$\therefore\ \boxed{F_t = 40}$ $\qquad \boxed{F_{t\,(dB)} \cong 16\,dB}$!! It is very noisy device !!

$$F_t = \frac{(S/N)_i}{(S/N)_o} \Rightarrow (S/N)_o = \frac{(S/N)_i}{F_t}$$

$$(S/N)_i = 10^{52/10} = 158489.32$$

$$\therefore\ (S/N)_o = \frac{158489.32}{40} \Rightarrow \boxed{(S/N)_o = 3962.23}$$

$$(S/N)_{o\,(dB)} = 10 \log (S/N)_o \Rightarrow \boxed{(S/N)_{o\,(dB)} \cong 36\,dB}$$

IF T_{et} is required then: $T_o = 17^\circ + 273 = 290\,^\circ K$

$$T_{et} = (F_t - 1)\,T_o \Rightarrow T_{et} = 11310\ ^\circ K$$

Example: Satellite Ground Receiver.

$G_2 = 26dB$ $\qquad G_3 = 17dB$

$F_2 = 0.06dB$ $\qquad F_3 = 6dB$

IF $(S/N)_{i\,(dB)} = 20.5dB$, then

a) Find $F_{t\,(dB)}$, T_{et}, $(S/N)_{o\,(dB)}$.

b) Replace the positions of the cable and the low noise amplifier and then re-calculate $F_{t\,(dB)}$, T_{et}, and $(S/N)_{o\,(dB)}$.

Sol.

a) $L = 10^{3/10} \cong 2$

$\therefore\ F_1 = 2$ $\qquad , \quad G_1 = \frac{1}{2}$

$F_2 = 10^{0.06/10} \cong 1.01$ $\qquad , \quad G_2 = 10^{26/10} \cong 398$

$F_3 = 10^{6/10} \cong 4$ $\qquad , \quad G_3 = 10^{17/10} \cong 50$

$$F_t = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

$$F_t = 2 + \frac{1.01 - 1}{\frac{1}{2}} + \frac{4 - 1}{\frac{1}{2} \times 398} \Rightarrow \boxed{F_t = 2.035}$$

$$\boxed{F_{t\,dB} = 3.086\ dB}$$

$$T_{et} = (F_t - 1)\,T_o \quad , \quad T_o = 17 + 273 = 290^\circ k$$

$$T_{et} = (2.035 - 1) \times 290^\circ k \qquad \therefore \boxed{T_{et} = 300.15\ ^\circ k}$$

$$(S/N)_i = 10^{20.5/10} = 112.2$$

$$F_t = \frac{(S/N)_i}{(S/N)_o} \Rightarrow (S/N)_o = \frac{112.2}{2.035}$$

$$\therefore \boxed{(S/N)_o = 55.14}$$

$$\boxed{(S/N)_{o\,(dB)} = 17.4\ dB}$$

b) When the cable and the low-noise amplifier is interchanged:

$F_1 = 1.01 \quad , \quad G_1 = 398$

$F_2 = 2 \quad , \quad G_2 = \frac{1}{2}$

$F_3 = 4 \quad , \quad G_3 = 50$

$$F_t = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} = 1.01 + \frac{2 - 1}{398} + \frac{4 - 1}{398 \times \frac{1}{2}}$$

$$\therefore \boxed{F_t \cong 1.03} \Rightarrow \boxed{F_{t\,(dB)} = 0.12\ dB}$$

It is clear that the noise figure is improved (decreased).

$$T_{et} = (F_t - 1)\,T_o \Rightarrow \boxed{T_{et} = 8.7\ ^\circ k}$$ it is reduced too.

$$F_t = \frac{(S/N)_i}{(S/N)_o} \Rightarrow (S/N)_o = \frac{112.2}{1.03}$$

$$\therefore \boxed{(S/N)_o \cong 109} \Rightarrow \boxed{(S/N)_{o\,(dB)} = 20.4\ dB}$$

S/N is improves (increased) by more than 3dB

6. Special Classes of Noise:

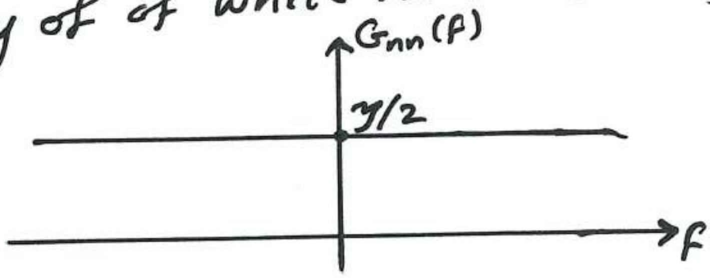
[A] White Noise:

In this type of noise, all frequency components appear with equal power. In other words, the power spectral density of white noise is independent of frequency.

White noise is analogous to "White Light" because the white color has an equal amount of power of all the other lights (frequencies).

The power spectral density of of white noise is given by:

$$\boxed{G_{nn}(f) = \frac{\eta}{2}}$$

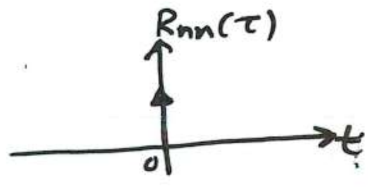


* The parameter (η) has a unit of (W/Hz), and usually measured at the input stage of receivers.
* The factor ($\frac{1}{2}$) has been included to indicate that half of the power is associated with positive frequencies and half with negative frequencies.

The auto correlation function of white noise is given by:

$$R_{nn}(\tau) = F.T^{-1}[G_{nn}(f)]$$

$$\boxed{R_{nn}(\tau) = \frac{\eta}{2}\,\delta(\tau)}$$

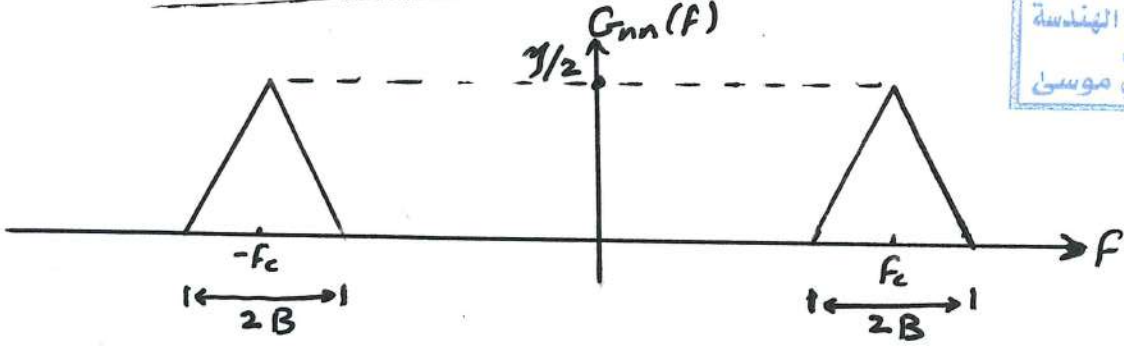


Notes:

*) Theoretically, white noise has infinite average power, but practically communication systems assumes white noise when the noise BW is appreciably larger than the system BW itself.

B Narrowband White Noise:

In many communication receivers, there are narrowband BPF.s whose BW are just large enough to pass the bandpass signal, but not so large as to pass excessive amount of noise. The noise appearing at the output of such a filter is called narrowband white noise.

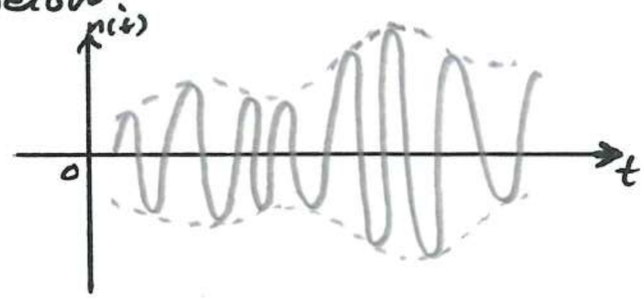


When this kind of noise is viewed on an oscilloscope, the observed waveform appears as a sinusoid with random amplitude and phase as shown below:

$n(t) = A(t)\,\cos(2\pi f_c t + \phi(t))$

$A(t)$ - random envelope function.

$\phi(t)$ - random phase function.



Inphase and Quadrature Components:

The narrowband noise can be represent in rectangular form instead of the magnitude-phase form.

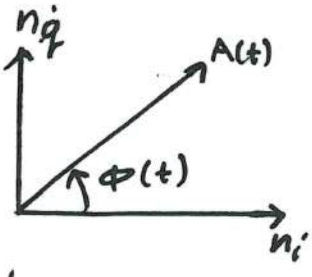
$$n(t) = A(t)\cos(2\pi f_c t)\cos(\phi(t)) - A(t)\sin(2\pi f_c t)\sin(\phi(t))$$

$$\boxed{n(t) = n_i(t)\cos(2\pi f_c t) - n_q(t)\sin(2\pi f_c t)}$$

where

$$\boxed{n_i(t) = A(t)\cos(\phi(t))}$$ inphase component

$$\boxed{n_q(t) = A(t)\sin(\phi(t))}$$ quadrature component



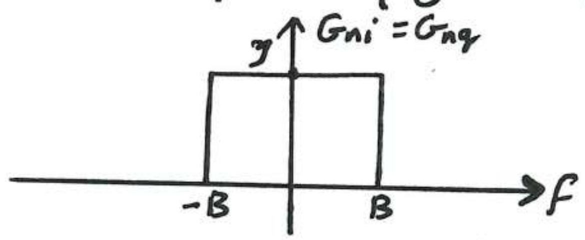
For any type of narrowband noise, the power spectral density of n_i and n_q are identical and related to that of $n(t)$ as follows:

$$G_{n_i}(f) = G_{n_q}(f) = \begin{cases} G_{nn}(f-f_c) + G_{nn}(f+f_c) & \text{for } |f| \leq B \\ 0 & \text{otherwise} \end{cases}$$

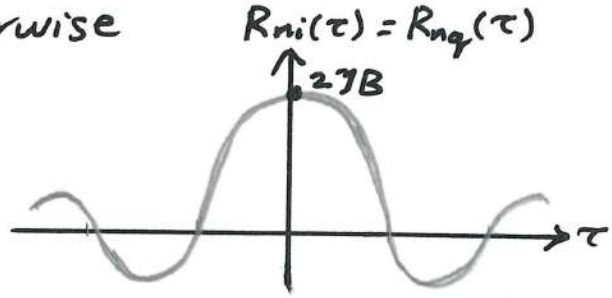
note that $n_i(t)$ and $n_q(t)$ are "<u>baseband signals</u>".

For narrow<u>band white noise</u>:

$$G_{n_i}(f) = G_{n_q}(f) = \begin{cases} \eta & \text{for } |f| \leq B \\ 0 & \text{otherwise} \end{cases}$$

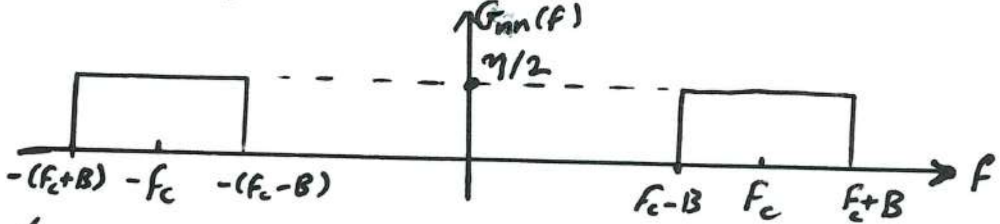


$G_{ni}(f) = G_{nq}(f) = \eta\, \Pi\left(\frac{f}{2B}\right)$



$R_{ni}(\tau) = R_{nq}(\tau) = 2\eta B\, \text{Sinc}(2B\tau)$

$n(t)$ is a bandpass signal, and its power spectral density is shown below (for narrowband white noise).



Note that:

if the noise power is denoted by (N), inphase noise power is denoted by (N_i), and the quadrature noise power is denoted by (N_q), then

$$N_i = \overline{n_i^2(t)} = R_{ni}(0)$$

$$N_q = \overline{n_q^2(t)} = R_{nq}(0)$$

$$N = \overline{n^2(t)} = R_{nn}(0)$$

but $P = \int_{-\infty}^{\infty} G_{xx}(f)\, df$

$$\therefore \; N_i = \int_{-\infty}^{\infty} G_{ni}(f)\, df = \int_{-B}^{B} \eta \cdot df = 2\eta B$$

$$N_q = \int_{-\infty}^{\infty} G_{nq}(f)\, df = \int_{-B}^{B} \eta \cdot df = 2\eta B$$

$$N = \int_{f_c-B}^{f_c+B} \frac{\eta}{2}\, df + \int_{-(f_c+B)}^{-(f_c-B)} \frac{\eta}{2}\, df = 2\eta B$$

$$\therefore \quad \boxed{N_i = N_q = N}$$

OR $\overline{n_i^2} = \overline{n_q^2} = \overline{n^2}$, $R_{ni}(0) = R_{nq}(0) = R_{nn}(0)$.

and also $\overline{n_i} = \overline{n_q} = \overline{n}$

<u>Example:</u> A noise signal $n(t)$ with power spectral density shown below in Fig. a is passed through a BPF with response shown in Fig. b. Find the power spectral density of the inphase and quadrature phase components. Then show that $N = N_i = N_q$.

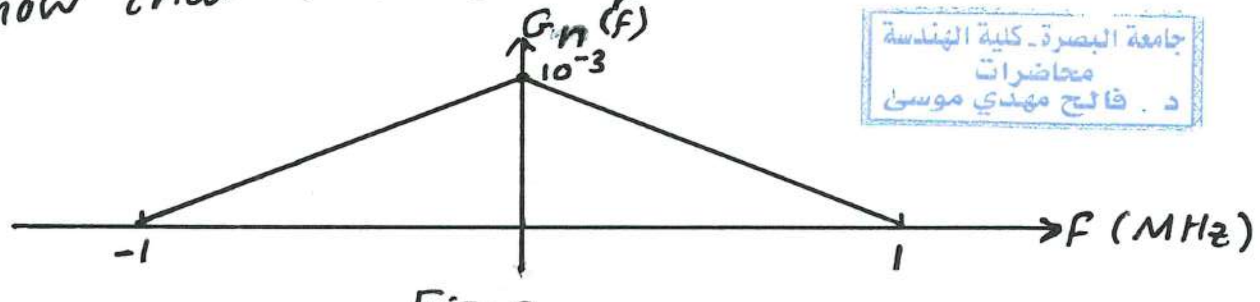


Fig. a

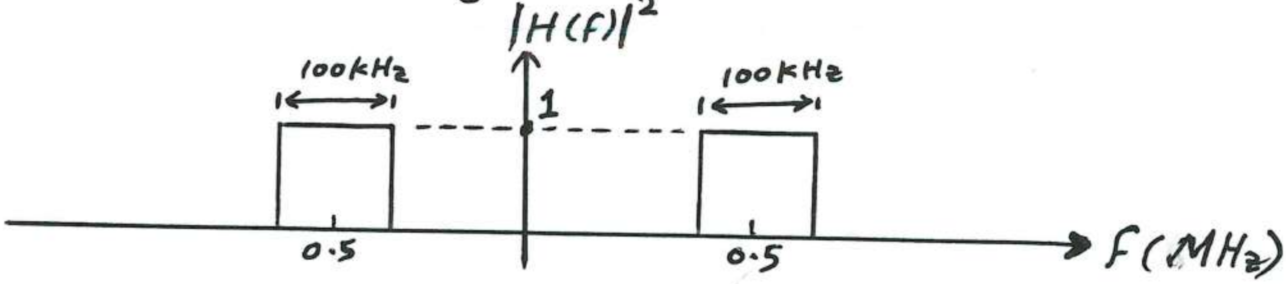


Fig. b

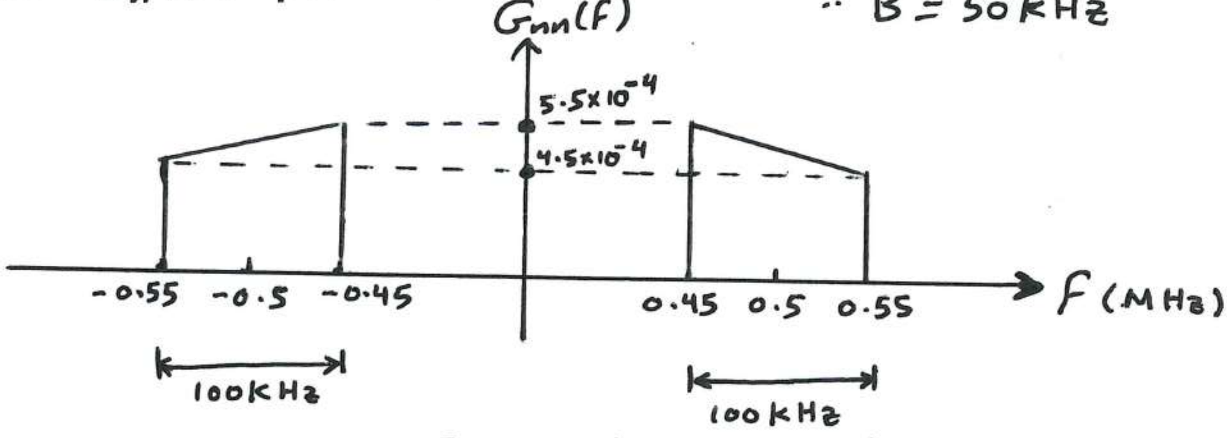
Sol.

$$G_n(f) = \begin{cases} -10^{-9}f + 10^{-3} & 0 \leq f \leq 1\,\text{MHz} \\ 10^{-9}f + 10^{-3} & -1\,\text{MHz} \leq f \leq 0 \end{cases}$$

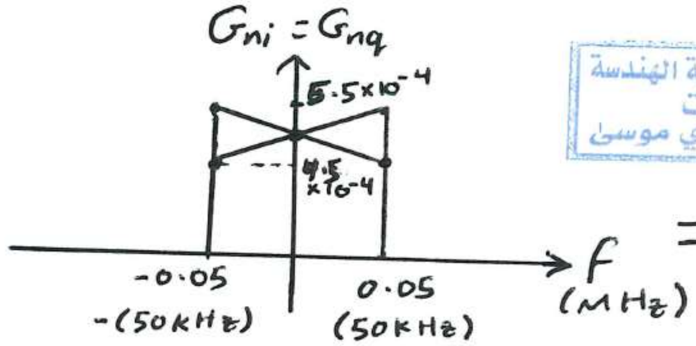
The out put of the BPF is narrowband noise $G_{nn}(f)$.

$$G_{nn}(f) = G_n(f)\,|H(f)|^2$$

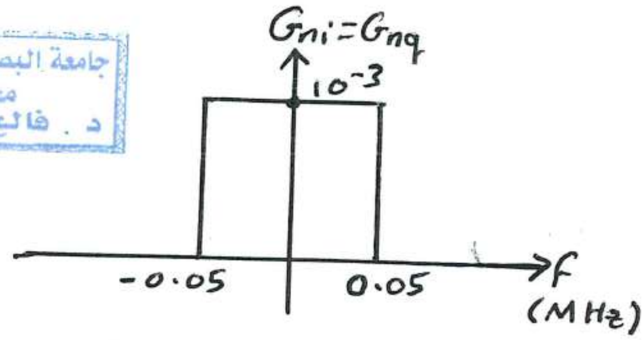
where $2B = 100\,\text{kHz}$
$\therefore B = 50\,\text{kHz}$



$$G_{ni}(f) = G_{nq}(f) = \begin{cases} G_{nn}(f-f_c) + G_{nn}(f+f_c) & |f| \leq 50\,\text{kHz} \\ 0 & \text{otherwise} \end{cases}$$



$\Rightarrow$



$$\therefore \; G_{ni}(f) = G_{nq}(f) = 10^{-3}\,\Pi\left(\frac{f}{0.1\,\text{MHz}}\right).$$

The power of the noise (N) is :

$$N = \int_{-0.55\times10^6}^{-0.45\times10^6} (10^{-9}f + 10^{-3})\,df + \int_{0.45\times10^6}^{0.55\times10^6} (-10^{-9}f + 10^{-3})\,df$$

this integration can easily be found by calculating the area under the curves.

$N = 2\,[\text{area of rectangle} + \text{area of triangle}]$

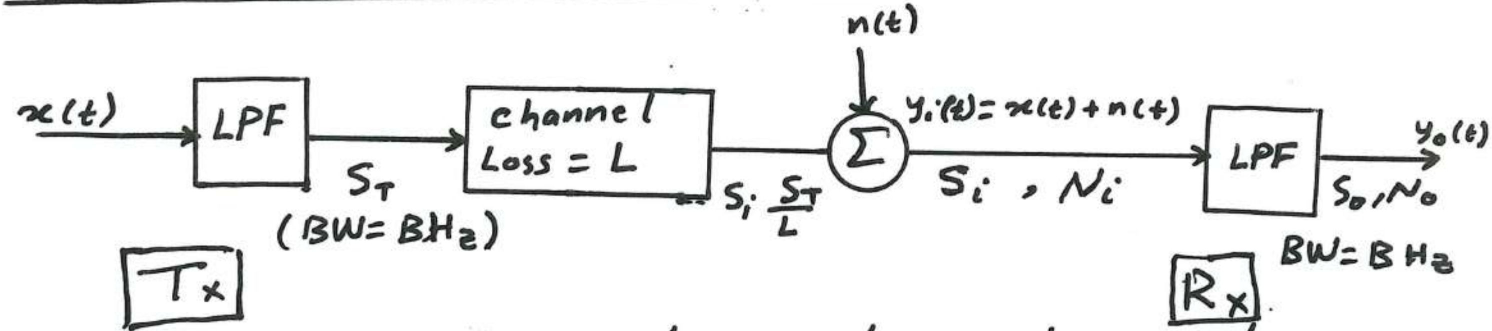
$$N = 2\left[100\times10^{3}\times 4.5\times10^{-4} + \tfrac{1}{2}\times100\times10^{3}\times(5.5\times10^{-4} - 4.5\times10^{-4})\right]$$

$$\boxed{N = 100\ \text{Watt}}$$

$$N_q = N_i = \int_{-0.05\times10^6}^{0.05\times10^6} 10^{-3}\,df = 10^{-3}\times 0.1\times10^{6} \Rightarrow \boxed{N_i = N_q = 100\,\text{W} = N}$$

Noise in Analog Modulation

1. Noise in Baseband Communication Systems:



In baseband communication systems, the signal is transmitted directly without any modulation. The channel is assumed to be distortionless.

$$(S/N)_o = \frac{\text{Output signal power } S_o}{\text{Output noise power } N_o}$$

For distortionless transmission $\boxed{S_o = S_i}$

and for white noise : $N_o = \int_{-\infty}^{\infty} G_{nn}(f)\,df = \int_{-B}^{B} \frac{\eta}{2}\,df$

$$\boxed{N_o = \eta B}$$

$$\therefore \frac{S_o}{N_o} = \frac{S_i}{\eta B} \Rightarrow \boxed{(S/N)_o = \frac{S_i}{\eta B}}$$

$$\therefore \boxed{\gamma = \frac{S_i}{\eta B}} \qquad \boxed{\gamma = (S/N)_o}$$

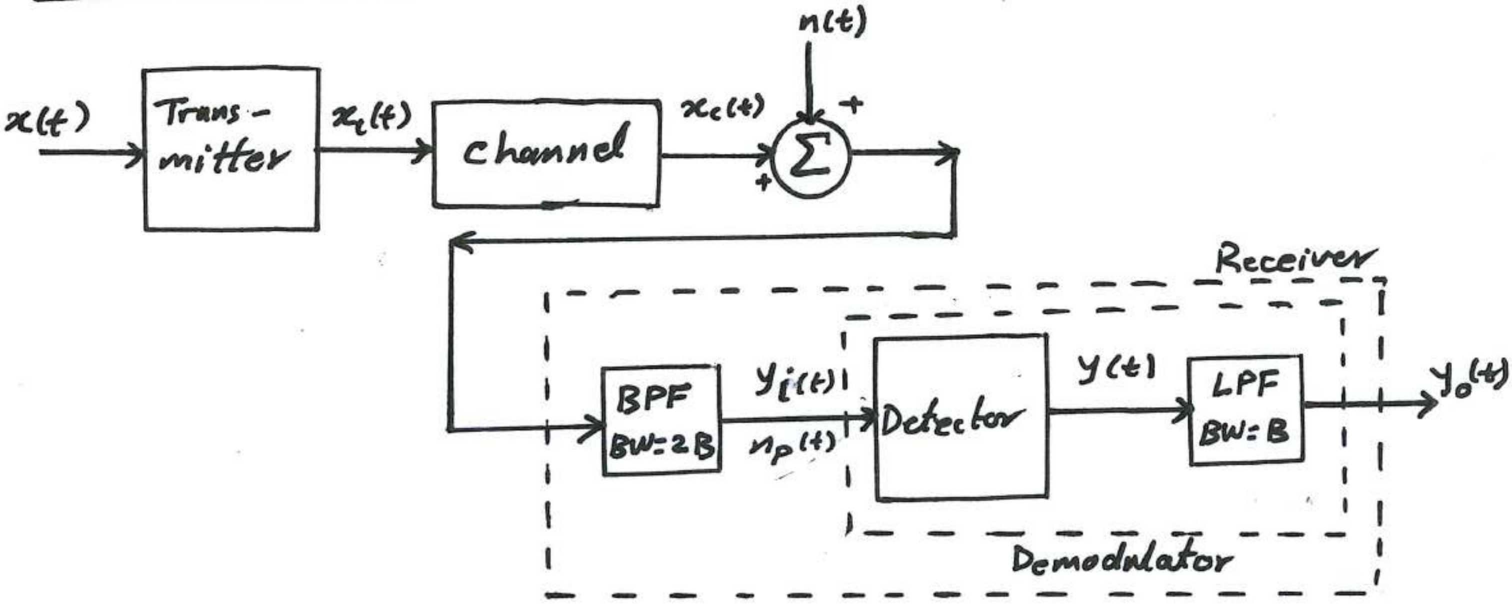
γ is a parameter denotes the output signal-to-noise ratio of the baseband communication systems. It is going to be used for comparison with SNR of different systems.

Since: $S_i = \frac{S_T}{L}$

$$\therefore \boxed{\gamma = \frac{S_T}{L \eta B}}$$

where S_T is the signal transmitted power.

2. Noise in Linear Modulation:



$$y_i(t) = x_c(t) + n_p(t)$$

The BPF limits the amount of noise to $n_p(t)$ which can be represented by the inphase-quadrature phase form as follows:

$$n_p(t) = n_i(t)\cos(2\pi f_c t) - n_q(t)\sin(2\pi f_c t)$$

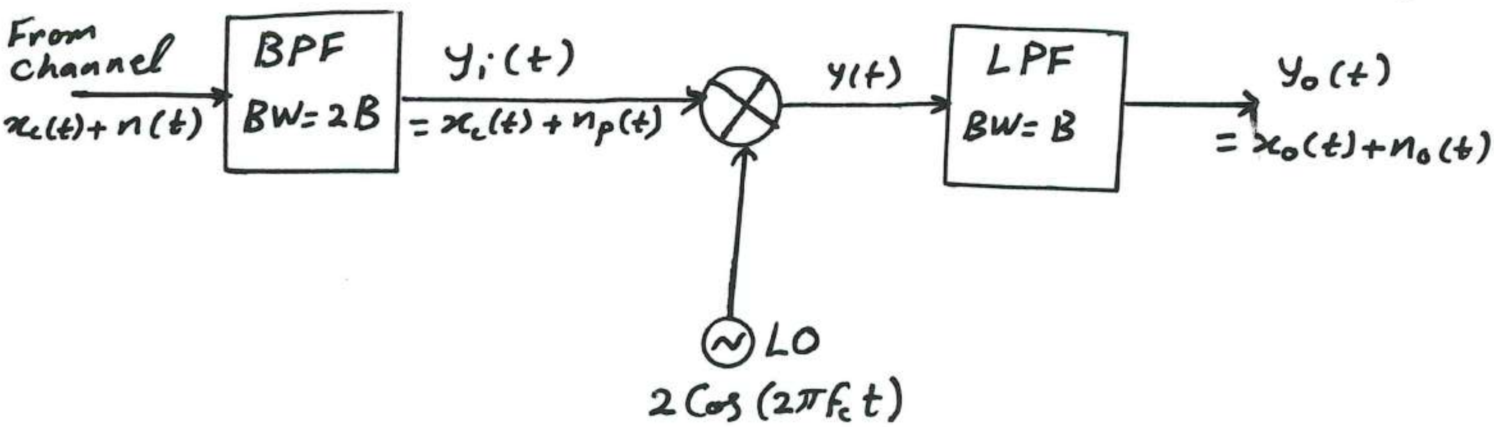
Remember that:

$$\boxed{\overline{n_p^2} = \overline{n_i^2} = \overline{n_q^2} = 2\eta B}$$

in otherwords $N_p = N_i = N_q = 2\eta B$

2.1 DSB-SC System:

The synchronous detection of DSB-SC system is shown below:



$$x_c(t) = A_c\, m(t) \, Cos(2\pi F_c t)$$

S_i is the input power content $\Rightarrow$ $\boxed{S_i = \frac{1}{2} A_c^2 \, \overline{m^2(t)}}$

$$y_i(t) = x_c(t) + n_p(t)$$

$$y_i(t) = A_c\, m(t) \, Cos(2\pi f_c t) + n_i(t) \, Cos(2\pi f_c t) - n_q(t) \, Sin(2\pi f_c t)$$

$$y_i(t) = (A_c\, m(t) + n_i(t)) \, Cos(2\pi f_c t) - n_q(t) \, Sin(2\pi F_c t)$$

$$y(t) = y_i(t) \cdot 2\, Cos(2\pi f_c t)$$

$$y(t) = 2[A_c\, m(t) + n_i(t)] \, Cos^2(2\pi F_c t) - 2 n_q(t) \, Sin(2\pi f_c t) \cdot Cos(2\pi F_c t)$$

$$y(t) = [A_c\, m(t) + n_i(t)] \, (1 + Cos^2(2\pi (2f_c) t)) - n_q(t) \, Sin(2\pi (2f_c) t)$$

After LPF with BW = B Hz

$$y_o(t) = \underbrace{A_c\, m(t)}_{\text{Signal}} + \underbrace{n_i(t)}_{\text{noise}}$$

$$\therefore (S/N)_o = \frac{S_o}{N_o} = \frac{S_o}{N_i}$$

but $\boxed{S_o = A_c^2 \, \overline{m^2(t)} = 2S_i}$

$$N_o = 2\eta B = N_i$$

$$(S/N)_o = \frac{2S_i}{2\eta B} \Rightarrow \boxed{(S/N)_o = \frac{S_i}{\eta B} = \gamma}$$

Synchronous detection of DSB-SC

<u>Note:</u>

$$(S/N)_i = \frac{S_i}{N_i} = \frac{S_i}{2\eta B} = \frac{1}{2} \frac{S_i}{\eta B} \Rightarrow \boxed{(S/N)_i = \frac{1}{2}\gamma}$$

γ_d is the detection gain of the detector.

$$\boxed{\gamma_d = \frac{(S/N)_o}{(S/N)_i} = 2}$$

For synchronus detection of DSB-SC