the geometry of Figure 3, and assuming that light leaves *P* at $t = 0$, we see that the time at which the ray arrives at *Q* is

$$t = \frac{r_1}{v_1} + \frac{r_2}{v_2} = \frac{\sqrt{a^2 + x^2}}{c/n_1} + \frac{\sqrt{b^2 + (d-x)^2}}{c/n_2}$$

To obtain the value of *x* for which *t* has its minimum value, we take the derivative of *t* with respect to *x* and set the derivative equal to zero:

$$\frac{dt}{dx} = \frac{n_1}{c}\frac{d}{dx}\sqrt{a^2 + x^2} + \frac{n_2}{c}\frac{d}{dx}\sqrt{b^2 + (d-x)^2}$$

$$= \frac{n_1}{c}\left(\tfrac{1}{2}\right)\frac{2x}{(a^2 + x^2)^{1/2}} + \frac{n_2}{c}\left(\tfrac{1}{2}\right)\frac{2(d-x)(-1)}{[b^2 + (d-x)^2]^{1/2}}$$

$$= \frac{n_1 x}{c(a^2 + x^2)^{1/2}} - \frac{n_2(d-x)}{c[b^2 + (d-x)^2]^{1/2}} = 0$$

Or

$$\frac{n_1 x}{(a^2 + x^2)^{1/2}} = \frac{n_2(d-x)}{[b^2 + (d-x)^2]^{1/2}}$$

………. (1)

From Figure 3,

$$\sin\theta_1 = \frac{x}{(a^2 + x^2)^{1/2}} \qquad \sin\theta_2 = \frac{d-x}{[b^2 + (d-x)^2]^{1/2}}$$

Substituting these expressions into Equation 1, we find that

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

which is Snell's law of refraction

Optical Path

To derive one of the most fundamental principles in geometric optics, it is appropriate to define a quantity called the *optical path*. The path d of a ray of light in any medium is given by the product *velocity* times *time*:

$$d = vt$$

Since by definition $n = c/v$, which gives $v = c/n$, we can write

$$d = (c/n)\, t \quad \text{or} \quad nd = ct$$

The product nd is called the *optical path* Δ:

$$\Delta = nd$$

The optical path represents the distance light travels in a vacuum in the same time it travels a distance d in the medium. If a light ray travels through a series of optical media of thickness d, d', d'', ... and refractive indices n, n', n'', ... , the total optical path is just the sum of the separate values:

$$A = nd + n'd' + n''d'' + ...$$