Special Case: Narrowband Angle Modulation (NB):

When $\phi(t)$ is small $\{|\phi(t)|_{max} << 1\}$, the high order terms of the general equation can be neglected.

$$x_c(t) \cong A_c \cos(2\pi f_c t) - A_c \phi(t) \sin(2\pi f_c t)$$

This equation can be re-written in term of PM and FM modulation as follows:

PM $$\boxed{x_{NBPM} \cong A_c \cos(2\pi f_c t) - A_c[K_p\, m(t)] \sin(2\pi f_c t)}$$

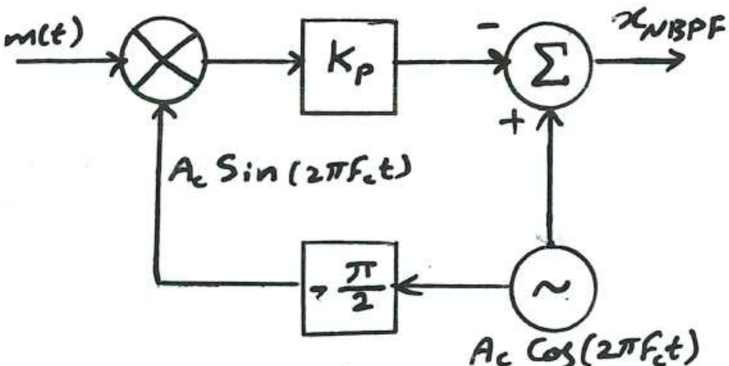
FM $$\boxed{x_{NBFM} \cong A_c \cos(2\pi f_c t) - A_c\left[K_f \int_{-\infty}^{t} m(\tau)\,d\tau\right] \sin(2\pi f_c t)}$$

The narrowband FM and PM is almost similar to the normal AM modulation with one difference: $\phi(t)$ is multiplied by 90° shifted cosine signal. Therefore, the bandwidth of the narrowband angle modulation is:

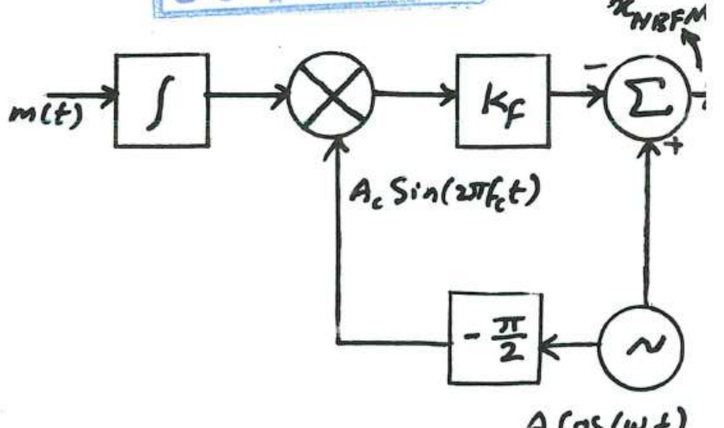
$$\boxed{BW_{NBPM} = BW_{NBFM} = 2f_m}$$

where f_m is the BW of the baseband signal (message).

The Generation of NBPM and NBFM are shown below:



Narrowband PM Modulator



Narrowband FM Modulator

4. Tone (Sinusoidal) Angle Modulation:

The single tone signal is always used by the system designers as a test function for the analog modulation. The message signal used for each modulation is given by:

$$m(t) = a_m \sin(2\pi f_m t) \qquad \text{For PM}$$

$$m(t) = a_m \cos(2\pi f_m t) \qquad \text{For FM}$$

for PM signal:

$$\phi(t) = k_p\, m(t) = k_p a_m \sin(2\pi f_m t)$$

for FM signal:

$$\phi(t) = k_f \int_{-\infty}^{t} m(\tau)\,d\tau = \frac{k_f a_m}{2\pi f_m} \sin(2\pi f_m t)$$

where k_p is the phase deviation constant, k_f is the frequency deviation constant, and f_m is the message bandwidth. The above two equations can be unified by one expression:

$$\boxed{\phi(t) = \beta \sin(2\pi f_m t)}$$

where β is the angle "modulation index".

$$\boxed{\beta = \begin{cases} k_p a_m & \text{for PM} \\ \dfrac{k_f a_m}{2\pi f_m} & \text{for FM} \end{cases}}$$

for FM signal: $k_f a_m = \Delta\omega = 2\pi \Delta f \quad \therefore \boxed{\Delta f = \frac{k_f a_m}{2\pi}}$

where Δf is the peak frequency deviation

$$\therefore \quad \beta = \frac{2\pi \Delta f}{2\pi f_m} \Rightarrow \boxed{\beta = \frac{\Delta f}{f_m}}$$

$$\boxed{\Delta f = |f_c - f_i|_{max} = \frac{k_f a_m}{2\pi f_m}}$$

5. Wideband Angle Modulation:

In term of modulation index of angle modulation:

Narrow Band $\beta \leq 0.2$

Wide Band $\beta > 0.2$

For tone message:

$$x_c(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

OR $x_c(t) = Re[A_c e^{j2\pi f_c t} \cdot e^{j\beta \sin(2\pi f_m t)}]$ — (*)

*The function $\{ e^{j\beta \sin(2\pi f_m t)} \}$ is periodic with period $T_o = \frac{1}{f_m}$ and can be expressed by exponential Fourier series as:

$$e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} D_n e^{j2\pi(nf_m)t}$$

where

$$D_n = \frac{1}{T_o}\int_{-T_o/2}^{T_o/2} x(t)\, e^{-j2\pi n f_m t}\, dt = \frac{1}{T_o}\int_{-T_o/2}^{T_o/2} e^{j\beta \sin(2\pi f_m)t} \cdot e^{j2\pi n f_m t}\, dt$$

$$D_n = \frac{1}{T_o}\int_{-T_o/2}^{T_o/2} e^{j(\beta \sin(2\pi f_m t) - 2\pi n f_m t)}\, dt$$

sub. $y = 2\pi f_m t \Rightarrow dy = 2\pi f_m\, dt$

$$\therefore dt = \frac{dy}{2\pi f_m}$$

when $t = T_o/2 = \frac{1}{2f_m}$ & $t = \frac{y}{2\pi f_m}$

$\therefore y = \pi$

similarly, when $t = -T_o/2 \Rightarrow y = -\pi$

$$\therefore D_n = \frac{1}{T_o}\int_{y=-\pi}^{\pi} e^{j(\beta \sin y - ny)} \frac{dy}{2\pi f_m}, \text{ when } \frac{1}{T_o} = f_m$$

$$\therefore \boxed{D_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta \sin y - ny)}\, dy = J_n(\beta)}$$ Bessel Integration

This integration is called "Bessel Function".

where

$J_n(\beta)$ is Bessel Function of the <u>1st kind</u> and <u>order (n)</u> and argument (β). Thus:

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi(nf_m)t}$$

sub. the above equation into equation ⊛ :

$$x_c(t) = R_e\left[A_c\, e^{j2\pi f_c t} \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi(nf_m)t}\right]$$

$$x_c(t) = A_c\, Re\left[\sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j(2\pi f_c t + 2\pi n f_m t)}\right]$$

$$x_c(t) = A_c\, Re\left[\sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi(f_c + nf_m)t}\right]$$

$$\therefore \quad \boxed{x_c(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\left(2\pi(f_c + nf_m)t\right)}$$

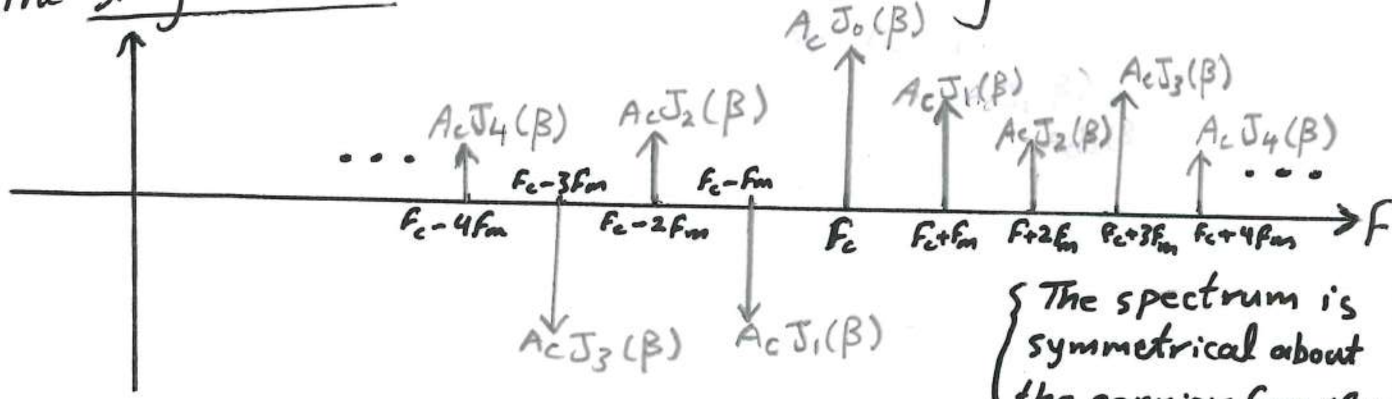
$$x_c(t) = \cdots + A_c J_{-2}(\beta) \cos\left(2\pi(f_c - 2f_m)t\right) + A_c J_{-1}(\beta) \cos\left(2\pi(f_c - f_m)t\right) +$$
$$+ A_c J_0(\beta) \cos(2\pi f_c t) + A_c J_1(\beta) \cos\left(2\pi(f_c + f_m)t\right) +$$
$$+ A_c J_2(\beta) \cos\left(2\pi(f_c + 2f_m)t\right) + \cdots$$

where $J_{-n} = \begin{cases} J_n(\beta) & \rightarrow n : \text{even} \\ -J_n(\beta) & \rightarrow n : \text{odd} \end{cases}$

$$\therefore \quad x_c(t) = \underbrace{A_c J_0(\beta) \cos(2\pi f_c t)}^{\text{carrier}} +$$
$$+ A_c J_1(\beta)\left[\cos\left(2\pi(f_c + f_m)t\right) - \cos\left(2\pi(f_c - f_m)t\right)\right] +$$
$$+ A_c J_2(\beta)\left[\cos\left(2\pi(f_c + 2f_m)t\right) + \cos\left(2\pi(f_c - 2f_m)t\right)\right] +$$
$$+ A_c J_3(\beta)\left[\cos\left(2\pi(f_c + 3f_m)t\right) - \cos\left(2\pi(f_c - 3f_m)t\right)\right] +$$
$$+ A_c J_4(\beta)\left[\cos\left(2\pi(f_c + 3f_m)t\right) + \cos\left(2\pi(f_c - 4f_m)t\right)\right] + \cdots$$

$$\vdots$$

The single-sided spectrum of this signal is:



{The spectrum is symmetrical about the carrier freq. (f_c)}

The value of $J_n(\beta)$ can be found from the following set of Bessel curves or from either of the tables given in page $\boxed{12}$ and $\boxed{13}$.

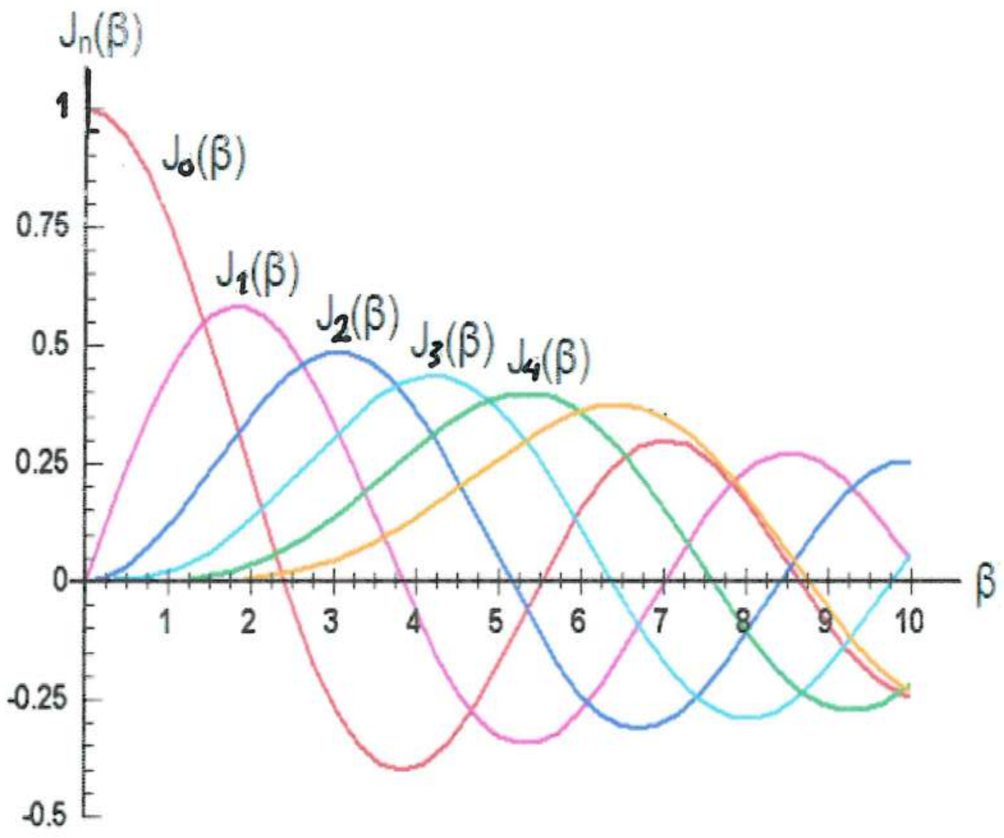


Table of Bessel Functions

β	$J_0(\beta)$	$J_1(\beta)$	$J_2(\beta)$	$J_3(\beta)$	$J_4(\beta)$	$J_5(\beta)$	$J_6(\beta)$	$J_7(\beta)$	$J_8(\beta)$	$J_9(\beta)$	$J_{10}(\beta)$
0	1	0	0	0	0	0	0	0	0	0	0
0.1	0.9975	0.0499	0.0012	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.9900	0.0995	0.0050	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.3	0.9776	0.1483	0.0112	0.0006	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.4	0.9604	0.1960	0.0197	0.0013	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.5	0.9385	0.2423	0.0306	0.0026	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.6	0.9120	0.2867	0.0437	0.0044	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.7	0.8812	0.3290	0.0588	0.0069	0.0006	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.8	0.8463	0.3688	0.0758	0.0102	0.0010	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
0.9	0.8075	0.4059	0.0946	0.0144	0.0016	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.7652	0.4401	0.1149	0.0196	0.0025	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
1.1	0.7196	0.4709	0.1366	0.0257	0.0036	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000
1.2	0.6711	0.4983	0.1593	0.0329	0.0050	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000
1.3	0.6201	0.5220	0.1830	0.0411	0.0068	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000
1.4	0.5669	0.5419	0.2074	0.0505	0.0091	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000
1.5	0.5118	0.5579	0.2321	0.0610	0.0118	0.0018	0.0002	0.0000	0.0000	0.0000	0.0000
1.6	0.4554	0.5699	0.2570	0.0725	0.0150	0.0025	0.0003	0.0000	0.0000	0.0000	0.0000
1.7	0.3980	0.5778	0.2817	0.0851	0.0188	0.0033	0.0005	0.0001	0.0000	0.0000	0.0000
1.8	0.3400	0.5815	0.3061	0.0988	0.0232	0.0043	0.0007	0.0001	0.0000	0.0000	0.0000
1.9	0.2818	0.5812	0.3299	0.1134	0.0283	0.0055	0.0009	0.0001	0.0000	0.0000	0.0000
2	0.2239	0.5767	0.3528	0.1289	0.0340	0.0070	0.0012	0.0002	0.0000	0.0000	0.0000
2.1	0.1666	0.5683	0.3746	0.1453	0.0405	0.0088	0.0016	0.0002	0.0000	0.0000	0.0000
2.2	0.1104	0.5560	0.3951	0.1623	0.0476	0.0109	0.0021	0.0003	0.0000	0.0000	0.0000
2.3	0.0555	0.5399	0.4139	0.1800	0.0556	0.0134	0.0027	0.0004	0.0001	0.0000	0.0000
2.4	0.0025	0.5202	0.4310	0.1981	0.0643	0.0162	0.0034	0.0006	0.0001	0.0000	0.0000
2.5	-0.0484	0.4971	0.4461	0.2166	0.0738	0.0195	0.0042	0.0008	0.0001	0.0000	0.0000
2.6	-0.0968	0.4708	0.4590	0.2353	0.0840	0.0232	0.0052	0.0010	0.0002	0.0000	0.0000
2.7	-0.1424	0.4416	0.4696	0.2540	0.0950	0.0274	0.0065	0.0013	0.0002	0.0000	0.0000
2.8	-0.1850	0.4097	0.4777	0.2727	0.1067	0.0321	0.0079	0.0016	0.0003	0.0000	0.0000
2.9	-0.2243	0.3754	0.4832	0.2911	0.1190	0.0373	0.0095	0.0020	0.0004	0.0001	0.0000
3	-0.2601	0.3391	0.4861	0.3091	0.1320	0.0430	0.0114	0.0025	0.0005	0.0001	0.0000
3.1	-0.2921	0.3009	0.4862	0.3264	0.1456	0.0493	0.0136	0.0031	0.0006	0.0001	0.0000
3.2	-0.3202	0.2613	0.4835	0.3431	0.1597	0.0562	0.0160	0.0038	0.0008	0.0001	0.0000
3.3	-0.3443	0.2207	0.4780	0.3588	0.1743	0.0637	0.0188	0.0047	0.0010	0.0002	0.0000
3.4	-0.3643	0.1792	0.4697	0.3734	0.1892	0.0718	0.0219	0.0056	0.0012	0.0002	0.0000
3.5	-0.3801	0.1374	0.4586	0.3868	0.2044	0.0804	0.0254	0.0067	0.0015	0.0003	0.0001
3.6	-0.3918	0.0955	0.4448	0.3988	0.2198	0.0897	0.0293	0.0080	0.0019	0.0004	0.0001
3.7	-0.3992	0.0538	0.4283	0.4092	0.2353	0.0995	0.0336	0.0095	0.0023	0.0005	0.0001
3.8	-0.4026	0.0128	0.4093	0.4180	0.2507	0.1098	0.0383	0.0112	0.0028	0.0006	0.0001
3.9	-0.4018	-0.0272	0.3879	0.4250	0.2661	0.1207	0.0435	0.0130	0.0034	0.0008	0.0002
4	-0.3971	-0.0660	0.3641	0.4302	0.2811	0.1321	0.0491	0.0152	0.0040	0.0009	0.0002
4.1	-0.3887	-0.1033	0.3383	0.4333	0.2958	0.1439	0.0552	0.0176	0.0048	0.0011	0.0002
4.2	-0.3766	-0.1386	0.3105	0.4344	0.3100	0.1561	0.0617	0.0202	0.0057	0.0014	0.0003
4.3	-0.3610	-0.1719	0.2811	0.4333	0.3236	0.1687	0.0688	0.0232	0.0067	0.0017	0.0004
4.4	-0.3423	-0.2028	0.2501	0.4301	0.3365	0.1816	0.0763	0.0264	0.0078	0.0020	0.0005
4.5	-0.3205	-0.2311	0.2178	0.4247	0.3484	0.1947	0.0843	0.0300	0.0091	0.0024	0.0006
4.6	-0.2961	-0.2566	0.1846	0.4171	0.3594	0.2080	0.0927	0.0340	0.0106	0.0029	0.0007
4.7	-0.2693	-0.2791	0.1506	0.4072	0.3693	0.2214	0.1017	0.0382	0.0122	0.0034	0.0008
4.8	-0.2404	-0.2985	0.1161	0.3952	0.3780	0.2347	0.1111	0.0429	0.0141	0.0040	0.0010
4.9	-0.2097	-0.3147	0.0813	0.3811	0.3853	0.2480	0.1209	0.0479	0.0161	0.0047	0.0012
5	-0.1776	-0.3276	0.0466	0.3648	0.3912	0.2611	0.1310	0.0534	0.0184	0.0055	0.0015

Bessel Functions of the First Kind, Jn(β)

β	J0	J1	J2	J3	J4	J5	J6	J7	J8	J9	J10
0.00	1.000	0.000									
0.20	0.990	0.100	0.005								
0.40	0.960	0.196	0.020	0.001							
0.60	0.912	0.287	0.044	0.004							
0.80	0.846	0.369	0.076	0.010	0.001						
1.00	0.765	0.440	0.115	0.020	0.002						
1.20	0.671	0.498	0.159	0.033	0.005	0.001					
1.40	0.567	0.542	0.207	0.050	0.009	0.001					
1.60	0.455	0.570	0.257	0.073	0.015	0.002					
1.80	0.340	0.582	0.306	0.099	0.023	0.004	0.001				
2.00	0.224	0.577	0.353	0.129	0.034	0.007	0.001				
2.20	0.110	0.556	0.395	0.162	0.048	0.011	0.002				
2.40	0.003	0.520	0.431	0.198	0.064	0.016	0.003	0.001			
2.60	-0.097	0.471	0.459	0.235	0.084	0.023	0.005	0.001			
2.80	-0.185	0.410	0.478	0.273	0.107	0.032	0.008	0.002			
3.00	-0.260	0.339	0.486	0.309	0.132	0.043	0.011	0.003			
3.20	-0.320	0.261	0.484	0.343	0.160	0.056	0.016	0.004	0.001		
3.40	-0.364	0.179	0.470	0.373	0.189	0.072	0.022	0.006	0.001		
3.60	-0.392	0.095	0.445	0.399	0.220	0.090	0.029	0.008	0.002		
3.80	-0.403	0.013	0.409	0.418	0.251	0.110	0.038	0.011	0.003	0.001	
4.00	-0.397	-0.066	0.364	0.430	0.281	0.132	0.049	0.015	0.004	0.001	
4.20	-0.377	-0.139	0.311	0.434	0.310	0.156	0.062	0.020	0.006	0.001	
4.40	-0.342	-0.203	0.250	0.430	0.336	0.182	0.076	0.026	0.008	0.002	
4.60	-0.296	-0.257	0.185	0.417	0.359	0.208	0.093	0.034	0.011	0.003	0.001
4.80	-0.240	-0.298	0.116	0.395	0.378	0.235	0.111	0.043	0.014	0.004	0.001
5.00	-0.178	-0.328	0.047	0.365	0.391	0.261	0.131	0.053	0.018	0.006	0.001
5.20	-0.110	-0.343	-0.022	0.327	0.398	0.287	0.153	0.065	0.024	0.007	0.002
5.40	-0.041	-0.345	-0.087	0.281	0.399	0.310	0.175	0.079	0.030	0.010	0.003
5.60	0.027	-0.334	-0.146	0.230	0.393	0.331	0.199	0.094	0.038	0.013	0.004
5.80	0.092	-0.311	-0.199	0.174	0.379	0.349	0.222	0.111	0.046	0.017	0.005
6.00	0.151	-0.277	-0.243	0.115	0.358	0.362	0.246	0.130	0.057	0.021	0.007
6.20	0.202	-0.233	-0.277	0.054	0.329	0.371	0.269	0.149	0.068	0.027	0.009
6.40	0.243	-0.182	-0.300	-0.006	0.295	0.374	0.290	0.170	0.081	0.033	0.012
6.60	0.274	-0.125	-0.312	-0.064	0.254	0.372	0.309	0.191	0.095	0.040	0.015
6.80	0.293	-0.065	-0.312	-0.118	0.208	0.363	0.326	0.212	0.111	0.049	0.019
7.00	0.300	-0.005	-0.301	-0.168	0.158	0.348	0.339	0.234	0.128	0.059	0.024
7.20	0.295	0.054	-0.280	-0.210	0.105	0.327	0.349	0.254	0.146	0.070	0.029
7.40	0.279	0.110	-0.249	-0.244	0.051	0.299	0.353	0.274	0.165	0.082	0.035
7.60	0.252	0.159	-0.210	-0.270	-0.003	0.266	0.354	0.292	0.184	0.096	0.043
7.80	0.215	0.201	-0.164	-0.285	-0.056	0.228	0.348	0.308	0.204	0.111	0.051
8.00	0.172	0.235	-0.113	-0.291	-0.105	0.186	0.338	0.321	0.223	0.126	0.061
8.20	0.122	0.258	-0.059	-0.287	-0.151	0.140	0.321	0.330	0.243	0.143	0.071
8.40	0.069	0.271	-0.005	-0.273	-0.190	0.092	0.300	0.336	0.261	0.160	0.083
8.60	0.015	0.273	0.049	-0.250	-0.223	0.042	0.273	0.338	0.278	0.178	0.096
8.80	-0.039	0.264	0.099	-0.219	-0.249	-0.007	0.241	0.335	0.292	0.197	0.110
9.00	-0.090	0.245	0.145	-0.181	-0.265	-0.055	0.204	0.327	0.305	0.215	0.125
9.20	-0.137	0.217	0.184	-0.137	-0.274	-0.101	0.164	0.315	0.315	0.233	0.140
9.40	-0.177	0.182	0.215	-0.090	-0.273	-0.142	0.122	0.297	0.321	0.250	0.157
9.60	-0.209	0.140	0.238	-0.040	-0.263	-0.179	0.077	0.275	0.324	0.265	0.173
9.80	-0.232	0.093	0.251	0.010	-0.245	-0.210	0.031	0.248	0.323	0.280	0.190
10.00	-0.246	0.043	0.255	0.058	-0.220	-0.234	-0.014	0.217	0.318	0.292	0.207

6. Bandwidth of Tone Angle Modulation:

It is clear that the spectrum of the angle modulation is extended to ∞. Therefore, the theoretical bandwidth of the angle modulation approaches to ∞. However, practically, there is an estimation for the effective spectrum bandwidth because the spectrum is getting smaller and smaller as it gets far from the carrier frequency f_c.

a Estimated BW Based on Significant Spectrum:

$J_n(\beta)$ for certain (β) is falling down as (n) increases. This bandwidth estimation based on the (n) values corresponding to $\boxed{|J_n(\beta)| \geq 0.01}$ and:

$$\boxed{BW = 2n f_m} \quad — (*)$$

where (n) is the largest Bessel order that gives $|J_n(\beta)| \geq 0.01$.

b Carson's Rule for BW Estimation:

It can be shown that 98% of the total power of the angle modulation is contained in a bandwidth approximately equal to:

$$\boxed{BW = 2(\beta + 1) f_m} \quad — (*)(*)$$

$$BW = 2(\beta f_m + f_m) \quad , \quad \text{but } \beta = \frac{\Delta f}{f_m}$$

$$\therefore \quad \boxed{BW = 2(\Delta f + f_m)}$$

This approximation is the most common one, so if the BW is required and the method is not specified "Carson's rule" should be used.

C Commercial broadcast Estimation:

Some commercial broadcast stations use the following approximation to estimate the angle modulation BW:

$$\boxed{BW = 2(\beta + 2) f_m} \quad \text{— ⊛⊛⊛}$$

Note: The spectrum of the angle modulation is sketched up to the value of (n) that gives $|J_n(\beta)| \geq 0.01$.

Example: An FM signal has a message signal frequency equal to 15kHz. Draw the single-sided spectrum of this signal then estimate its bandwidth using the significant spectrum method, Carlson's rule, and the commertial broadcast method for modulation index values: a) $\beta = 0.2$, b) $\beta = 1$, c) $\beta = 5$. and $f_c = 100\,kHz$.

Sol.

a) $\beta = 0.2$: { $\beta \leq 0.2$ narrow band FM }

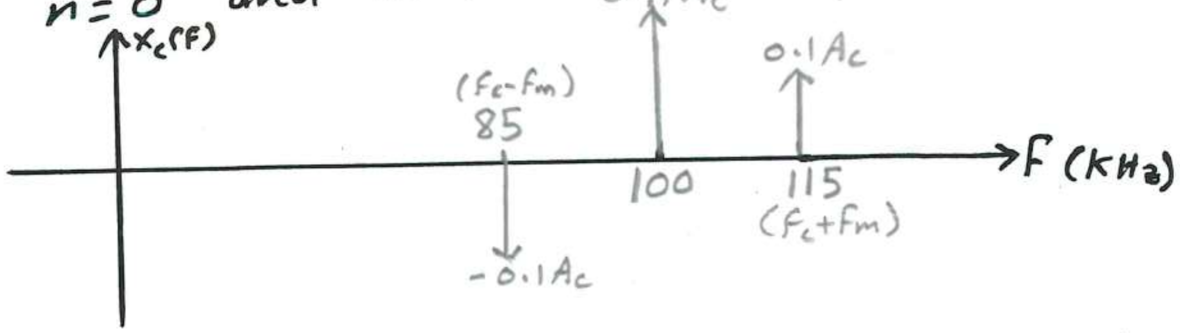
From Bessel function tables for $\beta = 0.2$

$J_0(0.2) \cong 0.99$ (n = 0) $|J_n(\beta)| > 0.01$

$J_1(0.2) \cong 0.1$ (n = 1) $|J_n(\beta)| > 0.01$

$J_2(0.2) = 0.005$ (n = 2) $|J_n(\beta)| < 0.01$ not used

Use only n = 0 and n = 1.



Estimation 1:

$BW = 2 n f_m$ where $n = 1$

$BW = 15 \times 2 = 30\,kHz$.

Estimation 2:

$$BW = 2(\beta+1) f_m$$

$$BW = 2(0.2+1)\times 15 \Rightarrow BW = 36\ kHz$$

Estimation 3:

$$BW = 2(\beta+2) f_m$$

$$BW = 2(0.2+2)\times 15 \Rightarrow BW = 66\ kHz.$$

b) $\beta = 1$:

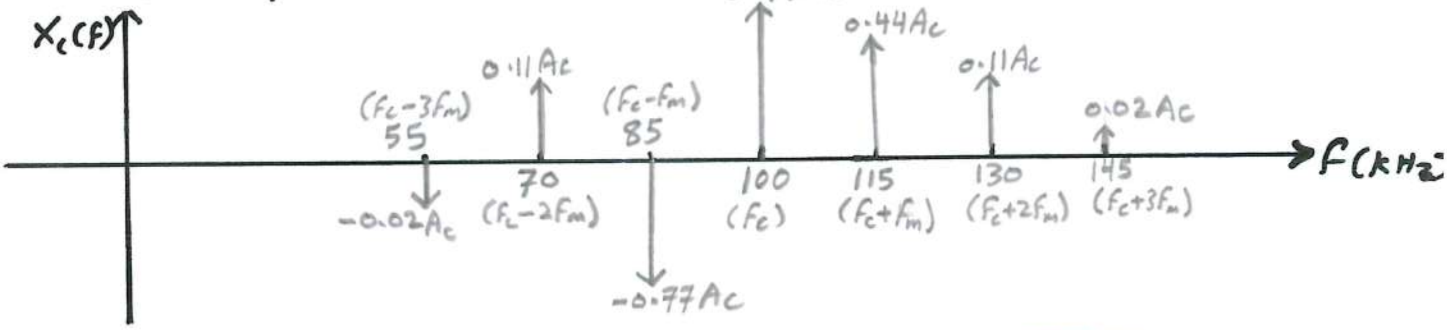
(n=0) $J_0(1) \cong 0.77 > 0.01$

(n=1) $J_1(1) \cong 0.44 > 0.01$

(n=2) $J_2(1) \cong 0.11 > 0.01$

(n=3) $J_3(1) \cong 0.02 > 0.01$

(n=4) $J_4(1) \cong 0.003 < 0.01$ (not used).



Estimation 1:

$$BW = 2n f_m \quad , \quad n = 3$$

$$BW = 2\times 3\times 15 \Rightarrow BW = 90\ kHz.$$

Estimation 2:

$$BW = 2(\beta+1) f_m$$

$$BW = 2(1+1)\times 15 \Rightarrow BW = 60\ kHz$$

Estimation 3:

$$BW = 2(\beta+2) f_m$$

$$BW = 2(1+2) f_m \Rightarrow BW = 90\ kHz$$