$$\therefore M_s(f) = f_s \sum_{n=-\infty}^{\infty} M(f) * \delta(f - nf_s)$$

$$\therefore \boxed{M_s(f) = f_s \sum_{n=-\infty}^{\infty} M(f - nf_s)}$$

* It is shown in the previous figure, $M_s(f)$ will be repeated periodically without overlap if:

$$f_s \geq 2f_m \quad \text{or} \quad T_s \leq \frac{1}{2f_m}$$

and $m(t)$ can be recovered by a Low-Pass Filtering of $m_s(t)$.

* If $m(t)$ is sampled at a rate less than (Nyquist rate) $\{f_s < 2f_m\}$, then the replicas of $M(f)$ overlap each other, and it is not possible to recover $m(t)$ from $m_s(t)$ by LPF. The distortion caused by the spectrum overlaping is called "Aliasing".

* Anti-aliasing filter: is a low-pass filter through which $m(t)$ is passed to limit its bandwidth (f_m) to values less than ($f_s/2$) to prevent the signal aliasing.

$$f_m < \frac{f_s}{2} \Rightarrow f_s > 2f_m .$$

2. Bandpass Sampling Theorem:

This theorem is applicable when bandpass (modulated) signals are to be sampled. It states that: [If a band-pass signal $m(t)$ has a spectrum of bandwidth $B_T = f_B$ and an upper frequency limit is (f_u) then $m(t)$ can be recovered from $m_s(t)$ by band-passing filter (BPF) if ($f_s = 2f_u/k$) where (k) is the largest integer not exceeding (f_u/f_B).

Example: Given that

$$m(t) = 10 \cos(2000\pi t) \cos(8000\pi t)$$

Find the sampling frequency (f_s) using:

a) Sampling theorem.

b) Bandpass sampling theorem.

Sol.

$$m(t) = \frac{10}{2}\left[\cos(8000\pi t - 2000\pi t) + \cos(8000\pi t + 2000\pi t)\right]$$

$$m(t) = 5\left[\cos(6000\pi t) + \cos(10000\pi t)\right]$$

a) $f_m = \dfrac{10000\pi}{2\pi} = 5\,kHz$

$f_s \geqslant 2 f_m \Rightarrow f_s \geqslant 10\,kHz$

Nyquist frequency $\boxed{f_N = 2 f_m = 10\,KHz}$

b) $f_u = f_m = 5\,kHz$

$$\frac{6000\pi}{2\pi}$$

$f_B = 5\,kHz - 3\,kHz = 2\,kHz$

$f_u / f_B = 2.5$

$\therefore\ k = \boxed{2}$

$\therefore\ f_s = 2 f_u / k \Rightarrow f_s = \dfrac{2 \times 5\,kHz}{2}$

$\therefore\ \boxed{f_s = 5\,kHz}$

* It is shown that the sample rate of the band-pass sampling is much smaller than that of the conventional sampling.
* Large sampling rate means high cost signal processing equipment.

3. Types of Sampling:

3.1 Ideal Sampling:

In this type of sampling, $m(t)$ is multiplied by a train of unit impulses ($\delta_{T_s}(t)$) which is given by:

$$\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

as discussed earlier in this chapter.

$$m_s(t) = m(t)\,\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\,\delta(t - nT_s)$$

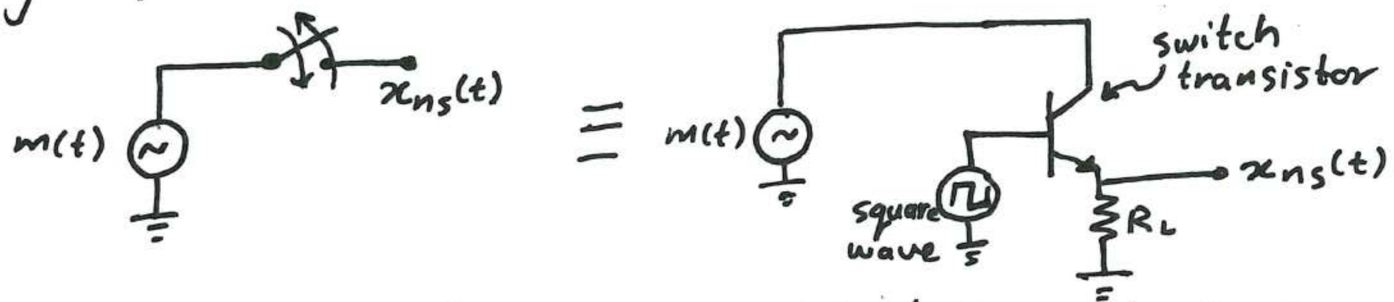
This signal is obtained using the property of :

$$x(t)\,\delta(t - t_o) = x(t_o)\,\delta(t - t_o)\ .$$

3.2 Practical Sampling:

$\boxed{A}$ Natural Sampling:

This type of sampling samples the signal $m(t)$ by using high-speed switching circuit as shown below:



The above circuit can be modeled theoretically by multiplying $m(t)$ by a train of gate function or (rectangular pulses) called $x_p(t)$.

$$x_{ns}(t) = m(t) \cdot x_p(t)$$

where $x_p(t) = \sum_{n=-\infty}^{\infty} \Pi\left(\frac{t - nT_s}{\tau}\right)$ with $\tau < T_s$

using Fourier series representation:

$$x_p(t) = \sum_{n=-\infty}^{\infty} D_n\, e^{+j2\pi(nf_s)t}$$

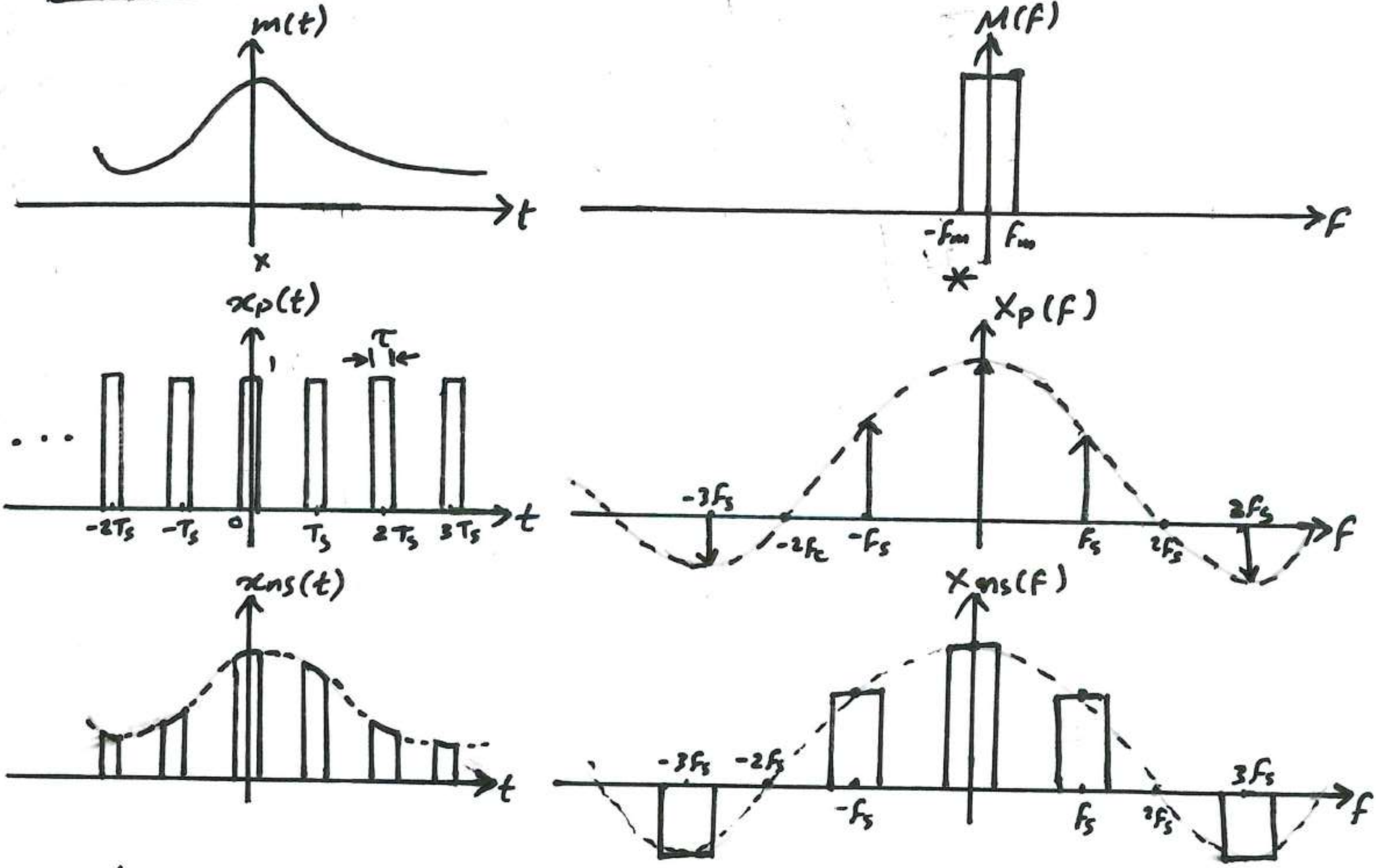
as calculated previously:

$$D_n = f_s \tau \,\mathrm{Sinc}(\tau n f_s)$$

$$x_{ns}(t) = m(t) \sum_{n=-\infty}^{\infty} D_n \, e^{-j2\pi (n f_s) t}$$

$$X_{ns}(f) = M(f) * \sum_{n=-\infty}^{\infty} D_n \, \delta(f - n f_s)$$

$$\boxed{X_{ns}(f) = \sum_{n=-\infty}^{\infty} D_n \, M(f - n f_s)}$$



<u>Notes:</u>

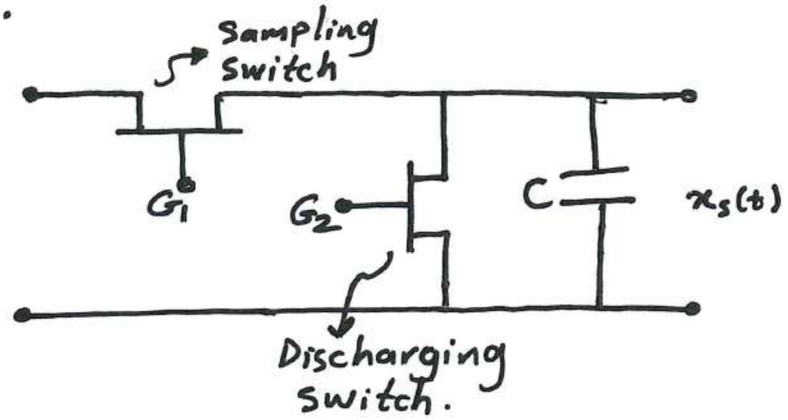
1. $X_{ns}(f)$ is weighted version of $M(f)$ centered on integer multiples of the sampling frequency (f_s).
2. If $f_s \geq 2 f_m$, then $m(t)$ can perfectly be recovered from $x_{ns}(t)$ by a low-pass filter (LPF)

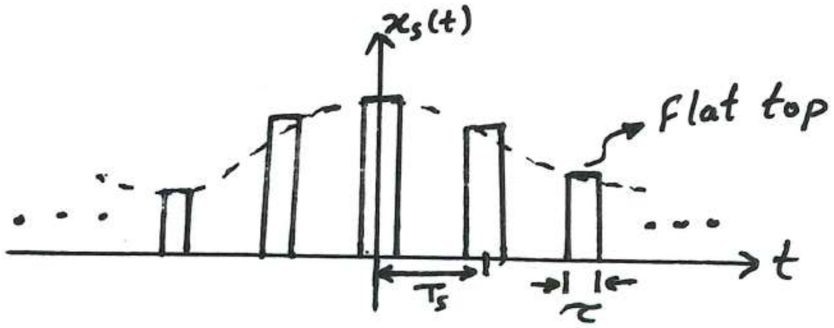
$$x_{ns}(t) \longrightarrow \boxed{\text{LPF}} \longrightarrow m(t)$$

$$f_m < f_0 \ll f_s$$

B Flat-Top Sampling:

It is performed by a sample and hold (S/H) circuit. This circuit produces flat-top pulses rather than covered-top pulses.





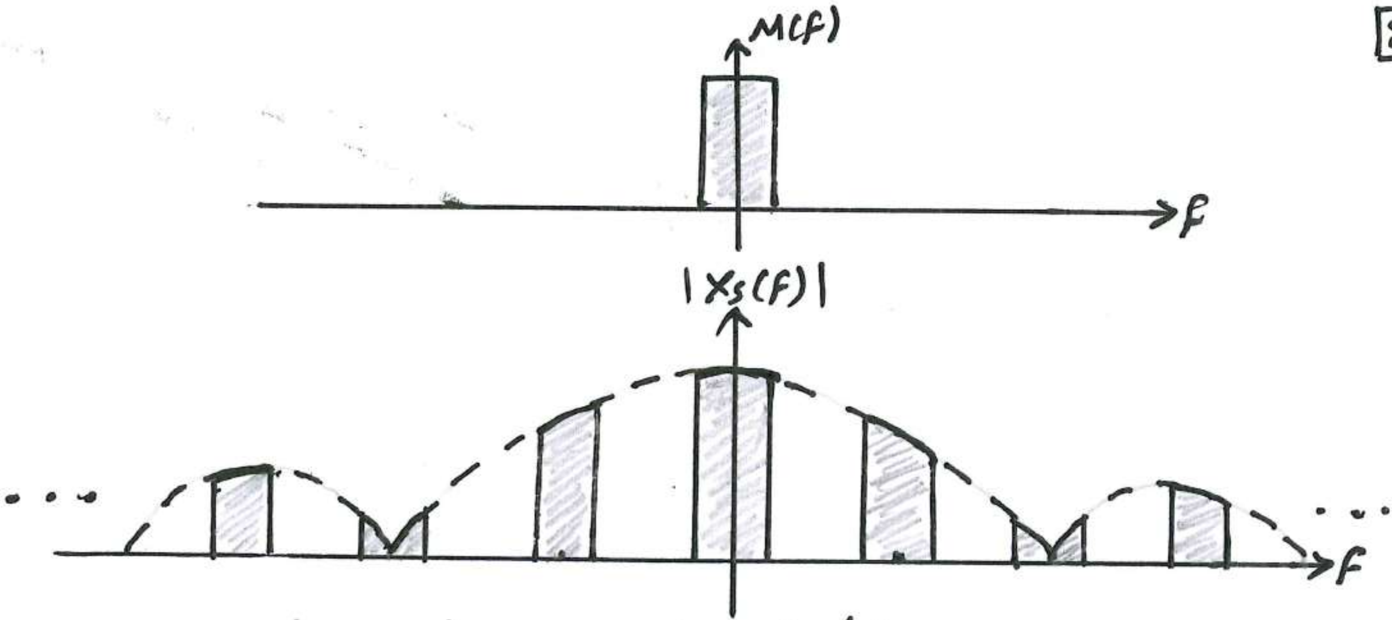
The (S/H) circuit consists of two FET switches and a capacitor. A gate pulse at G_1 closes the sampling switch, and the capacitor holds the sampled voltage until discharged by applying a pulse at G_2.

$$\boxed{x_s(t) = \Pi\left(\frac{t}{\tau}\right) * \left[m(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t-nT_s)\right]}$$

$$X_s(f) = \tau \operatorname{Sinc}(\tau f) \cdot \left[M(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s)\right]$$

$$\boxed{X_s(f) = \tau f_s \operatorname{Sinc}(\tau f) \cdot \sum_{n=-\infty}^{\infty} M(f-nf_s)}$$

* In this type of sampling the spectrum of $m(t)$ is repeated every (f_s) but it takes the shape of the sinc function as shown in the next page.



* The signal can be reconstructed by a (LPF) but the resulted signal is not exactly the same as the original one because of the frequency domain distortion caused by the sinc function.

4. Types of Analog Pulse Modulation:

In pulse modulation, the carrier is a train of pulses, and some-times parameters of the pulse train are varied in accordance with the instantaneous amplitude value of the baseband signal (information signal). The types of pulse modulation are:

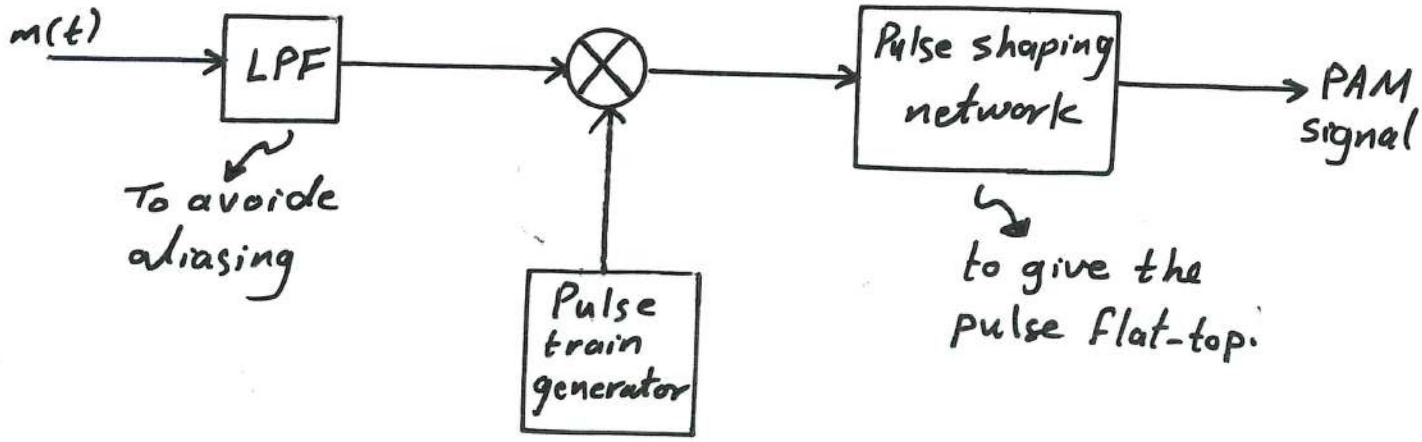
1. Pulse Amplitude Modulation (PAM)
2. Pulse Width Modulation (PWM)
3. Pulse Position Modulation (PPM)

This classification is suggested according to the pulse varying part. For example: in PAM, the amplitude of the pulse is varying in accordance with the message signal amplitude, and so on.

4.1 Pulse Amplitude Modulation (PAM):

In (PAM), the amplitude of the pulse train is varied in accordance with the information signal m(t). Simply, it is a flat-top sampling process, so it exactly follows the same process of the sampling generation and detection.

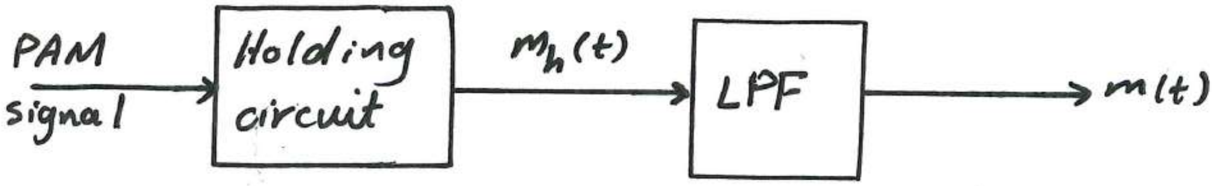
A Generation of PAM:



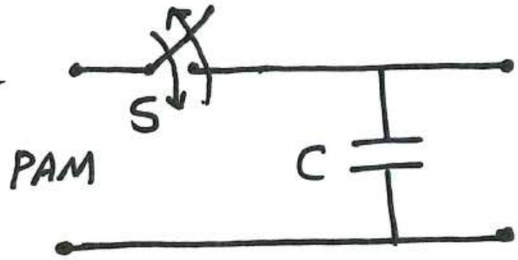
The above block diagram gives the details of PAM modulation. However, it can simply be generated using the (S/H) circuit explained previously.

B Demodulation of PAM:

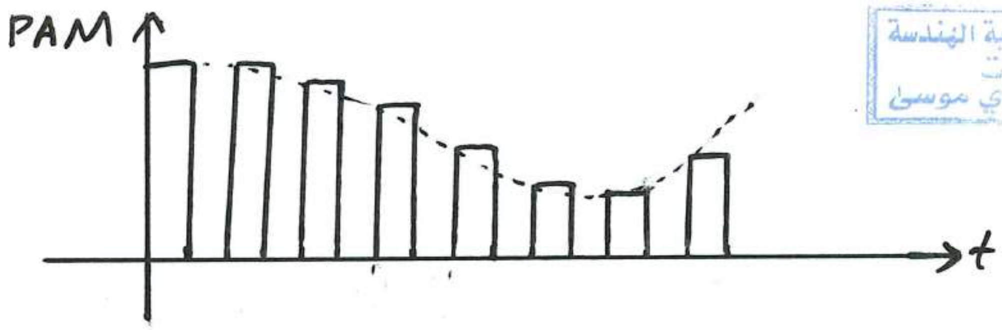
The message signal of PAM can be reconstructed by a LPF as mentioned earlier, but the flat-top sampling cause a distortion in the signal spectrum in the freq. domain. To reduce the effect of this distortion, holding circuit is being used in the detection of PAM modulation, as shown in the next figure.

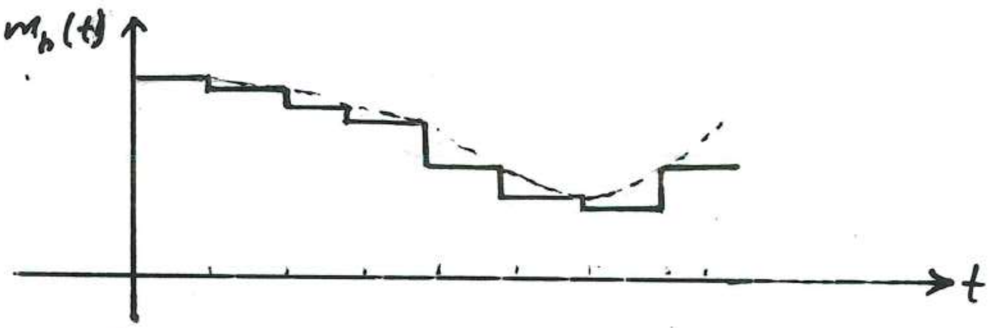


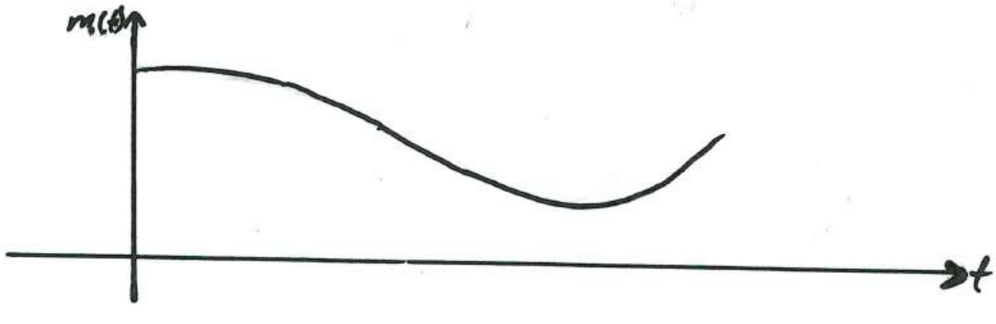
The holding circuit can be made as shown below using a switch and capacitor.



* The switch (S) is closed after the arrival of the pulse, and it is opened at the end of the pulse. Therefore, the capacitor (C) is charged up to the pulse amplitude when (S) is closed, and it holds this value when (S) is opened during the interval between two pulses. As a result, the detected signals are as shown below:







Notes:

1- The spectrum of PAM signal is a repeated version of the spectrum of m(t), so it has very wide BW ($B_T >> f_m$). Therefore, it is not used for transmission purposes.

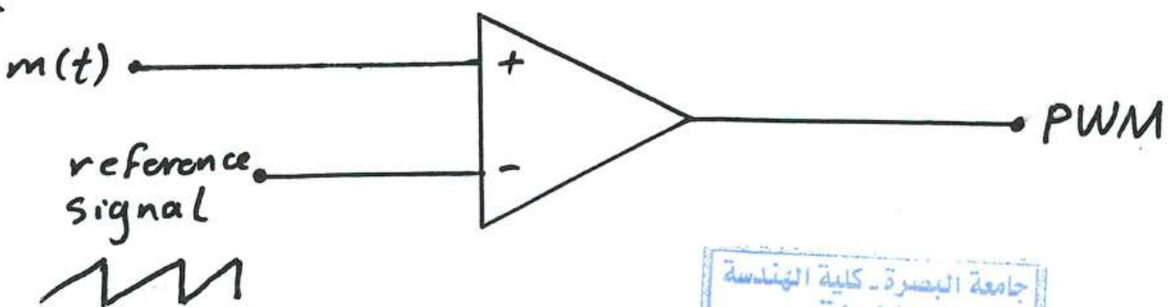
2- PAM is used as a first stage in converting the analog signals to digital signal (the 1st stage of the pulse code modulation (PCM) which will be studied in the next year in digital communications class).

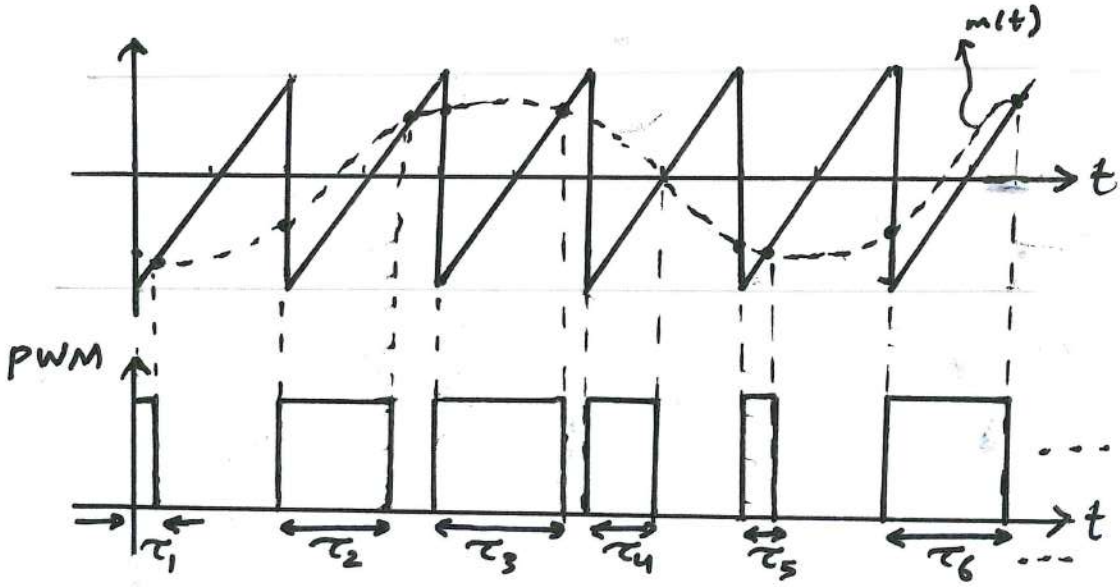
4.2 Pulse Width Modulation (PWM):

It is also known as Pulse Duration Modulation (PDM).
In (PWM), the amplitude of the pulse is kept constant, but the width of each pulse is made proportional to the amplitude of the message signal m(t).

[A] Generation of PWM:

PWM can simply be generated by a comparator that compares m(t) with a reference signal (usually a saw-tooth wave but not always).





* When $m(t) >$ Saw-tooth $\Rightarrow$ comparator o/p is logic (1).
  when $m(t) <$ saw-tooth $\Rightarrow$ comparator o/p is logic (0).

* It is clear that the width of each pulse is varied depending on the amplitude of $m(t)$.

* The transmission bandwidth of PWM is proportional to the $(1/\tau_{min})$. $\boxed{B_T \propto \frac{1}{\tau_{min}}}$ minimum value of pulse duration (τ).

B Demodulation of PWM:

The detection of PWM is achieved by converting it into PAM by an integrator then useing PAM demodulator to to get $m(t)$ back.