2.2 Normal AM System:

In this section the synchronous detection and the envelope detector is discussed to highlight the advantages of each of them.

A Synchronous Detection of AM:

$$x_c(t) = A_c[1+\mu m(t)] \cos(2\pi f_c t)$$

$$x_c(t) = A_c \cos(2\pi f_c t) + A_c \mu\, m(t) \cos(2\pi f_c t)$$

$$\therefore\ S_i = \frac{A_c^2}{2} + \frac{A_c^2 \mu^2}{2} \overline{m^2(t)} \Rightarrow \boxed{S_i = \frac{A_c^2}{2}\left(1+\mu^2 \overline{m^2(t)}\right)}$$

$$y_i(t) = A_c(1+\mu m(t)) \cos(2\pi f_c t) + n_p(t)$$

$$y_i(t) = A_c(1+\mu m(t)) \cos(2\pi f_c t) + [n_i(t) \cos(2\pi f_c t) - n_q(t) \sin(2\pi f_c t)]$$

$$y(t) = y_i(t) \times 2\cos(2\pi f_c t)$$

$$y(t) = 2[A_c(1+\mu m(t)) + n_i(t)] \cos^2(2\pi f_c t) - $$
$$-2n_q(t) \sin(2\pi f_c t) \cos(2\pi f_c t)$$

$$y(t) = [A_c(1+\mu m(t)) + n_i(t)]\,[1+\cos(2\pi(2f_c)t)] -$$
$$- n_q(t) \sin(2\pi(2f_c)t)$$

After LPF with BW = B Hz

$$y_o(t) = A_c(1+\mu m(t)) + n_i(t) = \underbrace{A_c}_{DC} + \underbrace{A_c \mu m(t)}_{\text{signal}} + \underbrace{n_i(t)}_{\text{noise}}$$

After Blocking capacitor:

$$y_o(t) = A_c \mu\, m(t) + n_i(t)$$

$$(S/N)_o = \frac{S_o}{N_o} = \frac{A_c^2 \mu^2 \overline{m^2(t)}}{2\eta B}$$

from the equation of S_i: $A_c^2 = \dfrac{2S_i}{1+\mu^2 \overline{m^2(t)}}$

$$\therefore (S/N)_o = \frac{2S_i}{1+\mu^2\overline{m^2(t)}} \times \frac{\mu^2\overline{m^2(t)}}{2\eta B}$$

$$\therefore (S/N)_o = \frac{\mu^2\overline{m^2(t)}}{1+\mu^2\overline{m^2(t)}} \cdot \frac{S_i}{\eta B}$$

$$\therefore \boxed{(S/N)_o = \frac{\mu^2\overline{m^2(t)}}{1+\mu^2\overline{m^2(t)}}\,\gamma}$$

but we knew that, $\mu^2\overline{m^2(t)} \leq 1$, thus

$$\therefore (S/N)_o \leq \frac{1}{1+1}\gamma$$

$$\therefore \boxed{(S/N)_o \leq \frac{1}{2}\gamma} \qquad (S/N)_{o\,max} = \frac{1}{2}\gamma$$

This means that the best condition for the synchronous detection of AM signal is when $\mu^2\overline{m^2(t)} = 1$, but it still 3dB lower than the DSB-SC detection.

The detection gain of the synchronous detection of normal AM modulation is given by:

$$\gamma_d = \frac{(S/N)_o}{(S/N)_i} = \frac{A_c^2\mu^2\overline{m^2(t)}/2\eta B}{\frac{A_c^2}{2}A_c^2[1+\mu^2\overline{m^2(t)}]/2\eta B}$$

$$\boxed{\gamma_d = \frac{2\mu^2\overline{m^2(t)}}{1+\mu^2\overline{m^2(t)}}}$$

The best case when $\mu^2\overline{m^2(t)} = 1$

$$\boxed{\gamma_{d\,max} = 1}$$ for synchronous detection of AM.

<u>Example:</u> A speech signal is transmitted with ~~power~~ r.m.s value of 0.2 ~~Watt~~ Volt using normal AM modulation with modulation index 0.5. The amplitude of the carrier signal is 10 Volt, and the white noise power spectral density <u>double side</u> level ($\eta/2 = 10^{-4}$ W/Hz). Find:

a) $(S/N)_o$ of the synchronous detection of the AM.

b) $(S/N)_o$ of the synchronous detection of DSB-SC modulation of the same signal. (Compare the resuts). Assume the signal $BW = B = 15\,kHz$.

Sol.

$\overline{m^2(t)} = (0.2)^2$ Watt , $\mu = 0.5$, $A_c = 10$ Volt.

$\eta/2 = 10^{-4} \Rightarrow \eta = 2\times10^{-4}$ W/Hz , $B = 15\,kHz$

a) Normal AM:

$$S_i = \frac{A_c^2}{2}\left(1+\mu^2\overline{m^2(t)}\right) = \frac{100}{2}\left(1+0.5^2\times0.2^2\right)$$

$$\therefore\ S_i = 50.5 \text{ Watt}$$

$$\gamma = \frac{S_i}{\eta B} = \frac{50.5}{2\times10^{-4}\times15\times10^3} \Rightarrow \gamma = 16.83$$

$$(S/N)_{o\,AM} = \frac{\mu^2\,\overline{m^2(t)}}{1+\mu^2\,\overline{m^2(t)}}\,\gamma$$

$$(S/N)_{o\,AM} = \frac{0.5^2\times0.2^2}{1+0.5^2\times0.2^2}\times16.83 \Rightarrow \boxed{(S/N)_{AM} = 0.167}$$

$\cong 0.1$

b) DSB-SC:

$$S_i = \frac{A_c^2}{2}\,\overline{m^2(t)} = 2 \text{ Watt}$$

$$\gamma = \frac{S_i}{\eta B} = \frac{2}{2\times10^{-4}\times15\times10^3}$$

$$\therefore\ \gamma = 0.667$$

$$(S/N)_{o\,DSB} = \gamma \Rightarrow \boxed{(S/N)_{o\,DSB} = 0.667} \approx 0.6$$

Comparison: Althoug the input signal power of DSB ($S_i = 2W$) is much smaller than that of AM ($S_i = 50.5W$), DSB still has $(S/N)_o$ six times larger than $(S/N)_o$ of AM signal. This means that DSB is not only providing better SNR, but also power-efficient kind of modulation.

B Envelope Detection:

At the detector input, the AM signal plus noise signal is given by:

$$y_i(t) = A_c\left[1+\mu m(t)\right]\cos(2\pi f_c t) + n_p(t)$$

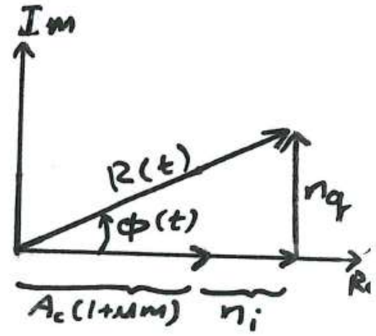
OR

$$y_i(t) = A_c\left[1+\mu m(t)\right]\cos(2\pi f_c t) + \left[n_i(t)\cos(2\pi f_c t) - n_q(t)\sin(2\pi f_c t)\right]$$

$$y_i(t) = \left[A_c(1+\mu m(t)) + n_i(t)\right]\cos(2\pi f_c t) - n_q(t)\sin(2\pi f_c t).$$

The above equation can be re-written using phasor form:

$$y_i(t) = \underbrace{R(t)}_{\text{envelope}}\cos(2\pi f_c t + \phi(t))$$



where $\underset{\text{envelope}}{R(t)} = \sqrt{\left[A_c(1+\mu m(t)) + n_i\right]^2 + n_q^2}$

$$\phi(t) = \tan^{-1}\frac{n_q}{A_c(1+\mu m(t)) + n_i}$$

The input of the envelope detector is $R(t)$:

$$|y_i(t)| = \sqrt{\left[A_c(1+\mu m(t)) + n_i\right]^2 + n_q^2}$$ envelope of the signal.

* If the signal is dominating the transmission:

$$\left(\frac{S}{N}\right)_i \gg 1 \quad \text{OR} \quad A_c(1+\mu m) \gg n_i \,\&\, n_q$$

$$R(t) = \left[A_c(1+\mu m(t)) + n_i\right]\sqrt{1 + \underbrace{\frac{n_q^2}{\left[A_c(1+\mu m(t)) + n_i\right]^2}}_{\simeq 0}}$$

$$R(t) \simeq A_c(1+\mu m(t)) + n_i$$

$$R(t) = \underbrace{A_c}_{DC} + \underbrace{A_c\mu m(t)}_{\text{signal}} + \underbrace{n_i}_{\text{noise}}$$

After the D.C blocking capacitor.

$$R(t) = A_c\mu m(t) + n_i$$

$\therefore \; y_o(t) = A_c \mu m(t) + n_i$

The above $y_o(t)$ is the same as that obtained by the synchronous detection of AM signal, so

$$\boxed{(S/N)_o = \frac{\mu^2 \overline{m^2(t)}}{1+\mu^2 \overline{m^2(t)}}\,\gamma} \quad \text{for } A_c(1+\mu m(t)) >> n_i \,\&\, n_q$$

* If the noise is dominating the signal envelope, the envelope will heavily be distorted, and the signal is lost. Therefore, $m(t)$ can not be extracted from the signal envelope in this case.

Threshold Effect:

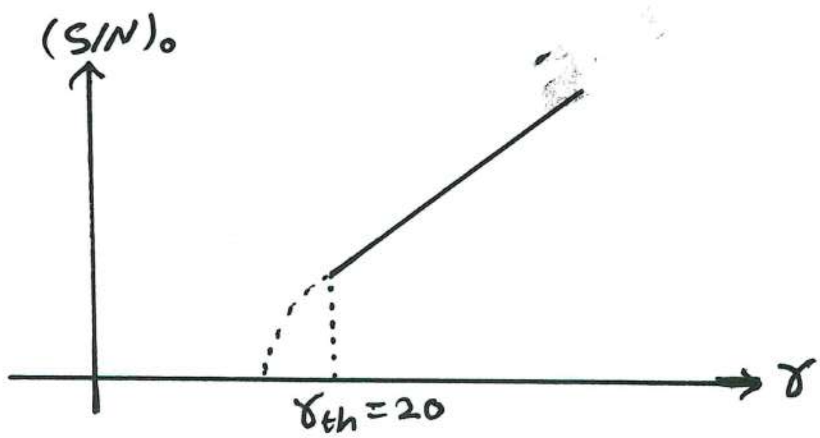
In some values of $(S/N)_i$ the signal distortion due to the noise is negligible.

* (γ_{th}) is the value of γ above which the condition $A_c(1+\mu m(t)) >> n_q \,\&\, n_i$ is applied.

Practically : $\boxed{\gamma_{th} = 20}$

the best value of $(S/N)_{i_{th}} = \frac{\gamma_{th}}{2} = \frac{20}{2}$

$$\boxed{\begin{aligned} (S/N)_{i\,th} &= 10 \\ (S/N)_{i\,th}\,(dB) &= 10\,dB \end{aligned}} \quad \text{for } \mu = 1$$



2.3 SSB Systems:

$$x_c(t) = A_c\left[m(t)\cos(2\pi f_c t) \pm \hat{m}(t)\sin(2\pi f_c t)\right]$$

For the synchronous detection of SSB, the BPF has a bandwidth of B Hz instead of 2B Hz in case of DSB-SC and AM systems.

$$S_i = \overline{x_c^2(t)} = \frac{A_c^2}{2}\left[\overline{m^2(t)} + \overline{\hat{m}^2(t)}\right]$$

but $\overline{m^2(t)} = \overline{\hat{m}^2(t)}$

$$\therefore \quad \boxed{S_i = A_c^2\,\overline{m^2(t)}}$$

$$y_i(t) = A_c\left[m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)\right] + n_p(t)$$

$$y_i(t) = A_c\left[m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)\right] + $$
$$+\, n_i(t)\cos(2\pi f_c t) - n_q(t)\sin(2\pi f_c t)$$

$$\therefore \quad y_i(t) = (A_c m(t) + n_i)\cos(2\pi f_c t) + (A_c \hat{m}(t) - n_q)\sin(2\pi f_c t)$$

$$y(t) = y_i(t) \times 2\cos(2\pi f_c t)$$

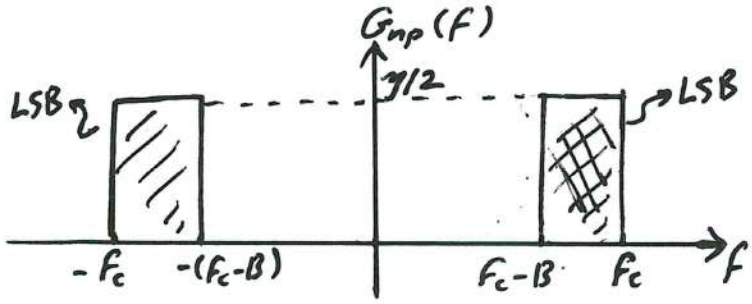
$$y(t) = 2(A_c m(t) + n_i)\cos^2(2\pi f_c t) + 2(A_c\hat{m}(t) - n_q)\sin(2\pi f_c t)\cos(2\pi f_c t)$$

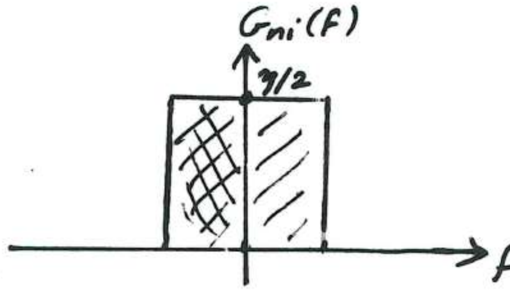
$$y(t) = (A_c m(t) + n_i) + (A_c m(t) + n_i)\cos(2\pi(2f_c)t) + $$
$$+\,(A_c\hat{m}(t) - n_q)\sin(2\pi(2f_c)t).$$

After LPF with BW = B Hz

$$y_o(t) = \underbrace{A_c\,m(t)}_{\text{signal}} + \underbrace{n_i(t)}_{\text{noise}}$$

$$\therefore \quad S_o = A_c^2\,\overline{m^2(t)} \Rightarrow \boxed{S_o = S_i}$$





$$N_o = \int_{-\infty}^{\infty} G_{ni}(f)\, df$$

$$N_o = \int_{-B}^{B} \frac{\eta}{2}\, df \Rightarrow \boxed{N_o = \eta B}$$

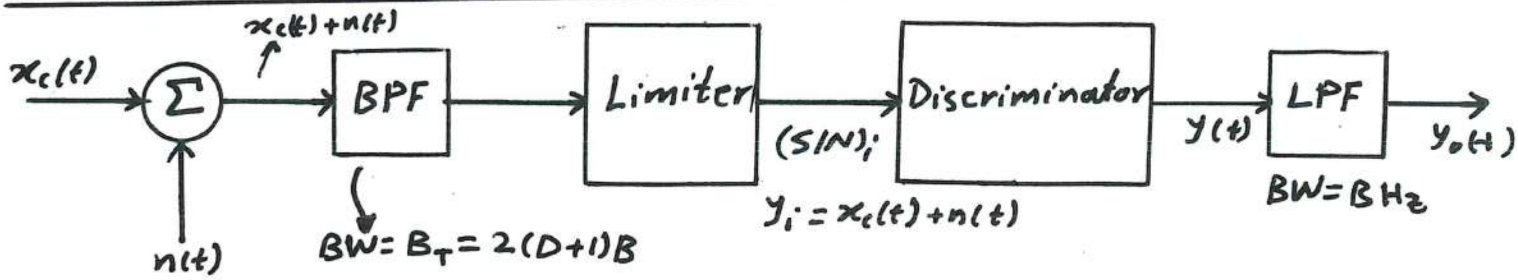
$$(S/N)_o = \frac{S_o}{N_o} = \frac{S_i}{\eta B}$$

$$\therefore \boxed{(S/N)_o = \gamma} = (S/N)_i$$

$$\therefore \quad \gamma_d = \frac{(S/N)_o}{(S/N)_i} \Rightarrow \boxed{\gamma_d = 1}$$

- SSB has the same $(S/N)_o$ as the DSB.
- VSB is exactly the same as SSB in noise calculations

3. Noise in Angle Modulation Systems:



$$x_c(t) = A_c \cos(2\pi f_c t + \phi(t)) = A_c \cos(\theta(t))$$

$$y_i(t) = x_c(t) + n(t) = A_c \cos(2\pi f_c t + \phi(t)) + n(t)$$

$$\boxed{S_i = \frac{A_c^2}{2}}$$

$$N_i = \overline{n^2(t)} = \int_{-(f_c+B_T/2)}^{-(f_c-B_T/2)} \frac{\eta}{2}\, df + \int_{f_c-B_T/2}^{f_c+B_T/2} \frac{\eta}{2}\, df$$

$$\therefore \boxed{N_i = \eta B_T} \Rightarrow \boxed{(S/N)_i = \frac{A_c^2}{2\eta B_T}}$$

Since the noise is narrow band, it can be written as:

$$n(t) = R_n(t) \cos(2\pi f_c t + \phi_n(t))$$

which is the phasor form of $\{n_i \cos(\omega_c t) - n_q \sin(\omega_c t)\}$

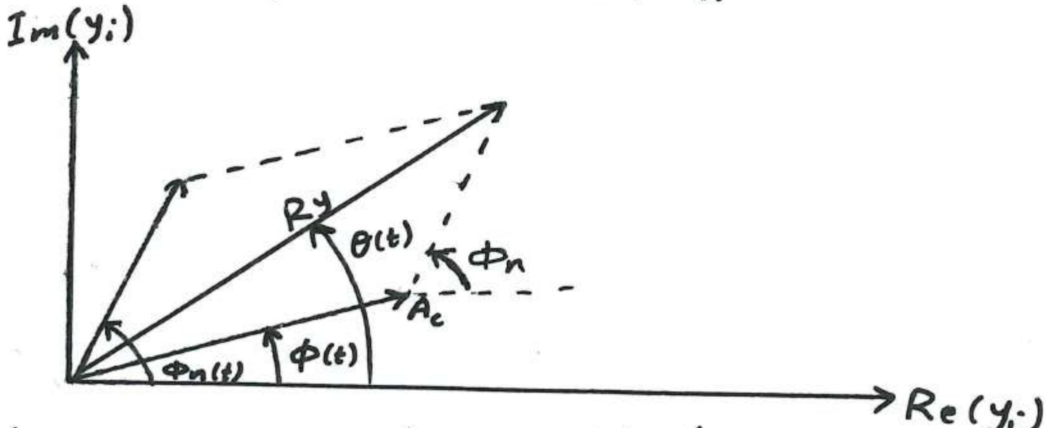
$$y_i(t) = A_c \cos[2\pi f_c t + \phi(t)] + R_n(t)\cos[2\pi f_c t + \phi_n(t)] \quad \text{—} (*)$$

$$\therefore \; y_i(t) = R_y(t)\,\cos[2\pi f_c t + \theta(t)]$$

where

$$R_y(t) = \sqrt{(A_c \cos\phi + R_n \cos\phi_n)^2 + (A_c \sin\phi + R_n \sin\phi_n)^2}$$

$$\theta(t) = \tan^{-1}\left[\frac{A_c \sin\phi + R_n \sin\phi_n}{A_c \cos\phi + R_n \cos\phi_n}\right]$$

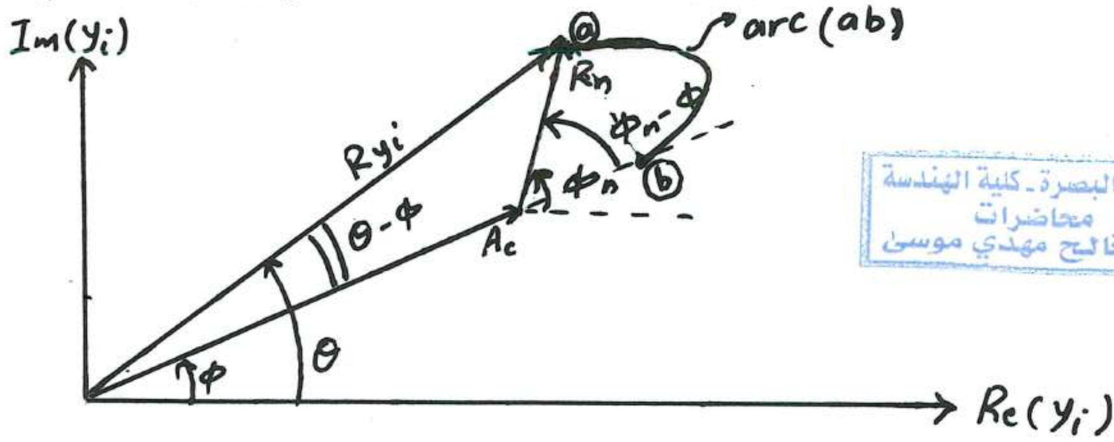


The limitter suppresses the amplitude $R_y(t)$ and keep it constant. Therefore, the $(S/N)_o$ is derived taking into account $\theta(t)$ only.

$$y_o(t) = \begin{cases} \theta(t) \longrightarrow PM \\ \dfrac{d\theta(t)}{dt} \longrightarrow FM \end{cases}$$

* If the signal is dominating the transmission:

$$(S/N)_i \gg 1 \quad , \quad A_c \gg R_n$$



eq: (*) can be written as follows:

$$y_i(t) = Re\left[A_c e^{jw_ct} \cdot e^{j\phi(t)} + R_n(t)\, e^{jw_ct} \cdot e^{j\phi_n(t)}\right]$$

$$y_i(t) = Re\left[\left(A_c e^{j\phi(t)} + R_n\, e^{j\phi_n(t)}\right) e^{jw_ct}\right]$$

$$y_i(t) = Re\left[R_{y_i}(t)\, e^{j2\pi f_c t}\right]$$

where $R_{y_i}(t)$ is the envelope of $y_i(t)$.

$$\boxed{R_{y_i}(t) = A_c\, e^{j\phi(t)} + R_n(t)\, e^{j\phi_n(t)}} \quad \text{. but } A_c >> R_n(t)$$

The length of the arc (ab) is (L):

$$L = R_{yi}(t)\left[\theta(t) - \phi(t)\right] \qquad \{\text{arc length} = \text{radius} \times \text{angle}\}$$

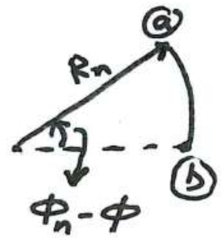
Since $A_c >> R_n(t)$

$$\therefore R_{yi}(t) \approx A_c + R_n \cos(\phi_n - \phi) \approx A_c$$

$$\therefore L \cong A_c\left[\theta(t) - \phi(t)\right]$$

From the noise triangle

$$L \cong R_n \sin(\phi_n - \phi)$$



$$\therefore A_c\left[\theta(t) - \phi(t)\right] = R_n \sin(\phi_n - \phi)$$

or $$\theta(t) \cong \frac{R_n}{A_c}\sin(\phi_n - \phi) + \phi(t)$$

$$\therefore \boxed{\theta(t) \cong \phi(t) + \frac{n_s(t)}{A_c}} \qquad n_s(t) = \frac{R_n}{A_c}\sin(\phi_n - \phi)$$

the output of the system is $y_o(t) = \begin{cases} \theta(t) & \text{PM} \\ \dfrac{d\theta}{dt} & \text{FM} \end{cases}$

$$\therefore y_o(t) = k_p\, m(t) + \frac{n_s(t)}{A_c} \qquad \text{for PM}$$

$$y_o(t) = \underbrace{k_f\, m(t)}_{\text{Signal}} + \underbrace{\frac{n_s'(t)}{A_c}}_{\text{Noise}} \qquad \text{for FM}$$

A (S/N)$_o$ in PM Systems:

$$y_o(t) = \underbrace{k_p m(t)}_{\text{Signal}} + \underbrace{\frac{n_s(t)}{A_c}}_{\text{Noise}}$$

$$\boxed{S_o = k_p^2 \overline{m^2(t)}}$$

$$N_o = \frac{\overline{n_s^2(t)}}{A_c^2} = \frac{1}{A_c^2} \cdot \int_{-B}^{B} \eta \, df \Rightarrow \boxed{N_o = \frac{2\eta B}{A_c^2}}$$

$$\therefore (S/N)_o = \frac{S_o}{N_o} = \frac{k_p^2 \overline{m^2(t)}}{\eta B} \cdot \frac{A_c^2}{2}$$

but $\frac{A_c^2}{2} = S_i$

$$\therefore (S/N)_o = k_p^2 \overline{m^2(t)} \frac{S_i}{\eta B}$$

remember that $\frac{S_i}{\eta B} = \gamma$

$$\therefore \boxed{(S/N)_o = k_p^2 \overline{m^2(t)} \gamma}$$

It is clear that the improvement of the (S/N)$_o$ depends on the phase deviation constant (k_p) and the power of the message signal $\overline{m^2(t)}$.

where $k_{p\,max} = \pi$.

B (S/N)$_o$ in FM Systems:

$$y_o(t) = \underbrace{k_f m(t)}_{\text{Signal}} + \underbrace{\frac{n_s'(t)}{A_c}}_{\text{Noise}}$$

$$\boxed{S_o = k_f^2 \overline{m^2(t)}}$$

$$n_s'(t) = \frac{d}{dt}(n_s(t))$$

$$F.T[n_s'(t)] = j2\pi f \, N_s(f)$$

Power spectral density of $n_s'(t)$ is $G_{ns}'(f)$

$$G_{ns}'(f) = \left| F.T[n_s'(t)] \right|^2$$