

* Now, let's determine the modulation index (μ) in term of A_{max} and A_{min} . 7

envelope: $A_c [1 + \mu \cos(2\pi f_m t)]$

$A_{max} = A_c [1 + \mu]$, $\{\max(\cos) = 1\}$

$A_{min} = A_c [1 - \mu]$, $\{\min(\cos) = -1\}$

$\therefore \frac{A_{max}}{A_{min}} = \frac{1 + \mu}{1 - \mu}$

$A_{max} - \mu A_{max} = A_{min} + \mu A_{min}$

$\mu(A_{max} + A_{min}) = A_{max} - A_{min}$

$\therefore \mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$

- Power Content in AM:

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$P_t = P_c + P_{LSB} + P_{USB}$

where: P_t represents the total power.

P_c represents the carrier power.

P_{LSB} is the Lower Side Band power.

P_{USB} is the Upper Side Band power.

$P_t = \frac{V_c^2}{R} + \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R}$

The normalized power is calculated by setting $R = 1 \Omega$.

$\therefore P_t = V_c^2 + V_{LSB}^2 + V_{USB}^2$

$x_c(t) = \underbrace{A_c \cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{\mu A_c m_n(t) \cos(2\pi f_c t)}_{\text{Double Side Band (DSB)}}$

For sinusoidal signal $P = \frac{A^2}{2} = \left(\frac{A}{\sqrt{2}}\right)^2$

$P_c = \frac{A_c^2}{2}$

$P_{DSB} = \frac{\mu^2 A_c^2 P_m}{2}$

where P_m is the power of the message $m_n(t)$.

$$P_{LSB} = P_{USB} = \frac{P_{DSB}}{2}$$

$$\therefore \boxed{P_{LSB} = \frac{\mu^2 A_c^2 P_m}{4} = P_{USB}}$$

For tone signal $\{m(t) = a_m \cos(2\pi f_m t)\}$

$$m_n(t) = \cos(2\pi f_m t)$$

$$P_m = \frac{1}{2}$$

$$\therefore \boxed{P_{LSB} = \frac{\mu^2 A_c^2}{8}}$$

$$\boxed{P_{USB} = \frac{\mu^2 A_c^2}{8}}$$

\therefore For tone signal:

$$P_t = \frac{A_c^2}{2} + \frac{\mu^2 A_c^2}{8} + \frac{\mu^2 A_c^2}{8}$$

$$P_t = \frac{A_c^2}{2} \left[1 + \frac{\mu^2}{4} + \frac{\mu^2}{4} \right] = \frac{A_c^2}{2} \left[1 + \frac{\mu^2}{2} \right]$$

$$\therefore \boxed{P_t = P_c \left[1 + \frac{1}{2} \mu^2 \right]} \quad \text{For tone signal only}$$

* Efficiency of AM (γ):

$$\boxed{\gamma = \frac{P_{DSB}}{P_t} \times 100\%}$$

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For tone signal:

$$\gamma = \frac{\frac{1}{4} \mu^2 A_c^2}{\frac{A_c^2}{2} \left[1 + \frac{1}{2} \mu^2 \right]} \times 100\%$$

$$\therefore \boxed{\gamma = \frac{\mu^2}{2 + \mu^2} \times 100\%}$$

For tone signal only

γ_{\max} occurs when $\mu = 1$.

$$\gamma_{\max} = \frac{1}{2 + 1} \times 100\% \Rightarrow \boxed{\gamma_{\max} = 33.33\%}$$

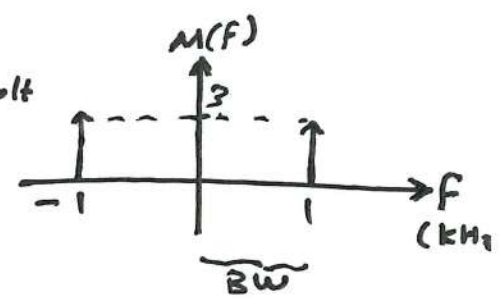
Example: A baseband signal $m(t) = 6 \cos(2 \times 10^3 \pi t)$ is used to modulate a carrier signal $10 \cos(2 \times 10^4 \pi t)$. If the modulation type is normal AM, then find:

- a) the bandwidth and the spectrum of the baseband signal
- b) the bandwidth and spectrum of the bandpass signal.
- c) the total power of the normal AM signal.
- d) the power of the single side band, and the modulation index.

Sol.

a) $m(t) = 6 \cos(2\pi(1 \text{ kHz})t)$, $a_m = 6 \text{ volt}$

$$M(f) = 3 [\delta(f - 1 \text{ kHz}) + \delta(f + 1 \text{ kHz})]$$



$\therefore \text{BW} = 1 \text{ kHz}$

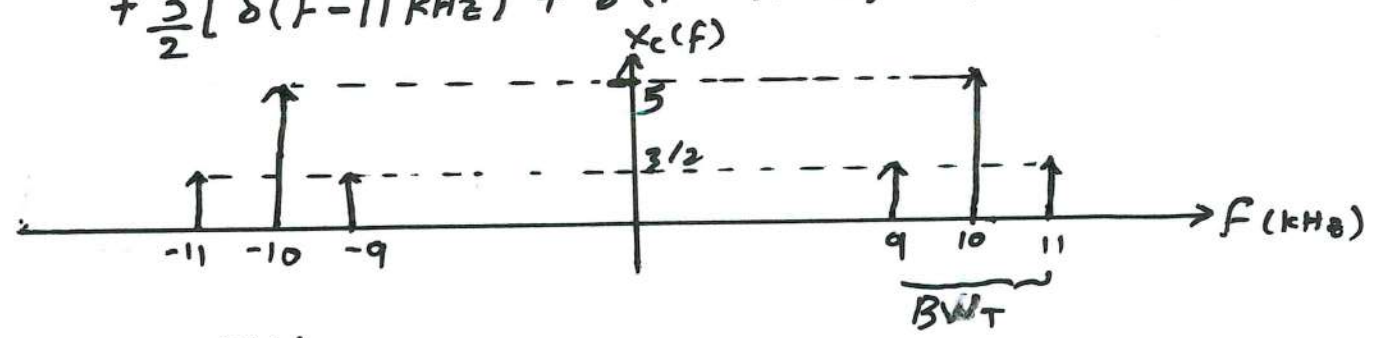
b) $x_c(t) = [A_c + m(t)] \cos(2\pi(f_c)t)$

$$x_c(t) = [10 + 6 \cos(2\pi f_m t)] \cos(2\pi(10 \text{ kHz})t)$$

$$x_c(t) = [10 + 6 \cos(2\pi(1 \text{ kHz})t)] \cos(2\pi(10 \text{ kHz})t)$$

$$X_c(f) = [10\delta(f) + 3[\delta(f - 1 \text{ kHz}) + \delta(f + 1 \text{ kHz})]] * \frac{1}{2} [\delta(f - 10 \text{ kHz}) + \delta(f + 10 \text{ kHz})]$$

$$X_c(f) = 5 [\delta(f - 10 \text{ kHz}) + \delta(f + 10 \text{ kHz})] + \frac{3}{2} [\delta(f - 11 \text{ kHz}) + \delta(f - 9 \text{ kHz}) + \delta(f + 11 \text{ kHz}) + \delta(f + 9 \text{ kHz})]$$



$\text{BW}_T = 2 \text{ BW}$

$\therefore \text{BW}_T = 2 \text{ kHz}$

$$c) m(t) = m_p m_n(t)$$

$$\therefore m_p = 6, \quad m_n(t) = \cos(200\pi t)$$

$$\mu = \frac{m_p}{A_c} = \frac{6}{10} = 0.6$$

$$P_T = P_c + P_{DSB}$$

$$P_c = \frac{A_c^2}{2} \Rightarrow P_c = 50 \text{ Watt}$$

$$P_{DSB} = \frac{A_c^2 \mu^2}{2} P_m, \quad \text{where } P_m = \frac{1}{2} \text{ \{power of } m_n(t)\}}$$

$$P_{DSB} = \frac{100 \times 0.6^2}{2} \times \frac{1}{2} = 9 \text{ Watt}$$

$$\therefore P_T = 50 + 9 \Rightarrow P_T = 59 \text{ Watt}$$

$$d) P_{SSB} = \frac{P_{DSB}}{2} \Rightarrow P_{SSB} = 4.5 \text{ watt}$$

$$\mu = \frac{m_p}{A_c} \Rightarrow \mu = 0.6$$

Example: A tone signal is used to modulate a certain carrier signal normal AM modulation. What is the suitable carrier power that leads to total power equal to 59 Watt at 60% modulation percentage.

Sol.

$$\mu = \frac{60}{100} = 0.6$$

$$P_T = 59 \text{ watt}$$

$$P_T = P_c + P_{DSB}$$

$$P_T = \frac{A_c^2}{2} + \frac{\mu^2 A_c^2}{2} P_m, \quad \text{tone signal } m_n(t) = \cos(2\pi f_m t)$$

$$\therefore P_m = \frac{1}{2}$$

$$P_T = \frac{A_c^2}{2} \left[1 + \frac{\mu^2}{2} \right]$$

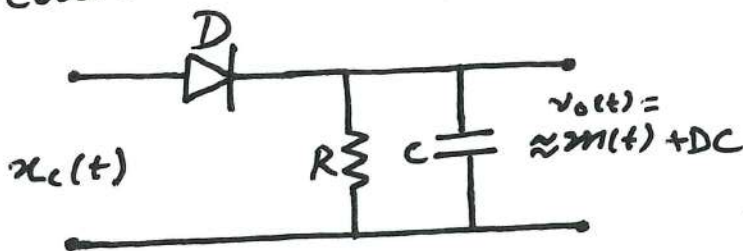
$$59 = \frac{A_c^2}{2} \left[1 + \frac{0.6^2}{2} \right] \Rightarrow A_c = 10 \text{ Volt}$$

$$\therefore P_c = \frac{A_c^2}{2} \Rightarrow P_c = 50 \text{ Watt}$$

- Demodulation (Detection) of AM:

a) Envelop Detector:

This detector is simply consists of a diode and (RC) circuit.



* During the +ve half cycle of the carrier, the diode (D) is forward biased and (C) charged up to the peak value of the carrier. As the signal falls below the max. value, the capacitor (C) slowly discharge through the resistance (R) until the next +ve half-cycle.

* The ripple of the output signal can be reduced by increasing the time constant of the RC-circuit.

$$\tau = R.C \gg \frac{1}{f_c}$$

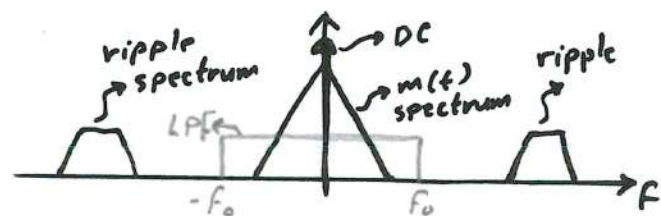
OR $\tau \gg T_c$

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But making (RC) too large would make it impossible for the capacitor voltage to follow the envelope, so

$$\tau = R.C < \frac{1}{f_m}$$

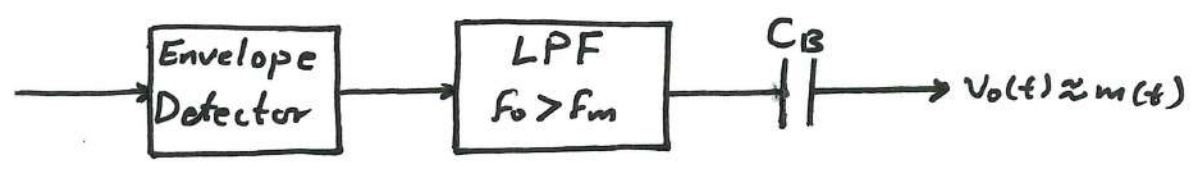
$$\therefore \frac{1}{f_c} \ll \tau < \frac{1}{f_m}$$



* Since the ripple is a high frequency component, it can also be eliminated by inserting a LPF with $f_0 > f_m$ after the detector.

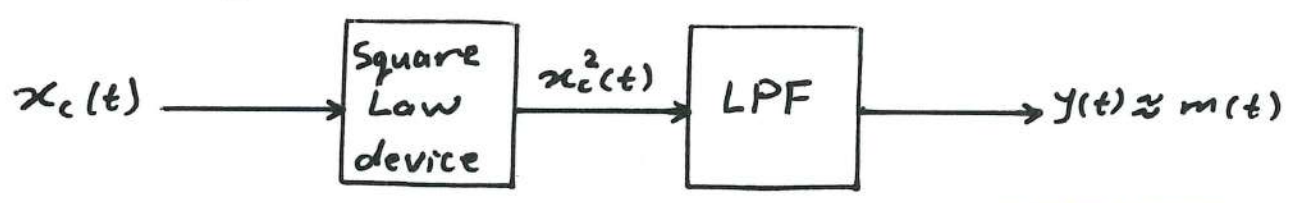


* It is clear that the output contains a DC component. This component can be canceled by a Blocking Capacitor (C_B) after the LPF.



b) The Square-Law Detector:

This circuit is used to detect $m(t)$ from $x_c(t)$ of the AM signal by squaring $x_c(t)$ then passing the result through a LPF.



$$x_c(t) = [A_c + m(t)] \cos(2\pi f_c t)$$

$$x_c^2(t) = [A_c + m(t)]^2 \cos^2(2\pi f_c t)$$

$$x_c^2(t) = [A_c^2 + 2A_c m(t) + m^2(t)] \times \frac{1}{2} (1 + \cos(4\pi f_c t))$$

$$x_c^2(t) = \frac{1}{2} [A_c^2 + 2A_c m(t) + m^2(t)] + \frac{1}{2} [A_c^2 + 2A_c m(t) + m^2(t)] \cos(4\pi f_c t)$$

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After passing this signal through a LPF with cut-off frequency f_0 slightly larger than f_m , then: the component that is multiplied by $\cos(2\pi(2f_0)t)$ is canceled $m^2(t)$ is also canceled. $\{f_m < f_0 < \text{frequency of } m^2(t)\}$

$$\therefore y(t) = \frac{1}{2} [A_c^2 + 2A_c m(t)]$$

$$y(t) = \frac{A_c^2}{2} [1 + 2\frac{m(t)}{A_c}] \text{ OR } \boxed{y(t) = \frac{A_c^2}{2} + A_c m(t)}$$

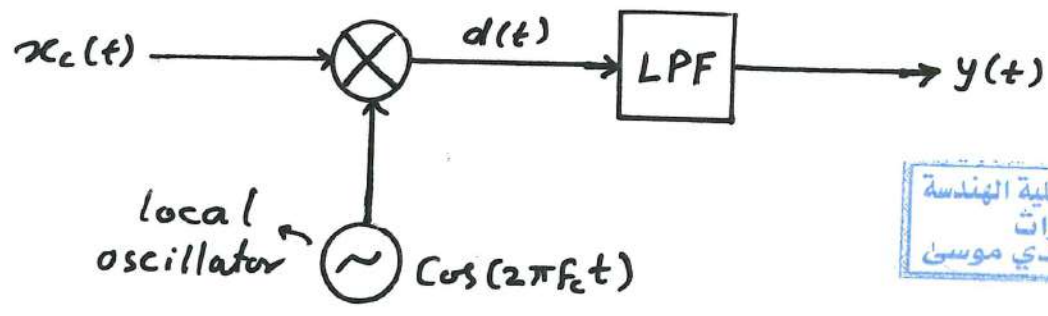
IF blocking capacitor is added at the output, the DC component will be canceled $\boxed{y(t) = A_c m(t)}$

c) Synchronous (Coherent) Detection:

This method requires local oscillator that generates a local carrier that is in frequency and phase coherence (synchronism) with the carrier signal.

{ local carrier has the same frequency and phase as the carrier signal }

* The synchronous detection will be studied in detail in the next section.



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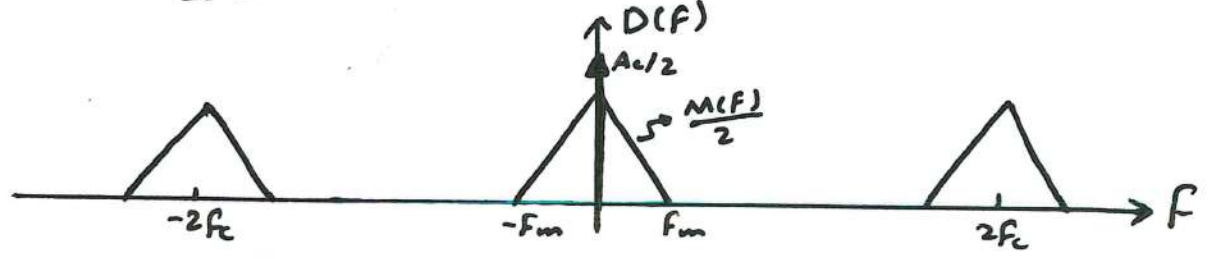
$$x_c(t) = [A_c + m(t)] \cos(2\pi f_c t)$$

$$d(t) = x_c(t) \cos(2\pi f_c t)$$

$$\therefore d(t) = [A_c + m(t)] \cos^2(2\pi f_c t)$$

$$d(t) = [A_c + m(t)] \frac{1}{2} (1 + \cos(2\pi(2f_c) t))$$

$$d(t) = \frac{1}{2} [A_c + m(t)] + \frac{1}{2} [A_c + m(t)] \cos(2\pi(2f_c) t)$$



by using LPF with $f_m < f_0 \ll f_c$ then:

$$y(t) = \frac{A_c}{2} + \frac{1}{2} m(t)$$

The DC component ($\frac{A_c}{2}$) can also be blocked by inserting a block capacitor at the output, so

$$y(t) = \frac{1}{2} m(t)$$

B Double-Sideband Suppressed Carrier (DSB-SC):

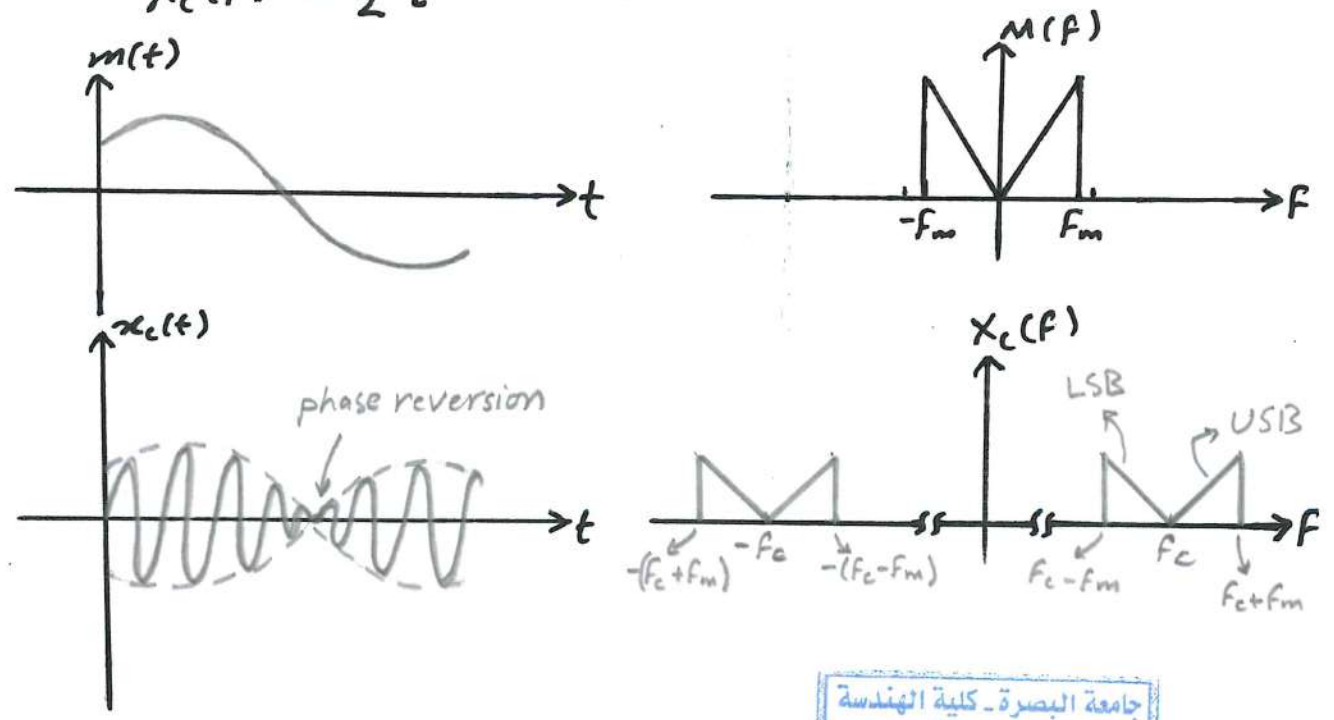
The modulated signal of DSB-SC is given by:

$$x_c(t) = A_c m(t) \cos(2\pi f_c t)$$

There is no carrier signal transmitted with the modulated signal, so it is called suppressed carrier.

The spectrum of the DSB-SC is given by:

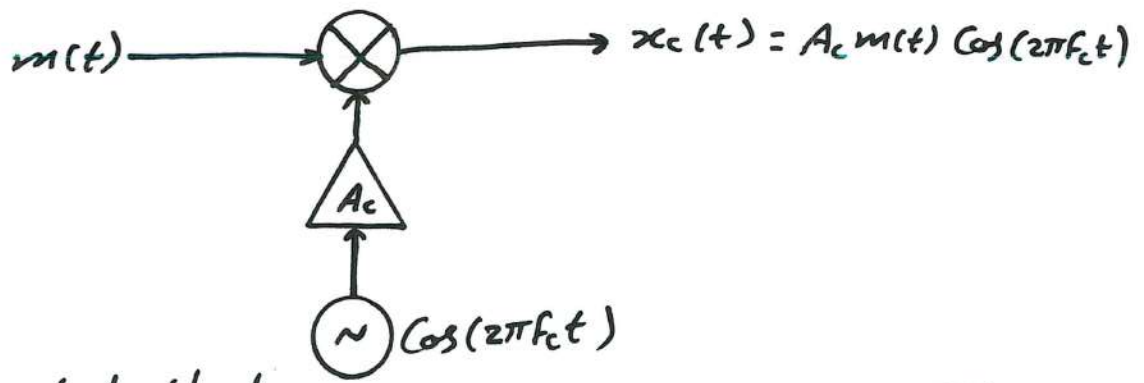
$$X_c(f) = \frac{A_c}{2} [M(f-f_c) + M(f+f_c)]$$



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Generation of DSB-SC:

This type of modulation can simply be generated by multiplying the message signal by the carrier signal.



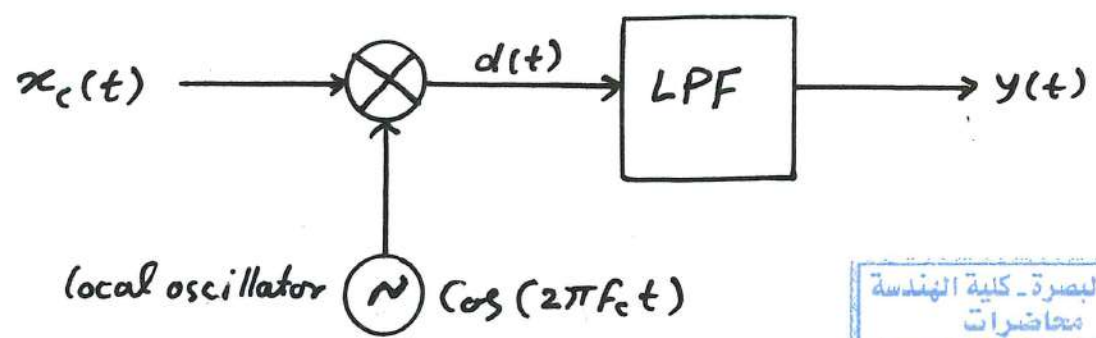
* It can be noted that, BW of $m(t) = f_m$

But !! BW $x_c(t) = 2f_m$

- Demodulation (Detection) of DSB-SC:

Since there is a phase reversion in DSB-SC, the information signal $m(t)$ resides in the phase and the envelope of $x_c(t)$. Therefore, Envelope Detector cannot be used for DSB-SC.

Synchronous (Coherent) Detection:



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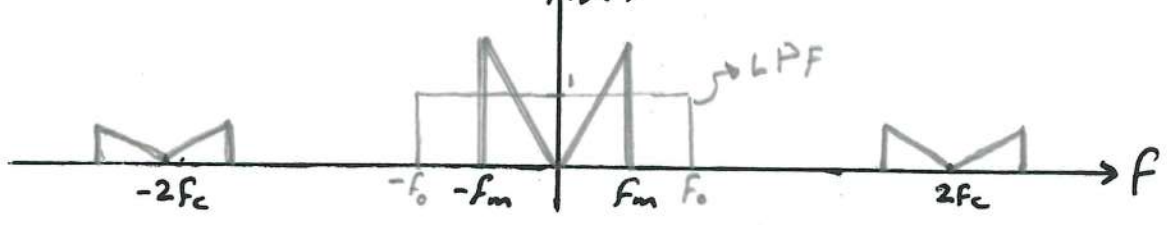
$x_c(t) = A_c m(t) \cos(2\pi f_c t)$

$d(t) = [A_c m(t) \cos(2\pi f_c t)] \cos(2\pi f_c t)$

$d(t) = \frac{A_c}{2} m(t) [1 + \cos(2\pi (2f_c) t)]$

$d(t) = \frac{A_c}{2} m(t) + \frac{A_c}{2} m(t) \cos(2\pi (2f_c) t)$

$D(f) = \frac{A_c}{2} M(f) + \frac{A_c}{4} [M(f-2f_c) + M(f+2f_c)]$



The LPF with $f_m \leq f_0 \ll f_c$ will cancel out the high frequency component and keep the baseband signal.

$y(t) = \frac{A_c}{2} m(t)$

As mention earlier, Local Oscillators should have:
The same phase and the same frequency as the carrier signal.

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Under these conditions, the local oscillator is said to be synchronized with the carrier.

Phase Error Problem:

Suppose the local oscillator with a phase error ϕ , then

$$d(t) = x_c(t) \cdot \cos(2\pi f_c t + \phi)$$

$$d(t) = A_c m(t) \cos(2\pi f_c t) \cos(2\pi f_c t + \phi)$$

$$d(t) = \frac{1}{2} A_c m(t) [\cos(\phi) + \cos(2\pi(2f_c)t + \phi)]$$

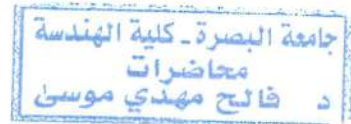
$$d(t) = \frac{1}{2} A_c m(t) \cos(\phi) + \frac{1}{2} A_c m(t) \cos(2\pi(2f_c)t + \phi)$$

After Low-pass filtering:

$$y(t) = \frac{1}{2} A_c m(t) \cos(\phi)$$

if ϕ is small value, the output is attenuated slightly.
 ϕ is large value, the attenuation is increased.

$$\phi = \pm \frac{\pi}{2}, \quad y = 0$$



Frequency Error Problem:

Suppose the local oscillator has frequency error of Δf .

$$d(t) = x_c(t) \cdot \cos(2\pi(f_c + \Delta f)t)$$

$$d(t) = A_c m(t) \cos(2\pi f_c t) \cos(2\pi(f_c + \Delta f)t)$$

$$d(t) = \frac{A_c}{2} m(t) [\cos(2\pi \Delta f t) + \cos(2\pi(2f_c + \Delta f)t)]$$

$$d(t) = \frac{A_c}{2} m(t) \cos(2\pi \Delta f t) + \frac{A_c}{2} m(t) \cos(2\pi(2f_c + \Delta f)t)$$

After LPF:

$$y(t) = \frac{A_c}{2} m(t) \cos(2\pi \Delta f t)$$

The output is the message $m(t)$ multiplied by low-frequency sinusoidal waveform which cause undesirable distortion called "beating".