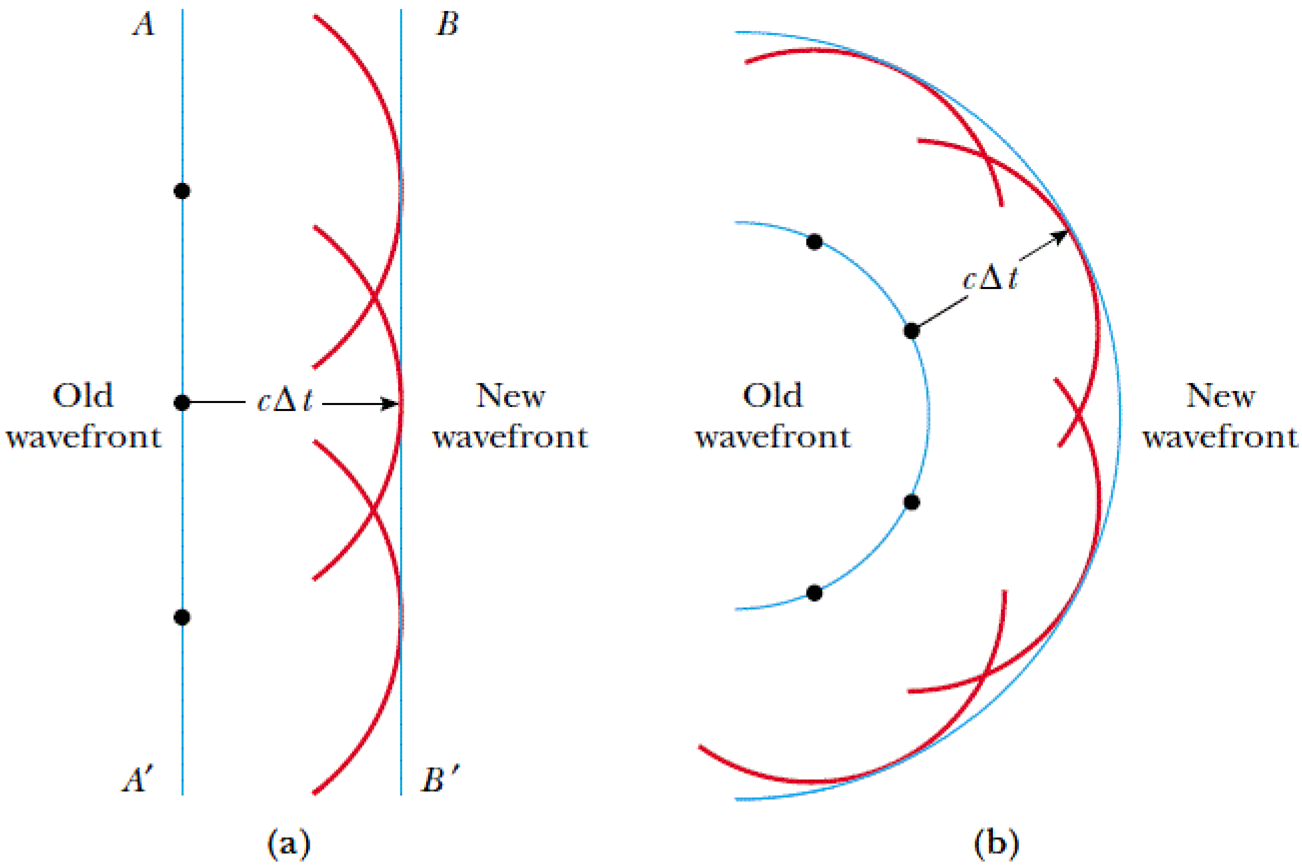


Figure 1- Huygens's construction for (a) a plane wave propagating to the right and (b) a spherical wave propagating to the right.

Index of Refraction

In general, the speed of light in any material is less than its speed in vacuum. In fact *light travels at its maximum speed in vacuum*. It is convenient to define the index of refraction n of a medium to be the ratio

$$n \equiv \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v} \qquad \text{............ (1)}$$

From this definition, we see that the index of refraction is a dimensionless number greater than unity because v is always less than c. Furthermore, n is equal to unity for vacuum.

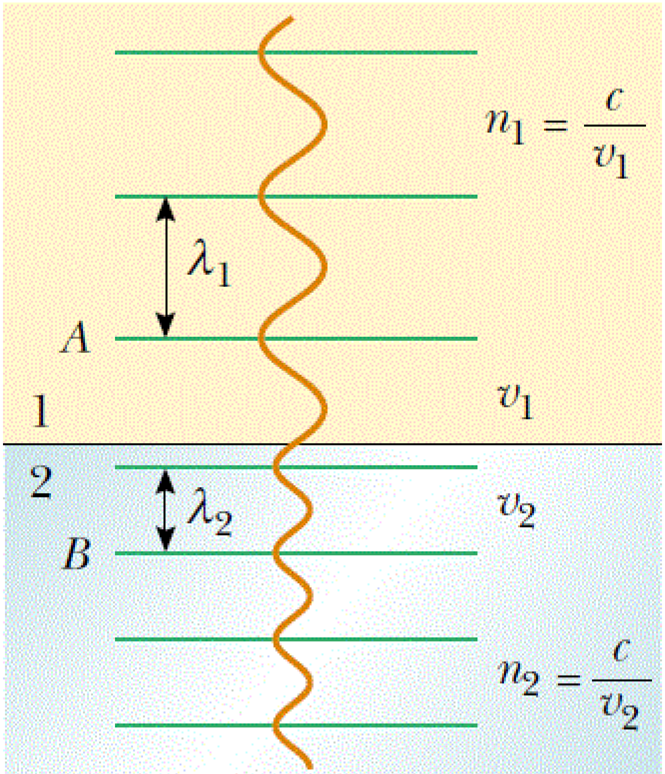


Fig.2

As light travels from one medium to another, its frequency does not change but its wavelength does. To see why this is so, consider Figure 2. Waves pass an observer at point *A* in medium 1 with a certain frequency and are incident on the boundary between medium 1 and medium 2. The frequency with which the waves pass an observer at point *B* in medium 2 must equal the frequency at which they pass point *A*. If this were not the case, then energy would be piling up at the boundary. Because there is no mechanism for this to occur, the frequency must be a constant as a light ray passes from one medium into another. Therefore, because the relationship $v = f\lambda$ must be valid in both media and because $f_1 = f_2 = f$, we see that

$$v_1 = f\lambda_1 \quad \text{and} \quad v_2 = f\lambda_2 \quad \ldots\ldots\ldots\ldots\ldots 2$$

Because $v_1 = v_2$, it follows that $\lambda_1 \neq \lambda_2$

We can obtain a relationship between index of refraction and wavelength by dividing the first Equation 2 by the second and then using Equation 1:

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots 3$$

This gives

$$\lambda_1 n_1 = \lambda_2 n_2$$

If medium 1 is vacuum, or for all practical purposes air, then $n_1 = 1$. Hence, it follows from Equation 3 that the index of refraction of any medium can be expressed as the ratio

$$n = \frac{\lambda}{\lambda_n} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 4$$

where λ is the wavelength of light in vacuum and λ_n is the wavelength of light in the medium whose index of refraction is n. From Equation 4, we see that because $n > 1$, $\lambda_n < \lambda$

We are now in a position to express Equation 35.3 in an alternative form.
If we replace the v_2/v_1 term with n_1/n_2 from Equation 3, we Obtain

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

The experimental discovery of this relationship is usually credited to Willebrord Snell (1591–1627) and is therefore known as Snell's law of refraction.

The Electromagnetic Spectrum:

There is no difference between light wave and other electromagnetic waves such as those from an oscillating electrical circuit.
There is no limit to the longest electromagnetic wave that can be produced .By operating an A.C. generator sufficiently slowly, the frequency f may be made as small as desired .The wave length of the waves radiated by a 60-cycle transmission line is

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{60} = 5\times10^4 = 5000 \quad Km$$

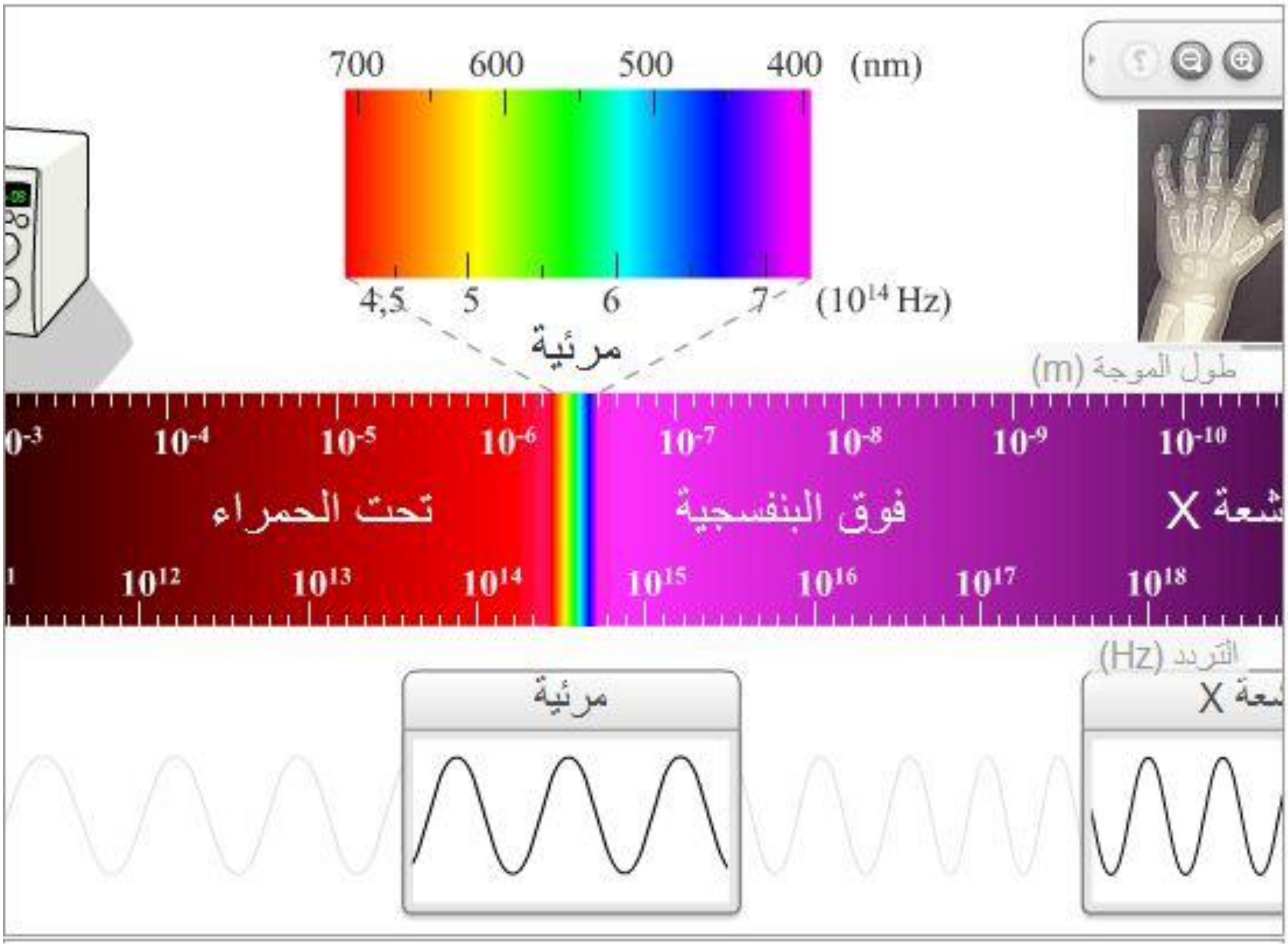
By increasing the generator speed and the number of poles, the frequency may be increased up to about 100,000cycles/sec , corresponding to a wave length of 3Km, but mechanical difficulties set a limit to this method. Higher frequencies may be developed by oscillating electrical circuits, the frequency of such an oscillation being given by

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

where L and C are respectively the inductance and capacitance of the circuit.
The waves from light sources are emitted by molecules and atoms and extend from the relatively long infrared waves through the visible spectrum and into the ultraviolet.
The deceleration of the bombarding electrons also gives raise to electromagnetic radiation. Wave produced in this way are called X-rays, and their wave lengths extend

from about $10^{-5}\,cm$ to about $10^{-10}\,cm$. The only limit at the short wave length end seems to be set by the difficulties of obtaining high speed electrons for the bombarding process. Wave of even short wave length a company the spontaneous disruptions of atomic nuclei in the processes of radioactive disintegration. These waves are called gamma rays.

A chart of the electromagnetic spectrum is given in Fig.3



Velocity of light

Electromagnetic theory predicts that the velocity of electromagnetic wave in free space is given by

$$c = \sqrt{\frac{1}{\mu_\circ \varepsilon_\circ}}$$

Where ,by definition magnetic permeability

$$\mu_\circ = 4\pi \times 10^{-7}\, newton - \sec^2 / coul^2$$

And the electric permittivity $\varepsilon_\circ = (\frac{1}{4\pi} \times 8.9875 \times 10^9) coul^2 / newton - m^2$

Hence from the equation above

$$c = \sqrt{\frac{4\pi \times 8.9875 \times 10^{9}}{4\pi \times 10^{-7}} \times \frac{newton - m^{2}}{coul^{2}} \times \frac{coul^{2}}{newton - \sec^{2}}}$$

$$c = 2.9979 \times 10^{8}\, m/\sec$$

Example: (a) What is the frequency of light waves of wave length 500 mμ .
(b) What is the frequency of X-rays of wave length 1A ?

Solution (a) $c = \upsilon\lambda \qquad \upsilon = \frac{c}{\lambda}$

$$\upsilon = \frac{3 \times 10^{8}}{500 \times 10^{-9}} = 6 \times 10^{14}\, Hz$$

Solution (b) $\upsilon = \frac{c}{\lambda} \qquad \upsilon = \frac{3 \times 10^{8}}{1 \times 10^{-10}} = 3 \times 10^{18} \quad Hz$

Example: What is the velocity of light of wave length 500 mμ in glass whose index at this wave length is 1.5? (b) What is the wave length of these waves in the glass?

Solution (a)

$$\upsilon = \frac{c}{\lambda} = \frac{3 \times 10^{8}}{1.5} = 2 \times 10^{8}\, m/\sec$$

Solution (b)

$$n = \frac{c}{\upsilon} = \frac{\nu\lambda}{\nu\lambda'}$$

Is the wavelength in the vacuum λ ,
Light frequency. ν
is the wave length in glass, λ'

$$n = \frac{\lambda}{\lambda'}$$

$$\lambda' = \frac{\lambda}{n} = \frac{500}{1.5} = 333.33 m\mu$$

Example : Use Snell’s law to solve for the sine of the angle of refraction.
$n_1 = 1, \qquad n_2 = 1.52, \qquad \theta_1 = 30^{\circ}$ Substitute

Solution

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$\sin \theta_2 = \left(\frac{n_1}{n_2}\right) \sin \theta_1$$
$$\theta_2 = \sin^{-1}\left(\left(\frac{n_1}{n_2}\right) \sin \theta_1\right)$$
$$= \sin^{-1}\left(\left(\frac{1.00}{1.52}\right) \sin 30.0^\circ\right)$$
$$= 19.2^\circ$$

Example Calculate the difference between the speed of light in kilometers per second in a vacuum and the speed of light in air if the refractive index of air is 1.0002340 .

Solution:

$$c = 3\times10^5\ Km/\sec$$
$$c = \upsilon\lambda \qquad \upsilon = \frac{c}{\lambda} = \frac{3\times10^5}{1.0002340} = 2.999298\times10^5\ km/\sec$$
$$\Delta = c - \upsilon = 70.184\ Km/\sec$$

Example: If the moon's distance from the earth is 3.840 x 10^5 km, how long will it take microwaves to travel from the earth to the moon and back again?

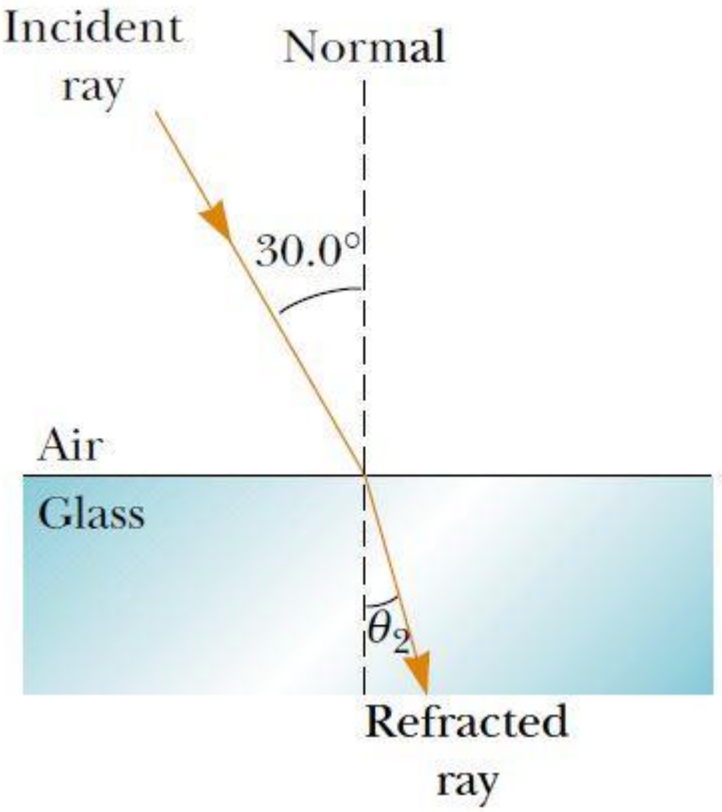
Solution: $$t = \frac{y}{c} = \frac{2\times3.840\times10^5}{3\times10^5} = 2.56\sec$$

Example: How long does it take light from the sun to reach the earth? Assume the earth's distance from the sun to be 1.50 x10^8 km.

Solution : $$t = \frac{y}{c} = \frac{1.5\times10^8}{3\times10^5} = 500\sec = 8\min \quad 20\sec$$

Example: A light ray of wavelength 589 nm traveling through air (n_1 = 1) is incident on a smooth, flat slab of crown glass(n_2 = 1.52 for crown glass) at an angle of 30° to the normal, as sketched in Figure below . Find the angle of refraction.

Solution :



We rearrange Snell's law of refraction to obtain

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

$$\sin\theta_2 = \left(\frac{1}{1.52}\right)\sin 30° = 0.329$$

$$\theta_2 = \sin^{-1}(0.329) = \boxed{19.2°}$$

Example: A beam of light of wavelength 550 nm traveling in air is incident on a slab of transparent material. The incident beam makes an angle of 40° with the normal, and the refracted beam makes an angle of 26° with the normal. Find the index of refraction of the material.

Solution:

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

$$n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2} = (1)\frac{\sin 40°}{\sin 26°}$$

$$= \frac{0.643}{0.438} = \boxed{1.47}$$

Example; A laser in a compact disc player generates light that has a wavelength of 780 nm in air.

(A) Find the speed of this light once it enters the plastic of a compact disc (n=1.55).

(B) What is the wavelength of this light in the plastic?

Solution (A) We expect to find a value less than 3x 10^8 m/s because n> 1. We can obtain the speed of light in the plastic by using Equation

$$v = \frac{c}{n} = \frac{3 \times 10^8\ \text{m/s}}{1.55}$$

$$v = \boxed{1.94 \times 10^8\ \text{m/s}}$$

Solution (B)

$$\lambda_n = \frac{\lambda}{n} = \frac{780\ \text{nm}}{1.55} = \boxed{503\ \text{nm}}$$