CHAPTER ONE: DIGITAL SYSTEMS AND BINARY NUMBERS

Introduction:

A digital system is a combination of devices {mechanical, electrical, photo electronic,…,etc.} arranged to perform certain functions in which quantities are represented digitally.

Digital systems are used in communication, business transactions, traffic control, spacecraft guidance, medical treatment, weather monitoring, the Internet, and many other commercial, industrial, and scientific enterprises.

Number Systems and Number-Base Conversions:

1. Decimal Number Systems

It is said to be of base (10) since it uses 10 digits {0, 1, 2, 3, ……, 9}.

Ex. 1: $[7392]_{10} = 2\times10^{0}+9\times10^{1}+3\times10^{2}+7\times10^{3} = 7392.$

Ex. 2: $[0.421]_{10} = 0\times10^{0}+4\times10^{-1}+2\times10^{-2}+1\times10^{-3} = 0.421.$

In General

For any numbers:

$$a_n a_{n-1}....a_1 a_0.a_{-1}a_{-2}.....a_m = a_0\times N^0 + a_1\times N^1 + a_{n-1}\times N^{n-1} + a_n\times N^n + a_{-1}\times N^{-1} + a_{-2}\times N^{-2} + a_m\times N^m$$

Where; N is the base of the system.

2. Binary Number Systems

It is said to be of base (2) since it uses 2 digits {0, 1}.

Binary to Decimal Conversion

Ex. : $[11010.11]_2 = 0\times2^{0}+1\times2^{1}+0\times2^{2}+1\times2^{3}+1\times2^{4}+1\times2^{-1}+1\times2^{-2} = (26.75)_{10}.$

In General

To convert r – base number system to decimal number:

$$a_2 a_1 a_0.a_{-1}a_{-2}a_{-3} = a_0\times r^0 + a_1\times r^1 + a_2\times r^2 + a_{-1}\times r^{-1} + a_{-2}\times r^{-2} + a_{-3}\times r^{-3}$$

Ex. : $[4021.2]_5 = 1\times5^0+2\times5^1+0\times5^2+4\times5^3+2\times5^{-1}=(511.4)_{10}$.

Decimal to Binary Conversion

Ex. 1: Convert the decimal number [41] to binary number

The arithmetic process can be manipulated more conveniently as follows:

Integer	Remainder
41÷ 2	
20 ÷ 2	1
10 ÷ 2	0
5 ÷ 2	0
2 ÷ 2	1
1	0
	1

$(101001)_2$ = answer

So, $[41]_{10} = (101001)_2$

Ex. 2: Convert the decimal number [27.15] to binary number

The arithmetic process can be manipulated more conveniently as follows:

Integer	Remainder
27÷ 2	
13 ÷ 2	1
6 ÷ 2	1
3 ÷ 2	0
1	1
	1

$(11011)_2$ = answer

$[27]_{10} = (11011)_2$

Fraction	Coefficient
0.15×2= 0.3	0
0.3 × 2=0.6	0
0.6 × 2=1.2	1
1.2 × 2=0.4	0
0.4 × 2=0.8	0
0.8 × 2=1.6	1
1.6 × 2=1.2	1

$(0.0010011)_2$ = answer

$[0.15]_{10} = (0.0010011)_2$

So, $[27.15]_{10} = (11011.0010011)_2$

Note: Conversion from decimal integers to any base-*r* system is similar to this example, except that division is done by *r* instead of 2.

Ex. : Convert the decimal number [22.5] to 4 base number system

The arithmetic process can be manipulated more conveniently as follows:

Integer	Remainder
22÷ 4	
5 ÷ 4	2
1	1
	1

$(112)_4$ = answer

$[22]_{10} = (112)_4$

Fraction	Coefficient
0.5×4= 2.0	2
2.0 × 4=0.0	0

$(0.20)_4$ = answer

$[0.5]_{10} = (0.2)_4$

So, $[22.5]_{10} = (112.2)_4$

H.W: Convert the following numbers:

1. $(645.34)_{10} \longrightarrow (\ )_2$
2. $(153.531)_{10} \longrightarrow (\ )_6$
3. $(11101.1101)_2 \longrightarrow (\ )_{10}$

3. Octal Number Systems

It is said to be of base (8) since it uses 8 digits {0, 1, 2,…..,7}.

Ex. 1: $[231]_8 = 1\times8^0 + 3\times8^1 + 2\times8^2 = (153)_{10}$.

Ex. 2: Convert the decimal number [245.5] to octal number system

The arithmetic process can be manipulated more conveniently as follows:

Integer	Remainder
245 ÷ 8	
30 ÷ 8	5
3	6
	3

←

$(365)_8$ = answer

$[245]_{10} = (365)_8$

Fraction	Coefficient
0.5×8= 4.0	4
4.0 × 8=0.0	0

→

$(0.40)_8$ = answer

$[0.5]_{10} = (0.4)_4$

So, $[245.5]_{10} = (365.4)_8$

4. Hexadecimal Number Systems

It is said to be of base (16) since it uses 16 digits {0, 1, 2,…..,9, A, B, C, D, E, F}.

Ex. 1 : $[2C.4A]_{16} = 12\times16^0 + 2\times16^1 + 4\times16^{-1} + 10\times16^{-2} = (44.2)_{10}$.

Ex. 2 : Convert the decimal number [165.25] to hexadecimal number system

The arithmetic process can be manipulated more conveniently as follows:

Integer	Remainder
165 ÷ 16	
(10) A	5
	A

←

$(A5)_{16}$ = answer

$[165]_{10} = (A5)_{16}$

Fraction	Coefficient
$0.25 \times 16 = 4.0$	4
$4.0 \times 16 = 0.0$	0

$\overrightarrow{(0.40)}_{16}$ = answer

$[0.25]_{10} = (0.4)_{16}$

So, $[165.25]_{10} = (A5.4)_{16}$

H.W 1: Convert the following numbers:

1. $(33.22)_4 \longrightarrow (\ )_8$
2. $(BA.C)_{16} \longrightarrow (\ )_7$

H.W 2: In base [13], list the numbers between (4 and 40).

Conversion between Binary & Octal systems

The conversion between Binary and Octal is accomplished by partitioning the binary number into groups of three digits.

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Ex. 1: Convert the octal number [63.4] to binary number system

$(63.4)_8 = (110011.100)_2$

Ex. 2: Convert the binary number (1011011.11011) to octal system

$(001011011.110110)_2 = (133.66)_8$

Conversion between Binary & Hexadecimal systems

The conversion between Binary and Hexadecimal is accomplished by partitioning the binary number into groups of four digits.

Hexadecimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Ex. 1: Convert the Hexadecimal number [F67.19] to binary number system

$(F67.19)_{16} = (111101100111.00011001)_2$

Ex. 2: Convert the binary number (10100111011.0110101) to hexadecimal system

$(010100111011.01101010)_2 = (53B.6A)_{16}$

H.W: Convert the following numbers:

1. $(11011.1001)_2 \longrightarrow (\ )_8$
2. $(DF3.C5)_{16} \longrightarrow (\ )_2$

Arithmetic operations

Binary Arithmetic

1) Addition operation

0 + 0 = 0

0 + 1 = 1

1 + 0 = 1

1 + 1 = 0; and carry 1 to the next column.

Ex. : Add; $(1011.01 + 101.101)_2$

1011.010 +
0101.101

$(10000.111)_2$

2) Subtraction operation

0 – 0 = 0
0 – 1 = 1; and barrow 1 from the next column.
1 – 0 = 1
1 – 1 = 0

Ex. : Subtract; $(1000.01 - 11.001)_2$

1000.010 –
0011.001

$(101.001)_2$

3) Multiplication operation

$0 \times 0 = 0$
$0 \times 1 = 0$
$1 \times 0 = 0$
$1 \times 1 = 1$

Ex. : Multiply; $(11.01 \times 1.01)_2$

1101 ×
101

1101 +
00000
110100

$(100.0001)_2$

4) Devision operation

$0 \div 0$ = undefined
$1 \div 0$ = undefined
$0 \div 1 = 0$
$1 \div 1 = 1$

Ex. : Divide; $(1101.01 \div 10)_2$

```
 110.101
 ________
 110101  | 10
 10   -   -----
 ______
   10
   10  -
 ______
    0 10
      10  -
 ______
     0 10
       10  -
 ______
       00
```

So, $1101.01 \div 10 = (110.101)_2$

R – Base Arithmetic

Ex. : Evaluate the following:

1. $(42.51 + 15.3)_8$ & $(42.51 - 15.3)_8$

42.51	42.51
+	–
15.30	15.30
$(60.01)_8$	$(25.21)_8$

2. $(B3 + 4D)_{16}$ & $(B3 - 4D)_{16}$

B3	B3
+	–
4D	4D
$(100)_{16}$	$(66)_{16}$

H.W: Perform the following operations

1. $(50.27)_9 \div (15.28)_9$
2. $(44.56)_7 + (12.5)_6$
3. $(B3)_{13} - (6.55)_{13}$
4. $(33.24)_5 \times (14.21)_5$

Complements of Numbers

In digital systems, the complements are used to simplify the subtraction operation. There are two types of complements for r – base system:

- r 's complement.
- (r – 1)'s complement.

In binary system, there are 2's complement and 1's complement which represent the negative form of binary number.

1. The first complement (1's) are changed zeros to ones and ones to zeros.

Ex. : $(01001.1101)_2 \xrightarrow{1's} (10110.0010)_2$

2. The second complement (2's) can be either leaving least significant zeros and ones digit unchanged then replacing 1's to 0's and 0's to 1's; or by forming 1's complement and adding {1} for the least significant bit.

Ex. : $(110101)_2 \xrightarrow{2's} (\ )_{2's}$

$(110101)_2 \xrightarrow{1's} (001010)_{1's} + 1 = (001011)_{2's}$

Subtraction using Complements

In digital computer, if the subtraction implemented, we use the complements and addition as shown:

1) Convert the second number using 1's or 2's complement.
2) Replace the subtraction operation to addition operation.

Ex. 1: Perform the following operation using 2's complement.
$(1010100)_2 – (1000100)_2$

$(1000100)_2 \xrightarrow{1's} (0111011)_{1's} + 1 = (0111100)_{2's}$

1010100
+
0111100

1 0010000 ….. $(0010000)_2$

Ignored

Ex. 2: Perform the following operation using 2's complement.
$(1010011.01)_2 - (0101100.10)_2$

$(0101100.10)_2 \xrightarrow{1's} (1010011.01)_{1's} + 1 = (1010011.10)_{2's}$

$$\begin{array}{r} 1010011.01 \\ + \quad 1010011.10 \\ \hline 1\;0100110.11 \end{array} \quad \ldots\ldots (0100110.11)_2$$

Ignored

Ex. 3: Perform the following operation using 1's complement.
$(1010111)_2 - (0110110)_2$

$(0110110)_2 \xrightarrow{1's} (1001001)_{1's}$

$$\begin{array}{r} 1010111 \\ + \quad 1001001 \\ \hline 1\;0100000 \\ + \quad 1 \\ \hline (100001)_2 \end{array}$$

Ex. 4: Find the 12's complement and 13's complement to the number $[B65.5C]_{13}$.

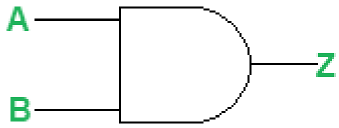
$$\begin{array}{r} CCC.CC \\ - \quad B65.5C \\ \hline (167.70)_{12's} \end{array} \quad + 1 = (167.71)_{13's}$$

Binary Logic Gates

1. AND Gate:

A	B	Z
0	0	0
0	1	0
1	0	0
1	1	1

AND Gate Truth Table

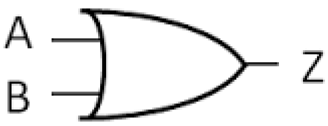


$Z = A \,.\, B$

2. OR Gate:

A	B	Z
0	0	0
0	1	1
1	0	1
1	1	1

OR Gate Truth Table

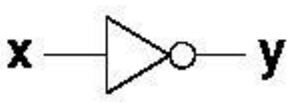


$Z = A + B$

3. NOT Gate:

x	y
0	1
1	0

NOT Gate Truth Table

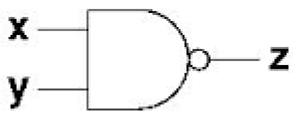


$y = \overline{x}$

4. NAND Gate:

x	y	Z
0	0	1
0	1	1
1	0	1
1	1	0

NAND Gate Truth Table

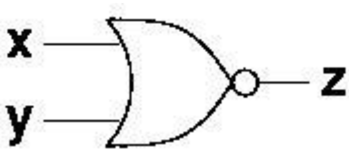


$z = \overline{x \,.\, y}$

5. NOR Gate:

x	y	z
0	0	1
0	1	0
1	0	0
1	1	0

NOR Gate Truth Table

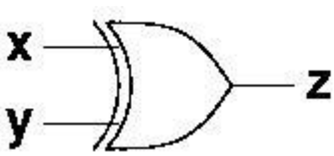


$z = \overline{x + y}$

6. Exclusive OR (Ex – OR) Gate:

x	Y	z
0	0	0
0	1	1
1	0	1
1	1	0

Ex - OR Gate Truth Table

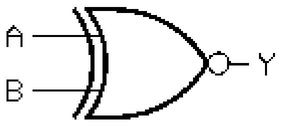


$z = x'y + xy' = x \oplus y$

7. Exclusive NOR (Ex – NOR) Gate:

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Ex - NOR Gate Truth Table



$$Y = AB + A'B' = \overline{A \oplus B}$$