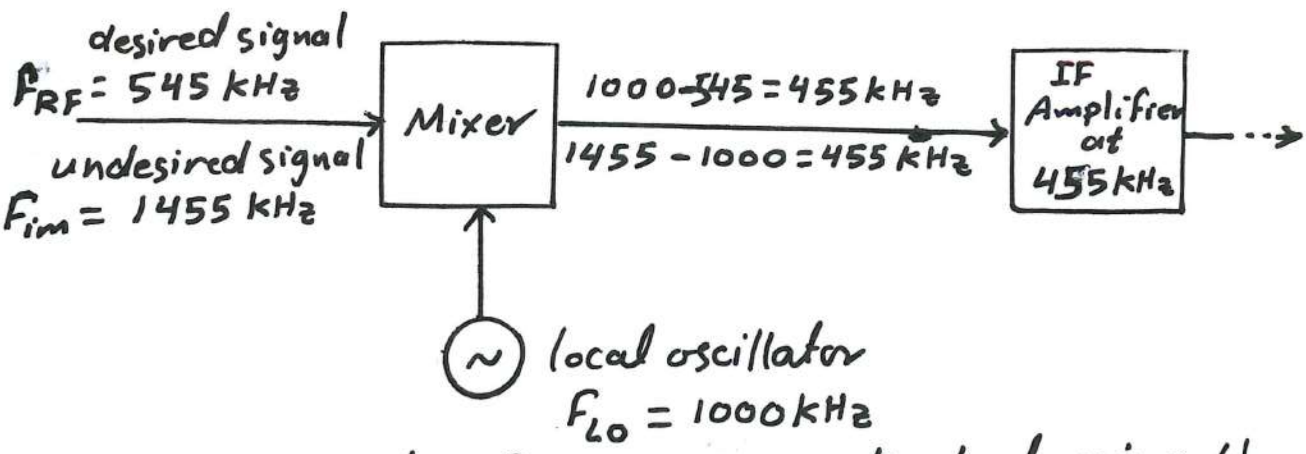


Local oscillator frequency is adjusted using the following equation :

$$f_{LO} = f_{RF} + f_{IF}$$

$$f_{LO} = 545 + 455 = 1000 \text{ kHz}$$

so that the difference between f_{OL} and f_{RF} is 455kHz.

However, the difference between the undesired signal (image signal) frequency is also 455kHz also.

$$f_{im} - f_{LO} = 455 \text{ kHz}$$

$$1455 - 1000 = 455 \text{ kHz}$$

where f_{im} denotes the image frequency.

where $\boxed{f_{im} = f_{RF} + 2f_{IF}}$

* Therefore, this component should be eliminated at the BPF of the receiver because it can not be eliminated after the mixer, and both the desired and undesired signal interfere each other and can not be separated.

Rejection Ratio (α):

The rejection of an image frequency by a BPF is defined as: a ratio of the gain at the RF frequency to the gain at the image frequency.

$$\alpha = \frac{\text{Gain (at } f_{RF})}{\text{Gain (at } f_{im})}$$

$$\boxed{\alpha = \sqrt{1 + Q^2 \rho^2}}$$

where $\rho = \dfrac{f_{im}^2 - f_{RF}^2}{f_{RF}\, f_{im}}$

Q is the quality factor of the BPF.

Example: Determine the image frequency for a standard AM broadcast receiver using IF frequency of 455 kHz and tuned to a station of 700 kHz, then find f_{LO} too.

Sol. $f_{IF} = 455\,kHz$, $f_{RF} = 700\,kHz$

$f_{im} = ?$, $f_{LO} = ?$

$$f_{im} = f_{RF} + 2f_{IF}$$
$$= 700 + 2 \times 455 \Rightarrow \boxed{f_{im} = 1610\,kHz}$$

$$f_{LO} = f_{RF} + f_{IF}$$
$$= 700 + 455 \Rightarrow \boxed{f_{LO} = 1155\,kHz}$$

Example: For a broadband superheterodyne receiver, the loaded quality factor Q of the BPF of the receiver is 200, and the IF frequency is 455 kHz. Determine the frequency of the local oscillator, the image frequency, and the rejection ratio at the incoming frequency 1000 kHz.

Sol.

$f_{IF} = 455\,kHz$, $f_{RF} = 1000\,kHz$, $Q = 200$

$f_{LO} = ?$, $f_{im} = ?$, $\alpha = ?$

$$f_{LO} = f_{RF} + f_{IF} \Rightarrow \boxed{f_{LO} = 1455\,kHz}$$

$$f_{im} = f_{RF} + 2f_{IF} \Rightarrow \boxed{f_{im} = 1910\,kHz}$$

$$\rho = \frac{f_{im}^2 - f_{RF}^2}{f_{RF}\, f_{im}} = \frac{(1910)^2 - (1000)^2}{1000 \times 1910}$$

$$\therefore \quad \rho = 1.386$$

$$\alpha = \sqrt{1 + Q^2 \rho^2}$$

$$\alpha = \sqrt{1 + 200^2 \times 1.386^2} \Rightarrow \boxed{\alpha = 277.2}$$

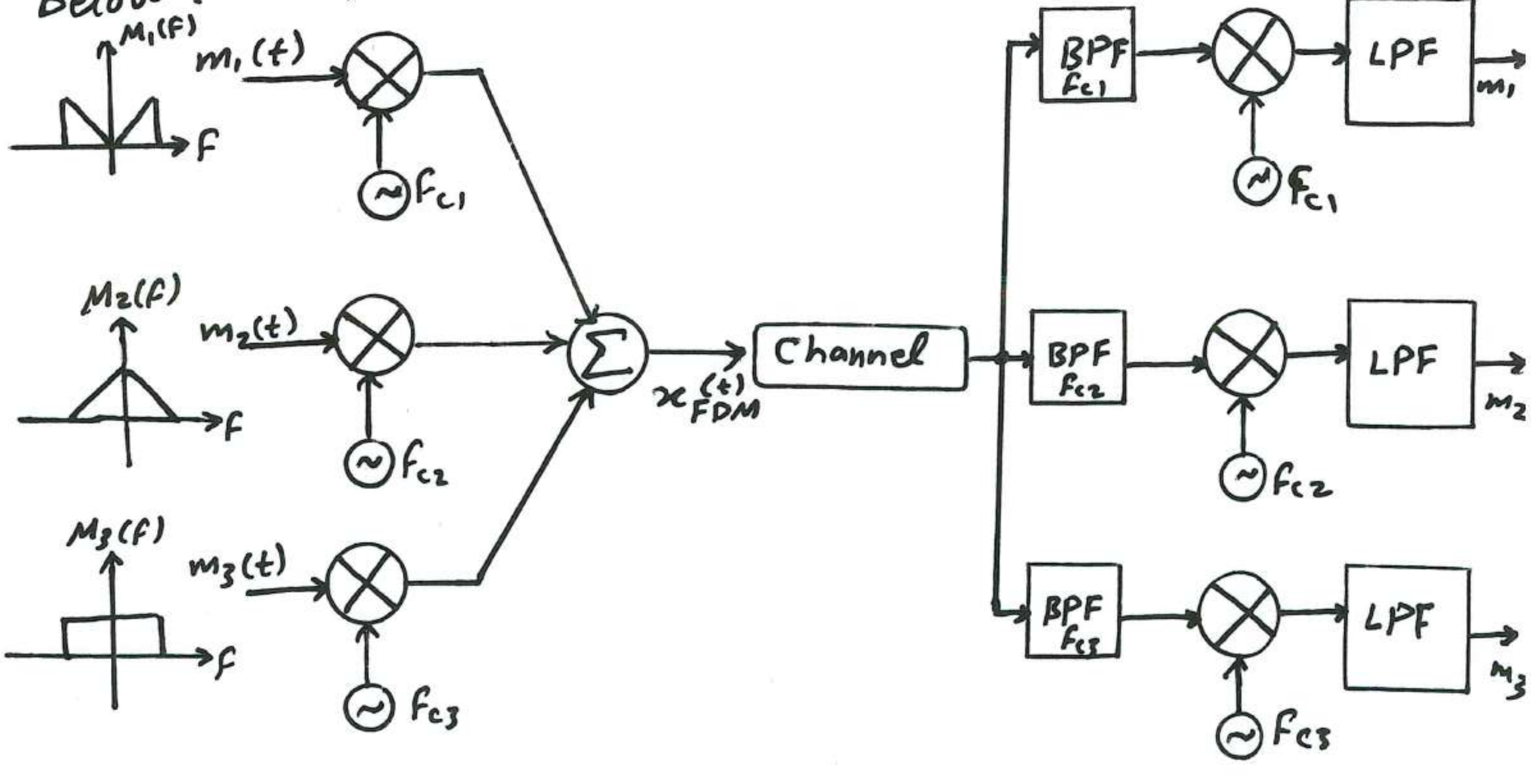
6. Frequency-Division Multiplexing (FDM):

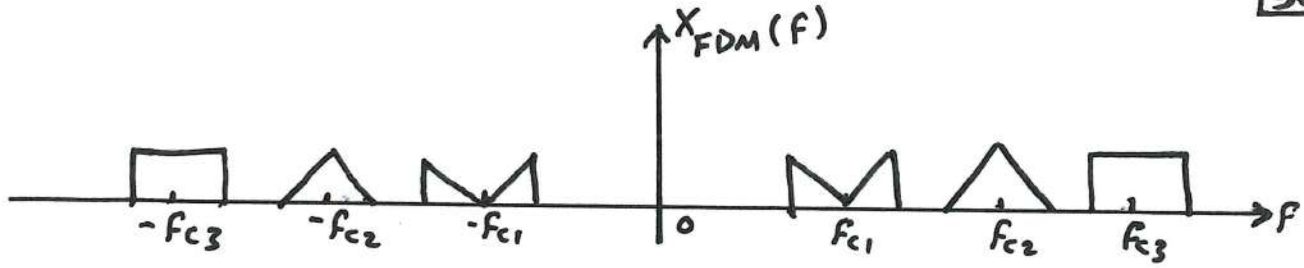
It is used when a large number of messages are to be sent over a common channel. The messages must be separated from each other in two ways:

1- Separated in time, so it is called Time Division multiplexing TDM. (will be studied later on).

2- Separated in frequency, and called Frequency Division Multiplexing FDM.

The block diagram of three messages FDM is shown below:





* In the previous system, DSB-SC is used to modulate each signal. However, any kind of modulation can be used in FDM as long as the carrier spacing is sufficient to avoid spectral overlap.

* At the receiver, the signals are separated by a BPF and then demodulated.

* FDM is used in Telephone systems, commercial broadcast TV, Satellite communications, and radio broadcasting.

Linear Modulation has:

1) Efficient utilization of the BW. The minimum BW between all the other modulation types.

2) High power consumption. Sometimes its transmitted power reaches to 10kW - 100kW.

3) Bad imunity against noise.

4) It is not used for data transmission.

Angle Modulation

1. Introduction:

Angle modulation (or exponential modulation) is the process by which the phase angle of the carrier signal is varied in accordance with the instantaneous value of the baseband signal $m(t)$.

Angle Modulation { Phase Modulation (PM) / Frequency Modulation (FM)

- In AM modulation, the spectrum of the bandpass signal is a shifted version of the spectrum of the baseband signal with bandwidth never exceed twice the message bandwidth.
- In angle modulation, the spectrum of the bandpass signal is not related to the message spectrum, and the bandwidth of the bandpass signal is much greater than twice the message bandwidth. However, it has better immunity against the noise and interference than the linear modulation.

2. Mathematical Representation of Angle Modulation

The amplitude of the carrier signal is constant, while either the phase or the time derivative of the phase of the carrier is varied linearly with the message signal.

$$x_c(t) = A_c \cos(w_c t + \Phi(t))$$

OR $$x_c(t) = A_c \cos(2\pi f_c t + \Phi(t))$$

$$x_c(t) = A_c \cos[\theta(t)]$$

where the overall phase (instantaneous phase) is given by:

$$\boxed{\begin{aligned}\theta(t) &= 2\pi f_c t + \phi(t)\\ \theta(t) &= \omega_c t + \phi(t)\end{aligned}}$$

* The instantaneous radian frequency (ω_i) is:

$$\omega_i(t) = \frac{d\theta(t)}{dt} = \omega_c + \frac{d\phi(t)}{dt}$$

$$\therefore \quad f_i(t) = \frac{1}{2\pi}\,\frac{d\theta(t)}{dt}$$

$$\boxed{f_i(t) = f_c + \frac{1}{2\pi}\,\frac{d\phi(t)}{dt}}$$

where $f_i(t)$ is the instantaneous frequency.

* $\phi(t)$ is also called the phase deviation, and $\frac{d\phi(t)}{dt}$ is called the frequency deviation.

* The peak (maximum) frequency deviation (Δf) is given by:

$$\boxed{\Delta f = |f_i - f_c|_{max}} \quad \text{OR} \quad \Delta\omega = |\omega_i - \omega_c|_{max}$$

$$\therefore \quad \boxed{\Delta f = \frac{1}{2\pi}\,\frac{d\phi(t)}{dt}\bigg|_{max}} \qquad \Delta\omega = \frac{d\phi(t)}{dt}\bigg|_{max}$$

- For PM:

The phase deviation $\phi(t)$ of the carrier is proportional to the message $m(t)$:

$$\boxed{\phi(t) = k_p\, m(t)}$$

where k_p is the phase deviation constant (rad./volt)

- For FM:

The frequency deviation ($\frac{d\phi(t)}{dt}$) is proportional to the message signal.

$$\frac{d\phi(t)}{dt} = k_f\, m(t)$$

$$\therefore \quad \boxed{\phi(t) = k_f \int_{t_0}^{t} m(\tau)\, d\tau + \phi(t_0)}$$

where k_f is the frequency deviation constant $\left(\frac{\text{rad.}}{\text{sec. Volt}}\right)$ 3

Usually, the initial value of the phase $\phi(t_0)$ is set to zero and $t_0 \rightarrow -\infty$, so

$$\boxed{\phi(t) = k_f \int_{-\infty}^{t} m(\tau)\, d\tau}$$

As a result, the general equation of each modulation type is given by:

$$\boxed{x_{PM}(t) = A_c \cos(2\pi f_c t + k_p m(t))} \quad PM$$

$$\boxed{x_{FM}(t) = A_c \cos(2\pi f_c t + k_f \int_{-\infty}^{t} m(\tau)\, d\tau)} \quad FM$$

* To illustrate the PM and FM signals, the instantaneous frequency of each modulation should be calculated:

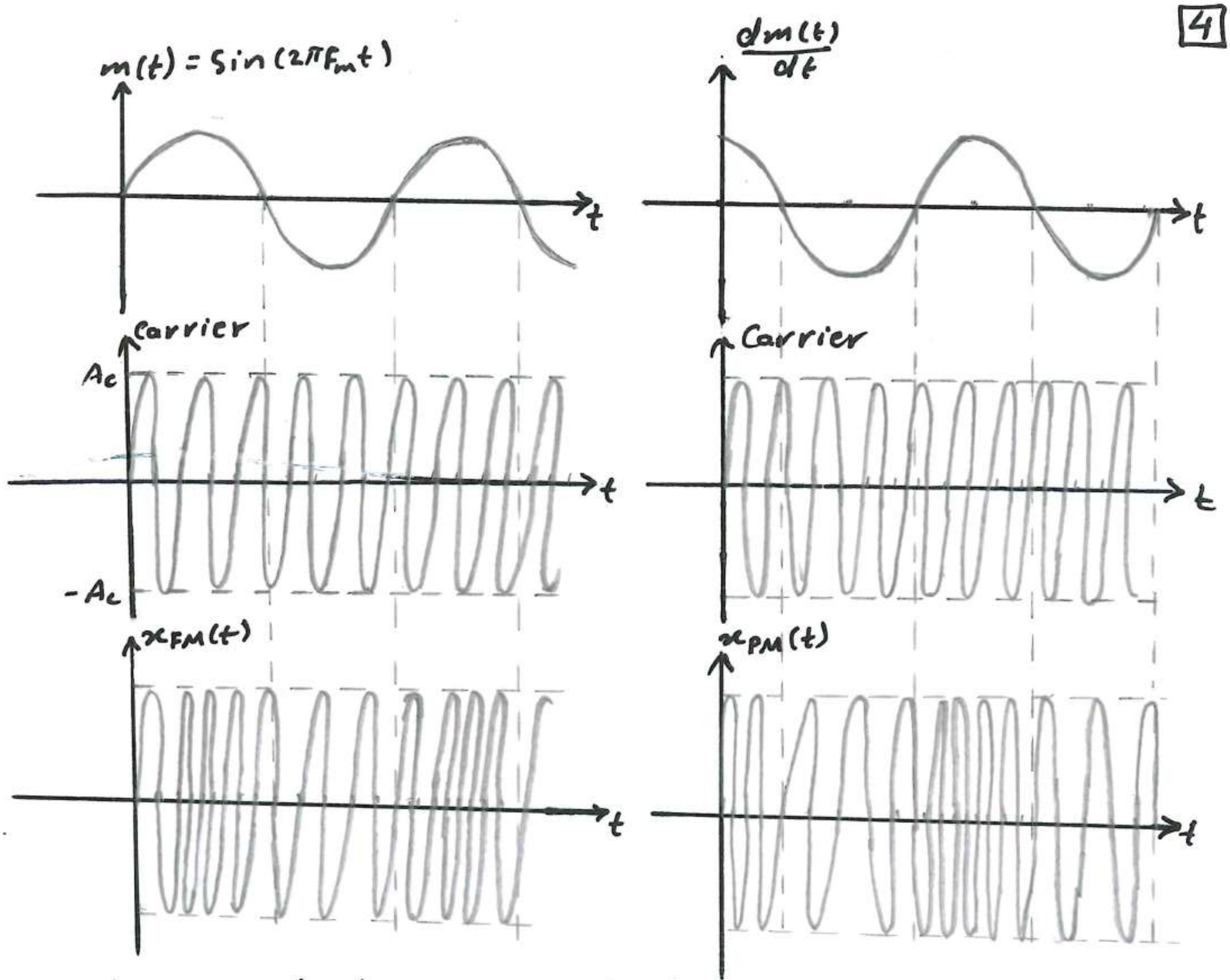
$$f_i(t) = f_c + \frac{1}{2\pi} k_p \frac{dm(t)}{dt} \Rightarrow \text{For PM}$$

$$f_i(t) = f_c + \frac{1}{2\pi} k_f\, m(t) \Rightarrow \text{For FM}$$

where $f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}$ as mentioned earlier. Therefore, to sketch the PM signal $m(t)$ need to be derived at first.

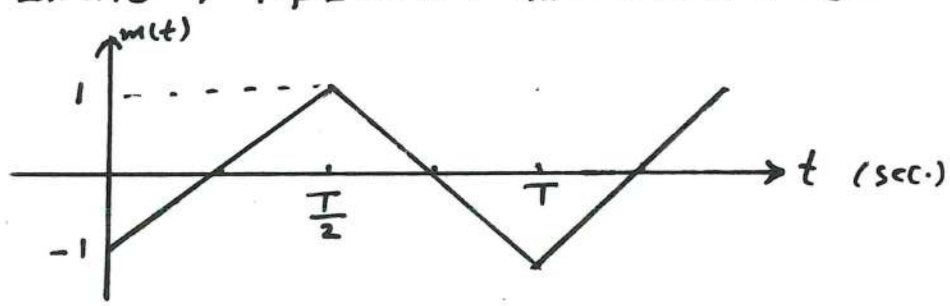
$$f_i(t) = \begin{cases} \text{linearly varies with } \frac{dm(t)}{dt} & \text{for PM} \\ \text{linearly varies with } m(t) & \text{for FM} \end{cases}$$

* It should be noted that if the signal $m(t)$ is not shown, it is not possible to distinguish between PM and FM signals.



Example: Find the FM and the PM for the baseband signal shown below, and find the maximum and minimum frequency and the maximum (peak) frequency and phase deviations.

where $k_f = 2\pi \times 10^5$, $k_p = 10\pi$, $f_m = 5\,kHz$, $f_c = 100\,MHz$



Sol.

$$T = \frac{1}{f_m} = \frac{1}{5\times10^3} = 2\times10^{-4}\ sec.$$

For FM:

$$f_i(t) = f_c + \frac{1}{2\pi} k_f\, m(t)$$

$$f_i(t) = 100\,MHz + \frac{1}{2\pi} \times 2\pi\times10^5\, m(t)$$

$$f_i(t) = 100\,MHz + 10^5\, m(t)$$

$$f_i|_{min} = 100\,MHz + 10^5 \times (-1) \Rightarrow \boxed{f_i|_{min} = 99.9\ MHz}$$

$$f_i|_{max} = 100\ MHz + 10^5 \times (1) \Rightarrow \boxed{f_i|_{max} = 100.1\ MHz}$$

$$\Delta f = |f_c - f_i|_{max} = |100\,MHz - 99.9| = |100 - 100.1|$$

$$\boxed{\Delta f = 0.1\ MHz}$$

For PM:

$$f_i(t) = f_c + \frac{1}{2\pi} k_p \frac{dm(t)}{dt}$$

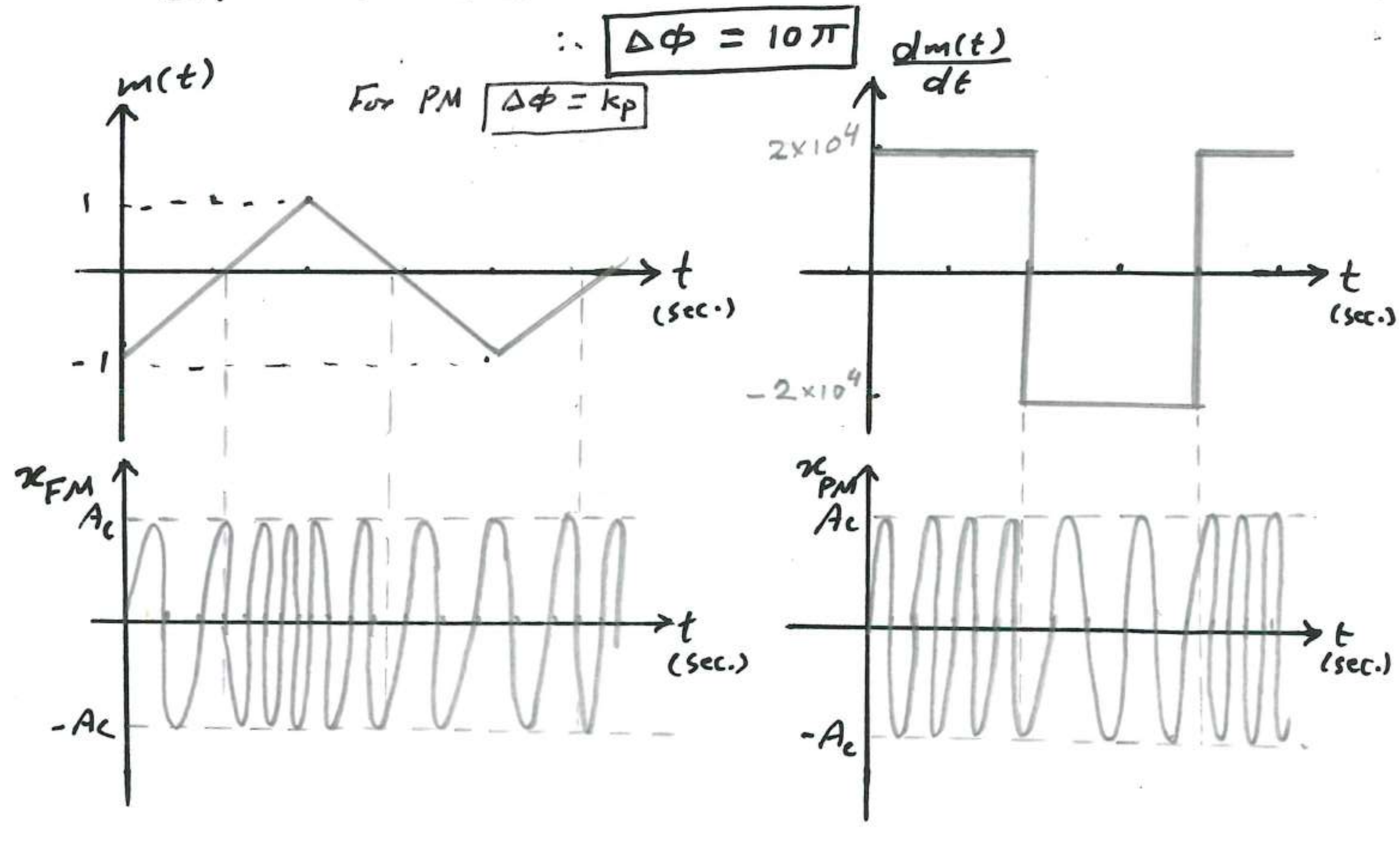
$$m(t) = \begin{cases} \frac{4t}{T} - 1 & 0 \leq t \leq \frac{T}{2} \\ \frac{-4t}{T} + 3 & \frac{T}{2} < t \leq T \end{cases} \Rightarrow \frac{dm(t)}{dt} = \begin{cases} \frac{4}{T} = 2\times10^4 & 0 \leq t \leq \frac{T}{2} \\ \frac{-4}{T} = -2\times10^4 & \frac{T}{2} < t \leq T \end{cases}$$

$$f_i|_{max} = 100\,MHz + \frac{1}{2\pi} \times 10\pi \times 2\times10^4 \Rightarrow \boxed{f_i|_{max} = 100.1\ MHz}$$

$$f_i|_{min} = 100\ MHz + \frac{1}{2\pi} \times 10\pi \times (-2\times10^4) \Rightarrow \boxed{f_i|_{min} = 99.9\ MHz}$$

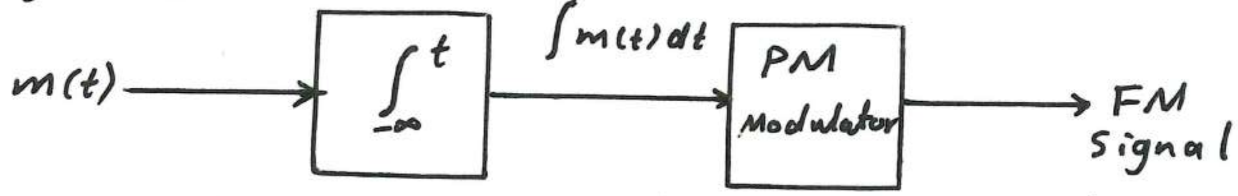
$$\Delta\phi = k_p m(t)|_{max} = 10\pi \times 1$$

$$\therefore \boxed{\Delta\phi = 10\pi}$$

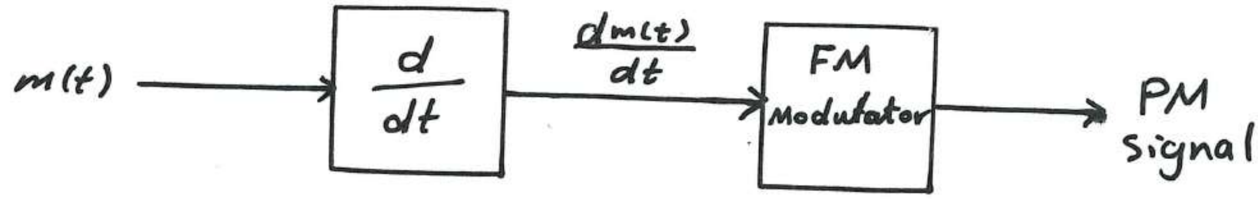


Notes:

- We can obtain FM signal from PM modulator by integrating $m(t)$.



- PM signal can be obtained from FM modulator by differentiating $m(t)$.



3. Frequency Spectra of Angle Modulation:

Recall that:

$$x_c(t) = A_c \cos[\omega_c t + \phi(t)]$$

OR

$$x_c(t) = Re\left[A_c e^{j(\omega_c t + \phi(t))}\right]$$

$$x_c(t) = Re\left[A_c e^{j\omega_c t} \cdot e^{j\phi(t)}\right]$$

but $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

by substituting $j\phi(t)$ instead of x

$$x_c(t) = Re\left[A_c e^{j\omega_c t}\left[1 + j\phi(t) - \frac{\phi^2(t)}{2!} - \frac{j\phi^3(t)}{3!} + \cdots + (j)^n \frac{\phi^n(t)}{n!} \cdots\right]\right]$$

$\therefore$ by substituting $e^{j\omega_c t} = \cos(\omega_c t) + j\sin(\omega_c t)$ and taking the real value of the resulted series.

$$x_c(t) = \underbrace{A_c \cos(\omega_c t)}_{\text{carrier}} \underbrace{- A_c \phi(t)\sin(\omega_c t) - A_c \frac{\phi^2(t)}{2!}\cos(\omega_c t) + \cdots}_{\text{side bands}}$$

Fourier Transform of the above signal consists of carrier signal plus the spectra of modulated $\phi(t), \phi^2(t), \phi^3(t), \cdots$