c) $\beta = 5$:

$J_0(5) \cong -0.18$ $(n=0)$ $|J_0(5)| > 0.01$

$J_1(5) \cong -0.33$ $(n=1)$ $|J_1(5)| > 0.01$

$J_2(5) \cong 0.05$ $(n=2)$ $|J_2(5)| > 0.01$

$J_3(5) \cong 0.36$ $(n=3)$ $|J_3(5)| > 0.01$

$J_4(5) \cong 0.39$ $(n=4)$ $|J_4(5)| > 0.01$

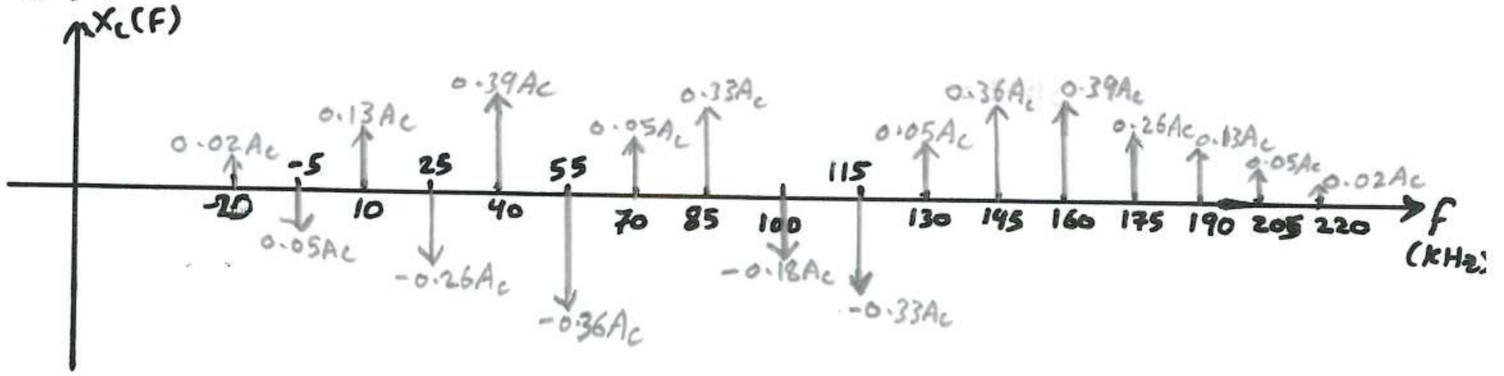
$J_5(5) \cong 0.26$ $(n=5)$ $|J_5(5)| > 0.01$

$J_6(5) \cong 0.13$ $(n=6)$ $|J_6(5)| > 0.01$

$J_7(5) \cong 0.05$ $(n=7)$ $|J_7(5)| > 0.01$

$J_8(5) \cong 0.02$ $(n=8)$ $|J_8(5)| > 0.01$

$J_9(5) \cong 0.006$ $(n=9)$ $|J_9(5)| < 0.01$ (not used)



Estimation 1:

$BW = 2 n f_m$ where $n = 8$

$BW = 2 \times 8 \times 15 \Rightarrow BW = 240\,kHz$

Estimation 2:

$BW = 2(\beta+1) f_m$

$BW = 2(5+1) \times 15 \Rightarrow BW = 180\,kHz$.

Estimation 3:

$BW = 2(\beta+2) f_m$

$BW = 2(5+2) \times 15 \Rightarrow BW = 210\,kHz$.

7. Bandwidth of Non-Sinusoidal Angle Modulation:

For non-sinusoidal message signal band-limited to (f_m), the bandpass signal has the deviation ratio (D) which is equivalent to the modulation index(β) in the tone angle modulation:

$$D = \frac{\text{Peak Freq. deviation}}{\text{messag BW } (f_m)}$$

$$\therefore \quad \boxed{D = \frac{\Delta f}{f_m}}$$

Thus, the BW can be estimated using Carson's rule:

$$\boxed{BW = 2(D+1) f_m} \quad *$$

or using the commercial broadcast estimation:

$$\boxed{BW = 2(D+2) f_m} \quad **$$

8. Power Content of Angle-Modulation:

As mentioned earlier:

$$x_c(t) = A_c \cos(2\pi f_c t + \phi(t))$$

The bandpass signal of the angle modulation is a sinusoidal signal with constant amplitude (A_c), so the normalized power of the angle modulation is:

$$\boxed{P = A_c^2/2}$$

The power in angle modulation is independent of the message signal $m(t)$.

9. Generation of Wideband Angle Modulation:

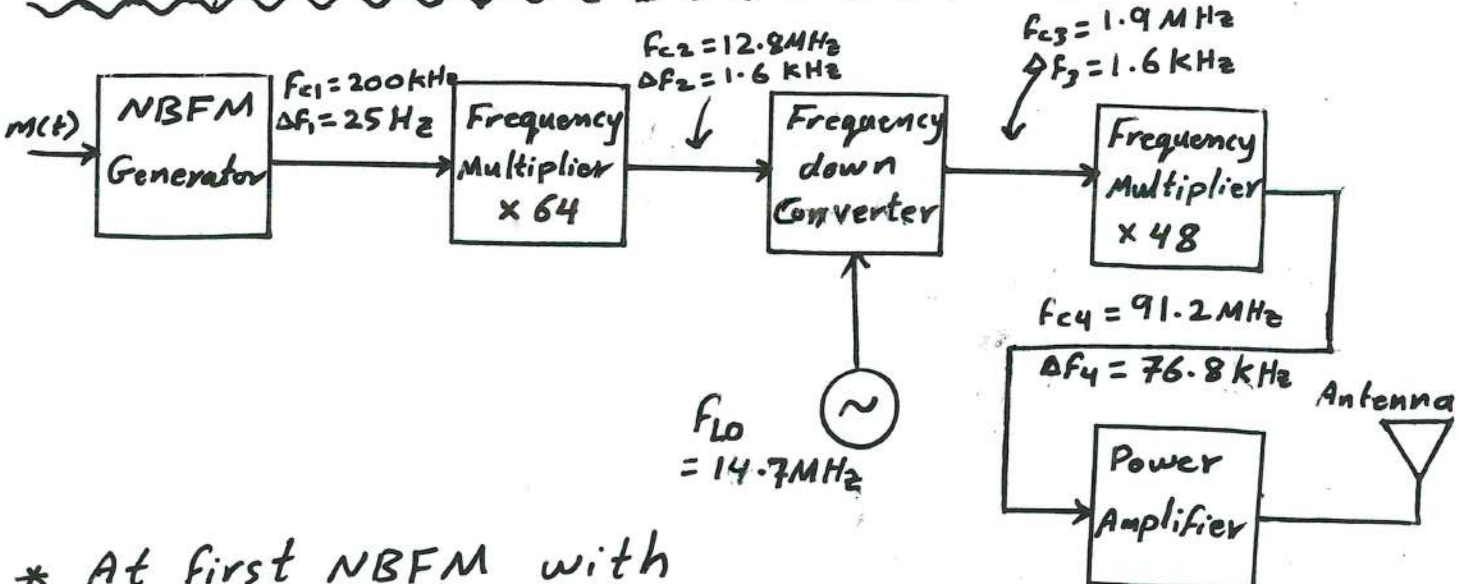
There are two methods of generating wideband angle-modulated signals:

[A] Indirect Method (Armstrong's Method):

In this method a narrowband (NB) angle-modulated signal is generated at first, then converted to wide-band (WB) angle modulated signal by using frequency multiplier. After that, the signal will be passed through a mixing circuit to shift the carrier frequency (f_c) down to a desirable and reasonable value.

Practical Example:

The Commercial Armstrong's FM transmitter:



* At first NBFM with $f_{c1} = 200\,kHz$ and $\Delta f_1 = 25\,Hz$ is generated

* About ($n = 3000$) frequency multiplier is hard to be implemented in practice, so two frequency multipliers with ($n_1 = 64$ and $n_2 = 48$) are used.

$$\therefore \quad n_1 \cdot n_2 = n$$
$$64 \times 48 = 3072$$

$$\therefore \quad \Delta f_4 = n_1 \cdot n_2 \times \Delta f_1$$
$$\Delta f_4 = 3072 \times 25 = 76.8\ kHz$$

* After the 1st multiplication

$$f_{c2} = 64 \times 200\,kHz = 12.8\ MHz$$
$$\Delta f_2 = 64 \times 25\,Hz = 1.6\,kHz$$

* It is required to down convert f_{c2} to $1.9\,MHz$, so the frequency of the local oscillator of the mixing circuit should be set as follows:

$$f_{LO} = f_{c2} + 1.9\,MHz = 14.7\,MHz$$

Thus, $f_{c3} = f_{LO} \pm f_{c2} = \begin{cases} 27.5\,MHz & \times \\ 1.9\,MHz & \checkmark \end{cases}$

$$\Delta f_3 = \text{unchanged} = \Delta f_2 = 1.6\,kHz$$

* After the 2nd multiplication:

$$f_{c4} = 48 \times f_{c3} = 48 \times 1.9\,MHz = 91.2\,MHz.$$
$$\Delta f_4 = 48 \times \Delta f_3 = 76.8\,kHz.$$

Notes: The indirect method of angle modulation generation has:

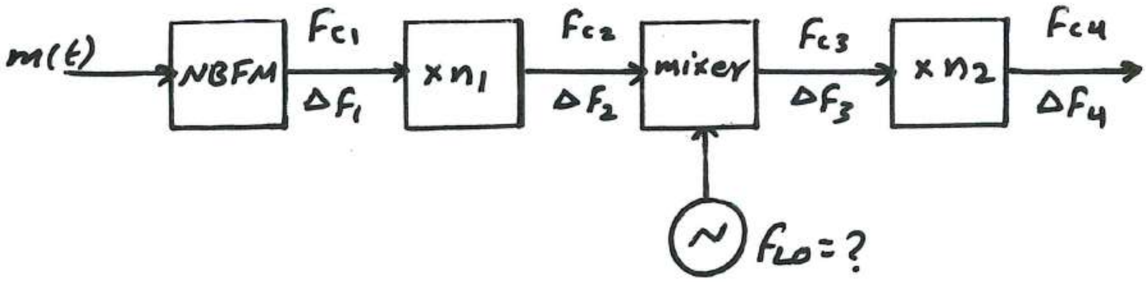
1- the advantage of frequency stability.

2- the disadvantage of the inherent noise caused by multiple stages as well as the excessive multiplications.

3- This method is widely used in commercial and military applications.

Example: Design an Armstrong's FM transmitter with NB carrier frequency equal to 200 kHz and NB deviation ratio (D = 0.2). The output carrier frequency is 108 MHz and the output peak frequency deviation is 75 kHz. Assume the message bandwidth is 50 Hz.

Sol.



Design Armstrong's transmitter means: find n_1, n_2, and f_{LO}

Given that: $f_{c1} = 200\,kHz$, $D = 0.2$, $f_m = 50\,Hz$

$\Delta f_4 = 75\,kHz$, $f_{c4} = 108\,MHz$

$$D = \frac{\Delta f_1}{f_m} \Rightarrow \Delta f_1 = D\, f_m = 0.2 \times 50$$

$$\Delta f_1 = 10\,Hz$$

but $\Delta f_4 = n_1 \times n_2 \times \Delta f_1$

$$\therefore n_1 \times n_2 = \frac{\Delta f_4}{\Delta f_1} \Rightarrow n_1 \times n_2 = \frac{75000}{10}$$

$$\therefore n_1 \times n_2 = 7500$$ {keep remember n_1 & n_2 are integers}

Let $\boxed{n_1 = 50}$

$$\therefore n_2 = \frac{7500}{50} \Rightarrow \boxed{n_2 = 150}$$

$f_{c2} = n_1 f_{c1}$

$f_{c3} = f_{LO} - f_{c2} = f_{LO} - n_1 f_{c1}$

$f_{c4} = n_2 (f_{c3}) = n_2 (f_{LO} - n_1 f_{c1})$

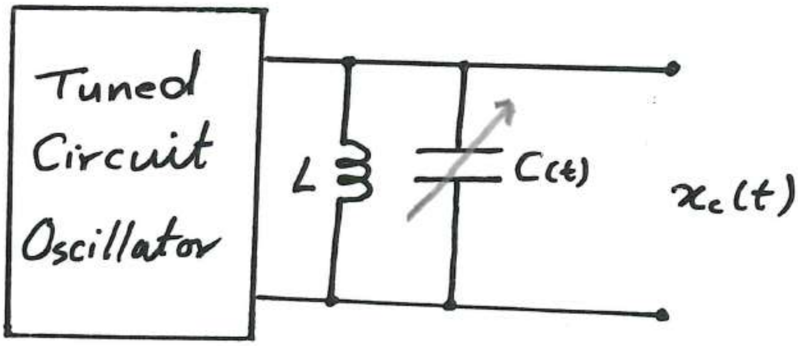
$\therefore 108\,MHz = 150\,(f_{LO} - 50 \times 0.2\,MHz)$, $0.2\,MHz = 200\,kHz$.

$$\boxed{f_{LO} = 10.72\,MHz}$$

B Direct Method:

In this method, the baseband signal $m(t)$ directly controls the carrier frequency. The common method that is used for generating FM directly is by varying the capacitance (C) of the tuned circuit oscillator in proportion to $m(t)$. This type of oscillators is called:

"Voltage Controlled Oscillator (VCO)"



* The variable capacitor that response to the applied voltage is called "voltage variable capacitor" or "varicap".
* Varactor Diode is a varicap responding to the amount or the applied reverse biasing voltage.
* The frequency of oscillation of the above circuit is given by:

$$f_i = \frac{1}{2\pi}\,\frac{1}{\sqrt{L\,C(t)}} \quad , \text{ where } C(t) = C_o + k\,m(t)$$

where C_o is the capacitor value when $m(t) = 0$, so

$$f_c = \frac{1}{2\pi}\,\frac{1}{\sqrt{L C_o}}$$

$$\therefore f_i = \frac{1}{2\pi}\,\frac{1}{\sqrt{L[C_o + k\,m(t)]}} \Rightarrow f_i = \frac{1}{2\pi}\,\frac{1}{\sqrt{L C_o}} \cdot \frac{1}{\sqrt{1 + \frac{k\,m(t)}{C_o}}}$$

$$\therefore f_i = \frac{1}{2\pi}\,\frac{1}{\sqrt{L C_o}}\left[1 + \frac{k}{C_o}\,m(t)\right]^{-1/2}$$

$$f_i = f_c\left[1 + \frac{k}{C_o}\right]^{-1/2}$$

but $\{(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots\}$

where $x = \frac{k}{C_o} m(t)$ and $n = -\frac{1}{2}$

$$f_i = f_c \left[1 - \frac{1}{2}\frac{k}{C_o} m(t) + \frac{3}{8}\left(\frac{k}{C_o} m(t)\right)^2 + \cdots\right]$$

but $\left|\frac{k}{C_o} m(t)\right| << 1$

$$\therefore \quad f_i \cong f_c \left[1 - \frac{1}{2}\frac{k}{C_o} m(t)\right]$$

$$\therefore \quad \boxed{f_i = f_c - \frac{k f_c}{2 C_o} m(t)} \equiv f_c + k_f m(t)$$

which is exactly equivalent to the instantaneous freq. of the FM signal.

Notes: The direct generation has:

1- Simpler circuit and lower inherent noise than the indirect method.

2- Frequency stabilization worse than that of the indirect method because it depends on variable capacitor whose value depends on the precision of the applied voltage and the capacitor age and quality.

3- In fact, the direct method can not produce BW as wide as the indirect method. Therefore, it is not used for modulation indeces $\beta > 1$.

10. Demodulation (Detection) of Angle Modulation:

The information of the angle modulation resides in the instantaneous frequency $\{f_i(t) = f_c + \frac{1}{2\pi}\frac{d\phi(t)}{dt}\}$. Therefore, the detection of FM signal requires a system that produces an output proportional to the instantaneous frequency.

The system that does this function is called "Frequency Discriminator".

A Ideal Discriminator:

It is known that:

$$x_c(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

$$f_i(t) = f_c + \frac{1}{2\pi} \frac{d\phi(t)}{dt}$$

then the output of the discriminator is

$$y_d(t) = K_d \frac{d\phi(t)}{dt}$$

where K_d is the discriminator sensitivity (Volt/Hz)

For FM modulation $\phi(t) = k_f \int_{-\infty}^{t} m(\tau)\, d\tau$

$$\therefore \boxed{y_d(t) = K_d k_f m(t)} \quad \text{for FM}$$

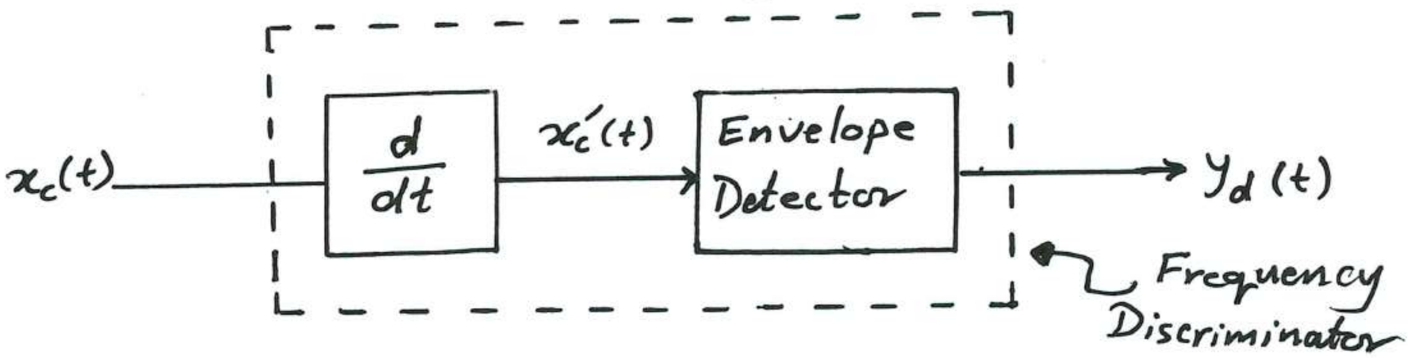
For PM modulation $\phi(t) = k_p m(t)$

$$\therefore \boxed{y_d(t) = K_d K_p \frac{dm(t)}{dt}} \quad \text{For PM}$$

and $m(t)$ can be obtained by integrating $y_d(t)$ for PM.

B Practical Implementation of Discriminator:

The discriminator can be implemented using a combination of differentiator followed by an evelope detector.



where $x_c(t) = A_c \cos[2\pi f_c t + \phi(t)]$

and $x_c'(t) = \underbrace{-A_c \left[2\pi f_c + \frac{d\phi(t)}{dt}\right]}_{\text{Envelope}} \cdot \sin(2\pi f_c t + \phi(t))$

$x_c'(t)$ is exactly a normal AM signal, so the envelope detector can be used extract the envelope $\left[A_c\left(2\pi f_c + \frac{d\phi(t)}{dt}\right)\right]$

$$\text{Envelope} = A_c\left[2\pi f_c + \frac{d\phi(t)}{dt}\right]$$

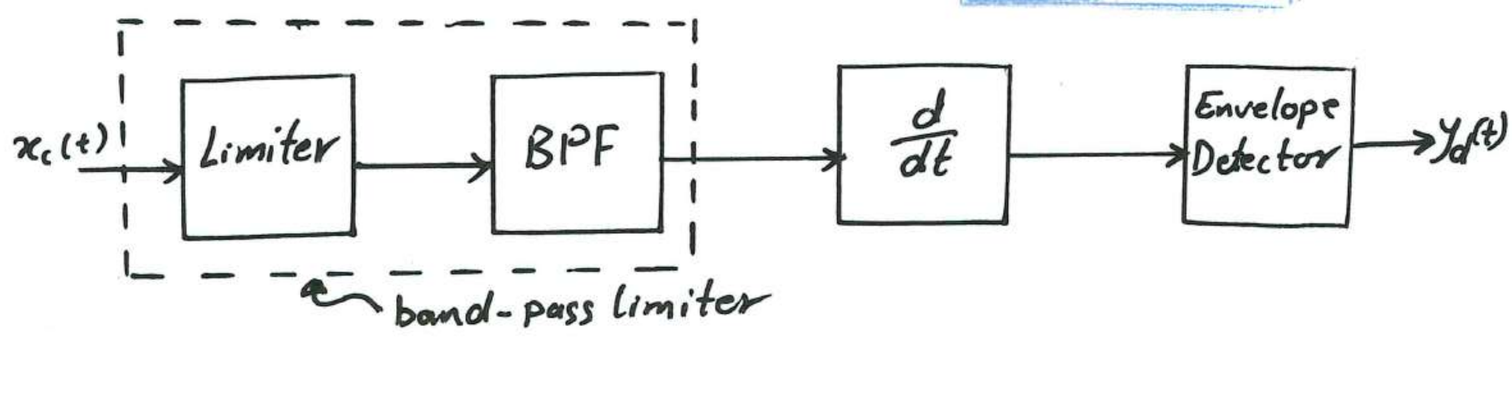
$$\text{Envelope} = \underbrace{2\pi f_c A_c}_{DC} + A_c \frac{d\phi(t)}{dt}$$

By using blocking capacitor, the dc would be removed

$$\boxed{y_d(t) = A_c \frac{d\phi(t)}{dt} = A_c k_f\, m(t)} \equiv k_d k_f\, m(t)$$

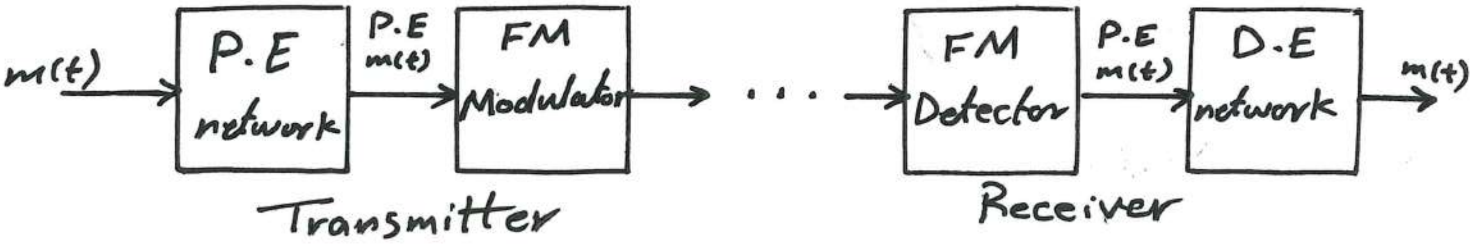
Note:

Unfortunately, A_c is not constant because of the channel noise, fading, and antenna noise. This variation can be solved by applying $x_c(t)$ to limiter placed before the discriminator. Since the limiter output is a square wave, a BPF having center frequency (f_c) is then placed after the limiter to convert the signal back to its original sinusoidal form which has constant A_c. The cascade combination of a limiter and BPF is known as Band-Pass Limiter.



11. Pre-Emphasis and De-Emphasis Networks:

In FM transmitter there is a circuit called Pre-emphasis (P.E) network, while the FM receiver has another network called de-emphasis (D.E) network.



* In commercial FM broadcasting, it is found that the speech and music having most of the energy at the lower frequencies and the noise rises parabolically with the frequency. Therefore, the noise spectral density is largest in the frequency range where the signal density is smallest.
* The P.E emphasizes (increases) the high frequency components of the signal before it undergoes the noise effect.

* The D.E de-emphasizes (decreases) the high frequency component of the received (signal + noise). As a result, the signal will get back its original components, while the noise is reduced. Consequently, the signal-to-noise ratio of the FM receiver is improved significantly.