$$G'_{ns}(f) = (2\pi f)^2 |N_s(f)|^2$$

$$G'_{ns}(f) = (2\pi)^2 f^2 G_{ns}(f) \quad , \quad G_{ns}(f) = \eta \quad -B \leq f \leq B$$

$$\therefore N_o = \frac{1}{A_c^2} \overline{[n'_s(t)]^2}$$

$$\overline{[n'_s(t)]^2} = \int_{-\infty}^{\infty} G'_{ns}(f)\, df = \int_{-B}^{B} (2\pi)^2 f^2 \eta\, df$$

$$\overline{[n'_s(t)]^2} = 2(2\pi)^2 \eta \int_0^B f^2\, df = \frac{2}{3}(2\pi)^2 \eta \left. f^3 \right|_0^B$$

$$\overline{[n'_s(t)]^2} = \frac{2}{3}(2\pi)^2 \eta B^3$$

$$\therefore N_o = \frac{1}{A_c^2} \times \frac{2}{3}(2\pi)^2 \eta B^3 \quad , \quad B^3 = B \times B^2$$

$$\boxed{N_o = \frac{2}{3}\frac{\eta B}{A_c^2} \times (2\pi B)^2}$$

$$\therefore (S/N)_o = k_f^2 \overline{m^2(t)} \times \frac{3}{2}\frac{A_c^2}{\eta B (2\pi B)^2}$$

but $\frac{A_c^2}{2} = S_i$

$$(S/N)_o = k_f^2 \overline{m^2(t)} \times 3 \frac{S_i}{\eta B (2\pi B)^2}$$

$$\frac{S_i}{\eta B} = \gamma$$

$$\therefore \boxed{(S/N)_o = 3 \frac{k_f^2 \overline{m^2(t)}}{(2\pi B)^2} \gamma}$$

In term of the deviation ratio D:

$$D = \frac{\Delta f}{f_m} = \frac{\Delta f}{B}$$

$$\Delta f = \frac{1}{2\pi} k_f \left. m(t) \right|_{max} \quad , \quad m(t) \text{ is normalized signal}$$

$$\left. m(t) \right|_{max} = 1$$

$$\Delta f = \frac{k_f}{2\pi}$$

$$\therefore (S/N)_o = 3 \underbrace{\left(\frac{k_f}{2\pi B}\right)^2}_{D^2} \overline{m^2(t)}\, \gamma$$

$$\boxed{(S/N)_o = 3\, D^2\, \overline{m^2(t)}\, \gamma}$$

* It is noticeable that, the improvement of $(S/N)_o$ for FM modulation is unlimited since it depends on D and D can take any value.
* Do not forget that: increasing D requires larger BW, so it should not take very large values
* For single tone message signal ($m(t) = \sin(2\pi f_m t)$), just replace D by β (modulation index).

$$\boxed{(S/N)_o = 3\beta^2\, \overline{m^2(t)}\, \gamma} \quad \text{where } \overline{m^2(t)} = \frac{1}{2} \text{ watt}$$

* For noise dominating FM systems, the signal cannot be separated from the noise, so the information is lost.
* Similar to the linear modulation, angle modulation has the following threshold signal-to-noise ratio:

$$\boxed{(S/N)_{i\,th} = 10 \equiv 10\,dB}$$

$$(S/N)_i = \frac{S_i}{\eta B_T} \quad , \quad \gamma = \frac{S_i}{\eta B}$$

$B_T = 2(1+\beta) f_m$, but we have $f_m = B$

$$B_T = 2(1+\beta) B$$

$$\therefore (S/N)_i = \frac{S_i}{2(1+\beta)\eta B} = \underbrace{\frac{S_i}{\eta B}}_{\gamma} \cdot \frac{1}{2(1+\beta)}$$

$$\therefore \gamma = 2(1+\beta)\,(S/N)_i$$

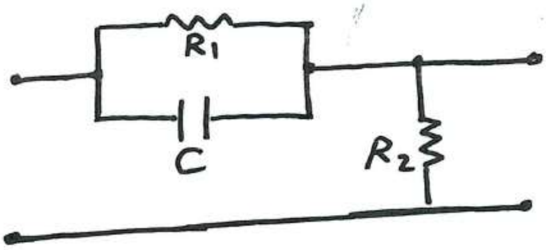
$$\therefore \gamma_{th} = 2(1+\beta)\,(S/N)_{i\,th} \Rightarrow \boxed{\gamma_{th} = 20(1+\beta)}$$

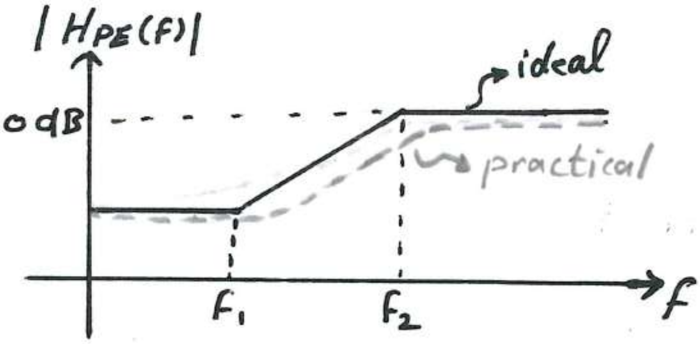
4. Preemphasis and Deemphasis in FM Broadcasting:

It is a technique used in commercial FM broadcasting to improve $(S/N)_o$. The high frequency components in the input signal is emphasized (increased) at the transmitter befor the noise is introduced. At the receiver, the high frequency components is de-emphasized (decreased) with the presence of noise.

After D.E filter, the original signal spectrum is restored to its original shape, but the noise is now reduced.

$\boxed{A}$ P.E Filter:





$$f_1 = \frac{1}{2\pi} \cdot \frac{1}{R_1 C} = \frac{1}{2\pi} \frac{1}{\tau_1}$$

$$f_2 = \frac{1}{2\pi} \frac{1}{R_2 C} = \frac{1}{2\pi} \frac{1}{\tau_2} \quad , \ R_1 >> R_2$$

$$H_{PE}(f) = \frac{R_2}{R_2 + (R_1 // j2\pi f C)} = \frac{R_2}{R_2 + \frac{R_1}{1 + j2\pi f R_1 C}}$$

$$H_{PE}(f) = \frac{R_2 (1 + j2\pi f R_1 C)}{R_1 + R_2 + j2\pi f R_1 R_2 C} = \frac{R_2}{R_1 + R_2} \cdot \frac{1 + j2\pi f R_1 C}{1 + j2\pi f \frac{R_1 R_2}{R_1 + R_2} C}$$

where $R_1 C = \tau_1 = \frac{1}{2\pi f_1}$

$$\frac{R_1 R_2}{R_1 + R_2} C \approx R_2 C = \tau_2 = \frac{1}{2\pi f_2} \quad \text{because } R_1 >> R_2$$

$$\frac{R_2}{R_1 + R_2} = K$$

$$\therefore H_{PE}(f) = K \frac{1+j\frac{f}{f_1}}{1+j\frac{f}{f_2}}$$

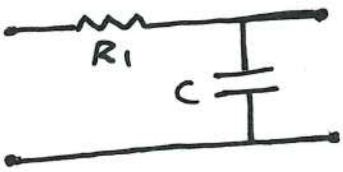
but $f_2 >> f_1$ and it is much larger than the cutoff freq. of the LPF of the transmitter, so:

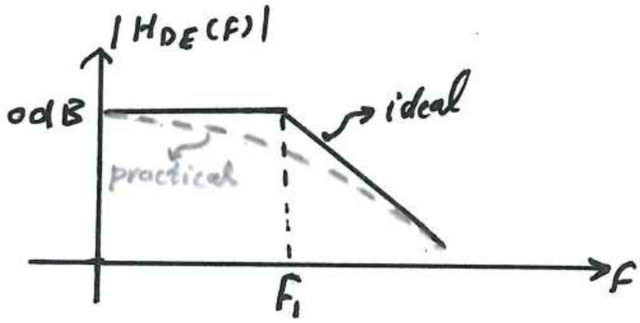
$$\boxed{H_{PE}(f) \simeq k\left(1+j\frac{f}{f_1}\right) = k\left(1+j2\pi f\tau_1\right)}$$

For commercial FM broadcasting $\tau_1 = 75\,\mu sec$.

$$f_1 = \frac{1}{2\pi\tau_1} = \frac{1}{2\pi\times 75\times 10^{-6}} = 2.1\ kHz.$$

B D.E Filter:





$$f_1 = \frac{1}{2\pi R_1 C} = \frac{1}{2\pi\tau_1}$$

$$H_{DE}(f) = \frac{1/j2\pi fC}{R_1 + \frac{1}{j2\pi fC}} = \frac{1}{1+j2\pi f R_1 C}$$

$$\boxed{H_{DE}(f) = \frac{1}{1+j\frac{f}{f_1}} = \frac{1}{1+j2\pi f\tau_1}}$$

$$|H_{DE}(f)| = \sqrt{\frac{1}{1+\left(\frac{f}{f_1}\right)^2}}$$

Now, in the presence of DE filter, the output noise N_o' can be calculated in a similar way as done by calculating (n_s') in FM system,

$$N_o' = \frac{1}{A_c^2}\overline{[n_s'(t)]^2} = \frac{1}{A_c^2}\int_{-B}^{B}(2\pi f)^2\eta \cdot |H_{DE}(f)|^2\, df$$

$$N_o' = \frac{2(2\pi)^2\eta}{A_c^2}\int_0^B \frac{f^2}{1+\left(\frac{f}{f_1}\right)^2}\, df$$

$$N_o' = \frac{2(2\pi)^2\eta}{A_c^2}\times f_1^2\left[B - f_1\tan^{-1}\frac{B}{f_1}\right]$$

Remember that, without D.E

$$N_o' = \frac{2\eta B^3}{3\,A_c^2}(2\pi)^2$$

Noise improvement Factor (Γ) is:

$$\Gamma = \frac{\text{output noise power without (PE-PD)}}{\text{output noise power with (PE-PD)}} = \frac{N_o}{N_o'}$$

$$\boxed{\Gamma = \frac{1}{3}\,\frac{(B/F_1)^3}{\left[\left(\frac{B}{F_1}\right) - \tan^{-1}\left(\frac{B}{F_1}\right)\right]}}$$

where $\tan^{-1}\left(\frac{B}{F_1}\right)$ in radian

Example: An FM system with D.E filter with $R = 75.8\,\Omega$ and $C = 1\,\mu F$. Find the noise improvement factor of this system if the signal bandwidth is 15 kHz.

Sol.

$$F_1 = \frac{1}{2\pi R_1 C} \cong 2.1\text{ kHz}$$

$$B = 15\text{ kHz}$$

$$\therefore\ (B/F_1) = \frac{15\times10^3}{2.1\times10^3} = 7.14$$

$$\tan^{-1}(B/F_1) = \tan^{-1}(7.14) = 1.43\ \text{rad.}$$

$$\Gamma = \frac{1}{3}\,\frac{(B/F_1)^3}{(B/F_1) - \tan^{-1}(B/F_1)} \Rightarrow \boxed{\Gamma = 21.25}$$

$$\Gamma_{(dB)} = 10\log(\Gamma)$$

$$\boxed{\Gamma_{(dB)} = 13.27\text{ dB}}$$

The $(S/N)_o$ is improved by 13.27 dB by using (PE-DE) filters. It is clear that by simple RC circuits the $(S/N)_o$ can be improved significantly without increasing the cost of the entire system.

Example:
Consider an FM system with peak frequency deviation of 75 kHz, signal bandwidth of 15 kHz. Assume single tone signal is used, find the output signal-to-noise ratio in term of the baseband signal-to-noise ratio (γ).

Sol.

$$\beta = \frac{\Delta f}{B} = \frac{75\ kHz}{15\ kHz} = 5$$

$$(S/N)_o = 3\,\beta^2\,\overline{m^2(t)}\,\gamma$$

$$m(t) = 1\,\sin(2\pi B t) \Rightarrow \overline{m^2(t)} = \frac{1}{2}$$

$$(S/N)_o = \frac{3}{2}\beta^2\gamma \Rightarrow \boxed{(S/N)_o = 37.5\,\gamma}$$

Example: An audio signal m(t) is to be transmitted over a noisy channel such that the output S/N is greater than 40 dB. Assume that the signal bandwidth is 15 kHz, $\eta/2 = 10^{-10}$ W/Hz, $\overline{m^2(t)} = 0.5$ watt, and the power loss of the channel is 50 dB. Calculate the transmission BW (B_T) and the required transmitted power (S_T) for:

a) DSB-SC

b) Normal AM with 100% modulation index.

c) PM with $k_p = 3$.

d) FM with $D = 5$.

Sol.

$$(S/N)_o^{(dB)} > 40\ dB \Rightarrow (S/N)_o > 10^{40/10} = 10^4$$

$$B = 15\ kHz, \quad \eta = 2\times10^{-10}\ W/Hz, \quad \overline{m^2(t)} = 0.5\ watt$$

$L(dB) = 50\,dB$

$L = 10^{50/10} = 10^5$

a) DSB-SC :

$B_T = 2B \Rightarrow B_T = 2 \times 15\,kHz$

$\therefore \boxed{B_T = 30\,kHz}$

$(S/N)_o = \gamma$

$(S/N)_o = \dfrac{S_i}{\eta B} > 10^4$

$\dfrac{S_i}{2 \times 10^{-10} \times 15 \times 10^3} > 10^4 \Rightarrow S_i > 0.03$ watt

$S_i = \dfrac{S_T}{L} \Rightarrow S_T = S_i L$

$\therefore \dfrac{S_T}{L} > 0.03 \Rightarrow S_T > 0.03 \times 10^5$

$\therefore \boxed{S_T > 3000\ \text{Watt}}$

b) Normal AM :

$\mu\% = 100\% \Rightarrow \mu = 1$

$(S/N)_o = \dfrac{\mu^2 \overline{m^2(t)}}{1 + \mu^2 \overline{m^2(t)}} \gamma = \dfrac{1 \times 0.5}{1 + 1 \times 0.5} \times \dfrac{S_i}{\eta B} > 10^4$

$\therefore \dfrac{0.5}{1+0.5} \times \dfrac{S_i}{2 \times 10^{-10} \times 15 \times 10^3} > 10^4$

$\therefore S_i > 0.09$ watt

$\dfrac{S_T}{L} > 0.09$ watt $\Rightarrow \boxed{S_T > 9000\ \text{watt}}$

c) PM with $k_p = 3$:

$B_T = 2(D+1)B$, where $D = k_p\, a_m$, $a_m = 1$

$D = k_p = 3$

$B_T = 2(3+1) \times 15\,kHz \Rightarrow \boxed{B_T = 120\,kHz}$

$(S/N)_o = k_p^2\, \overline{m^2(t)}\, \gamma > 10^4$

$k_p^2\, \overline{m^2(t)}\, \dfrac{S_i}{\eta B} > 10^4$

$$3^2 \times 0.5 \times \frac{S_i}{2\times10^{-4} \times 15\times10^3} > 10^4$$

$$S_i > 0.00667$$

$$\frac{S_T}{L} > 0.00667 \Rightarrow \boxed{S_T = 667 \text{ watt}}$$

d) FM with $D = 5$:

$$B_T = 2(D+1)B$$

$$B_T = 2(5+1)\times 15\text{ kHz} \Rightarrow \boxed{B_T = 180\text{ kHz}}$$

$$(S/N)_o = 3D^2\,\overline{m^2(t)}\,\gamma > 10^4$$

$$3D^2\,\overline{m^2(t)}\,\frac{S_i}{\eta B} > 10^4$$

$$\frac{3\times5^2\times0.5}{2\times10^{-10}\times15\times10^3}S_i > 10^4$$

$$S_i > 8\times10^{-4}$$

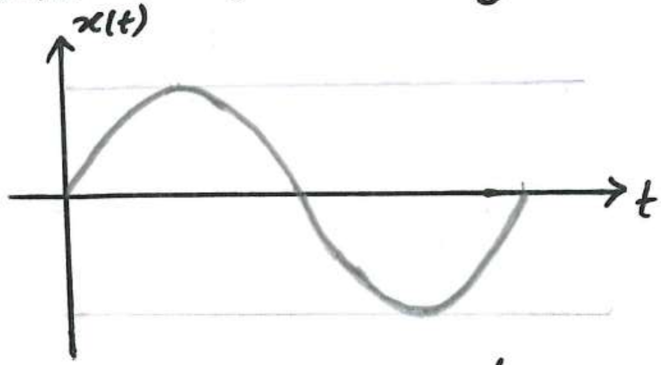
$$\frac{S_T}{L} > 8\times10^{-4}$$

$$\therefore \boxed{S_T > 80 \text{ watt}}$$

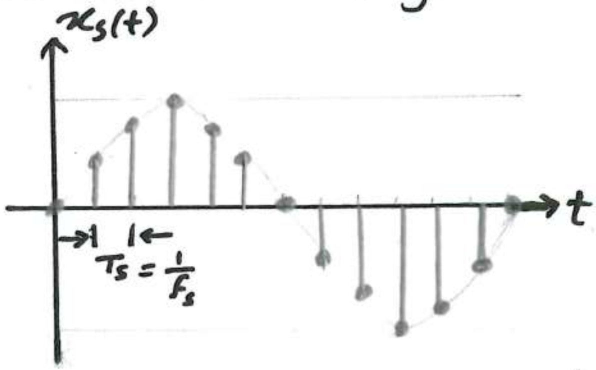
Note that FM has non-efficient utilization for the BW (largest BW), but very efficient efficient utilization for the power (smallest power).

Pulse Modulation

In pulse modulation, discrete time signals are used to represent the modulated signal of the pulse modulation. Sampling process is used to convert continuous time signal into discrete time signal.



Continuous signal



Discrete time signal
(sampled signal)

1. Sampling Theorem:

It states that: [Any band-limited signal $m(t)$, which has no frequency component higher than (f_M) Hz, can be recovered perfectly from a set of samples taken at the rate of $(f_s \geq 2 f_M)$ sample/sec.]

$$\boxed{f_s \geq 2 f_M} \quad \text{sample/sec.}$$

but $T_s = \frac{1}{f_s}$

$$\therefore \quad \boxed{T_s \leq \frac{1}{2 f_M}}$$

where f_s : sampling rate (sampling frequency).
T_s : sampling period.

Sampling Theorem is also called "Nyquist Theorem" and f_s some times called Nyquist rate.

Let $m(t)$ be a band-limited signal. This signal is sampled using a train of unit impulses. As we studied previously, Fourier transform of the train of impulses is given below:

$$F.T\left[\sum_{n=-\infty}^{\infty}\delta(t-nT_s)\right] = \frac{1}{T_s}\sum_{n=-\infty}^{\infty}\delta(f-nf_0)$$

$$= f_s \sum_{n=-\infty}^{\infty}\delta(f-nf_s)$$

$$M_s(f) = F.T\left[m(t)\sum_{n=-\infty}^{\infty}\delta(t-nT_s)\right] = M(f) * f_s\sum_{n=-\infty}^{\infty}\delta(f-nf_s)$$