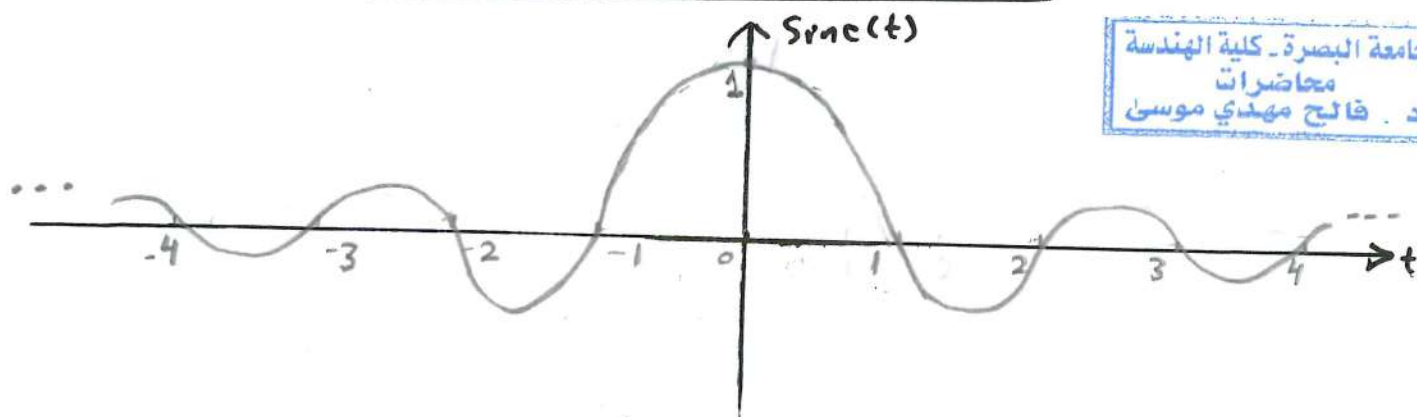


### d) Sinc Function :

It is very important signal especially in digital communication systems:

$$\text{Sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$



- \* It has even symmetry w.r.t  $t$ .
- \* Its max. value occurs at  $t=0$ .
- \* It has zero values at  $\pi t = \pm\pi, \pm 2\pi, \pm 3\pi, \dots$   
 $t = \pm 1, \pm 2, \pm 3, \dots$

for example, the first zero of  $\text{Sinc}(\frac{3}{7}\omega)$ :

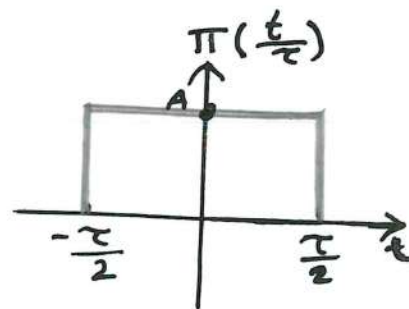
$$\text{Sinc}(\frac{3\omega}{7}) = \frac{\sin(\pi \frac{3\omega}{7})}{\pi \frac{3\omega}{7}}$$

$$\sin(\pi \frac{3\omega}{7}) = 0 \quad \text{when} \quad \frac{3\omega}{7} = 1 \Rightarrow \omega = \frac{7}{3}$$

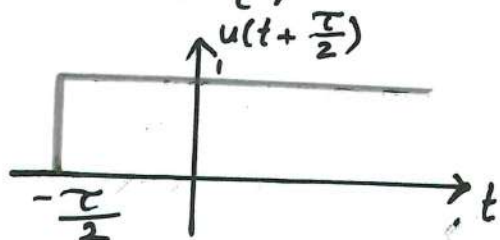
$$\therefore \frac{3\omega}{7} = 1 \Rightarrow \boxed{\omega = \frac{7}{3}}$$

### e) Rectangular Pulse $\Pi(\frac{t}{\tau})$ :

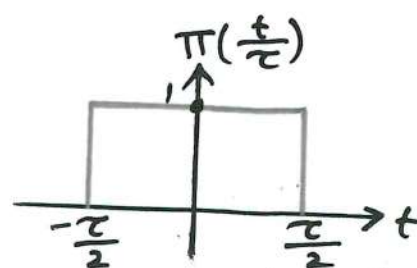
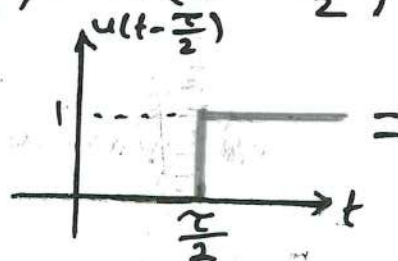
$$x(t) = A \Pi(\frac{t}{\tau}) = \begin{cases} A & |t| < \frac{\tau}{2} \\ 0 & |t| > \frac{\tau}{2} \end{cases}$$



$$\Pi(\frac{t}{\tau}) = u(t + \frac{\tau}{2}) - u(t - \frac{\tau}{2})$$

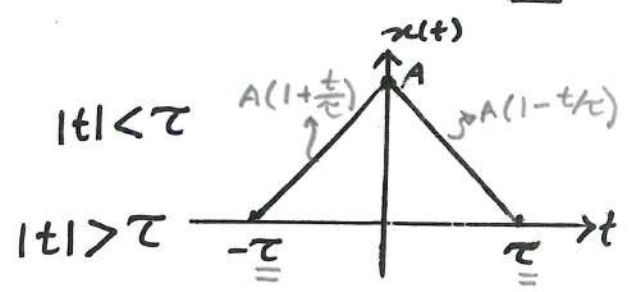


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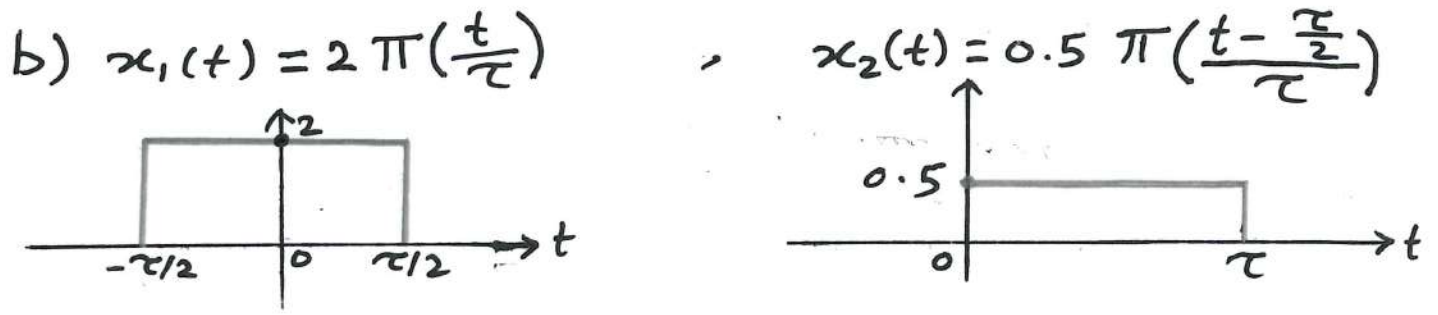
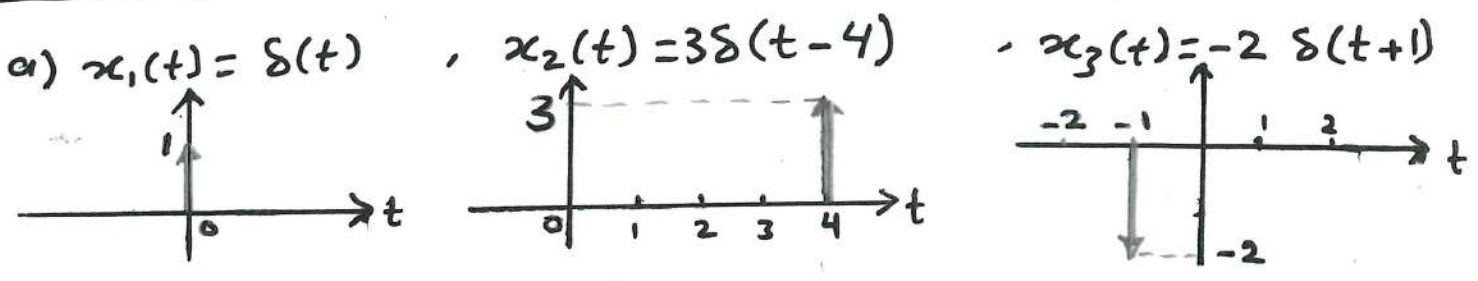


**F** Triangular Pulse  $\Lambda(\frac{t}{\tau})$ :

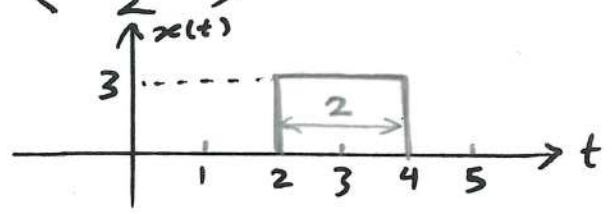
$$x(t) = A \Lambda(\frac{t}{\tau}) = \begin{cases} A(1 - \frac{|t|}{\tau}) & |t| < \tau \\ 0 & |t| > \tau \end{cases}$$



Example: Sketch the following signals:

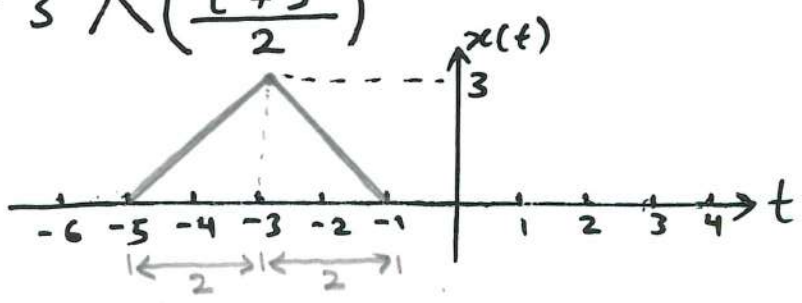


c)  $x(t) = 3\Pi(\frac{8t - 24}{16})$   
 $x(t) = 3\Pi(\frac{8(t - \frac{24}{8})}{16}) = 3\Pi(\frac{t - \frac{24}{8}}{(\frac{16}{8})})$   
 $x(t) = 3\Pi(\frac{t - 3}{2})$



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d)  $x(t) = 3\Lambda(\frac{8t + 24}{16})$   
 $x(t) = 3\Lambda(\frac{8(t + \frac{24}{8})}{16}) = 3\Lambda(\frac{t + \frac{24}{8}}{16/8})$   
 $x(t) = 3\Lambda(\frac{t + 3}{2})$



## 2. Phasor and Spectrum:

### [A] Single-Sided Spectrum:

Consider the conventional sinusoidal signal waveform:

$$x(t) = A \cos(\omega_0 t + \phi)$$

OR  $x(t) = A \cos(2\pi f_0 t + \phi)$

where  $A$  is the amplitude

$\phi$  is the phase angle

$\omega_0$  is the angular frequency (rad./sec.)

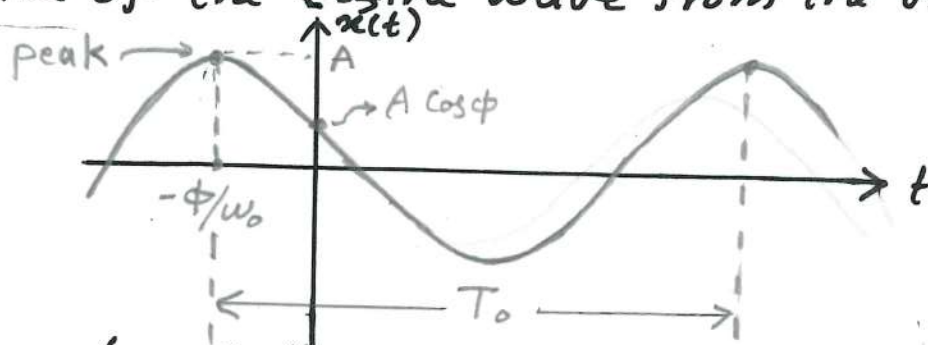
$f_0$  is the frequency in (Hz).  $\omega_0 = 2\pi f_0$



\* The sinusoidal signal repeats itself every time period ( $T_0$ ) which represents the reciprocal of the frequency:

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{T_0} \quad (\text{Hz})$$

\* The phase angle represents the shift of the peak value of the cosine wave from the origin.



The peak value of the cosine wave occurs when the angle of the cosine = 0, so:

$$\omega_0 t_{\max} + \phi = 0 \Rightarrow t_{\max} = -\frac{\phi}{\omega_0} \quad (\text{sec.})$$

OR  $\phi = -\omega_0 t_{\max} \quad (\text{rad.})$

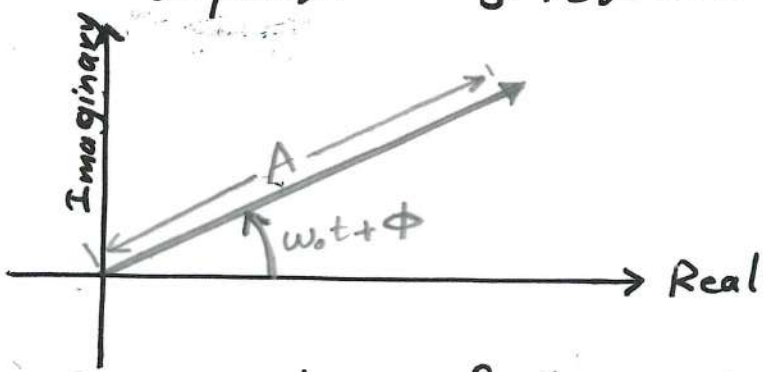
\* From Euler's Theorem:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$A \cos(\omega_0 t + \phi) = A \operatorname{Re}[e^{j(\omega_0 t + \phi)}]$$

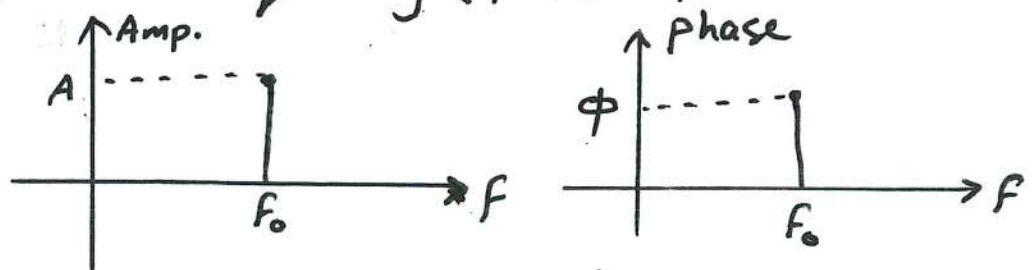
- The phasor representation is a time domain plot specified by
  1. Amplitude
  2. phase
  3. rotational frequency

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Time-domain phasor of  $(A \cos(\omega_0 t + \phi))$

- The line spectrum is a two-plot representation
  1. Amplitude vs Frequency (Amplitude Spectrum)
  2. phase vs frequency (Phase spectrum)



Freq. Domain line spectrum of  $(A \cos(\omega_0 t + \phi))$

Notes:

- The reference signal is the cosine wave, so the sine wave should be converted to the cosine using the following identity:

$$\sin \omega t = \cos(\omega t - 90^\circ)$$

- Negative amplitude should be converted to  $\mp 180^\circ$  phase shift.
  - $A \cos \omega t = A \cos(\omega t \mp 180^\circ)$ .

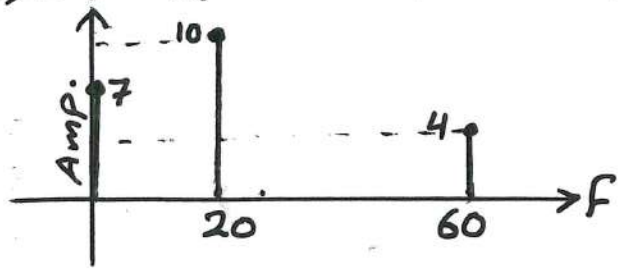
Example: Draw the line spectra of the following wave.

$$x(t) = 7 - 10 \cos(40\pi t - 60^\circ) + 4 \sin 120\pi t$$

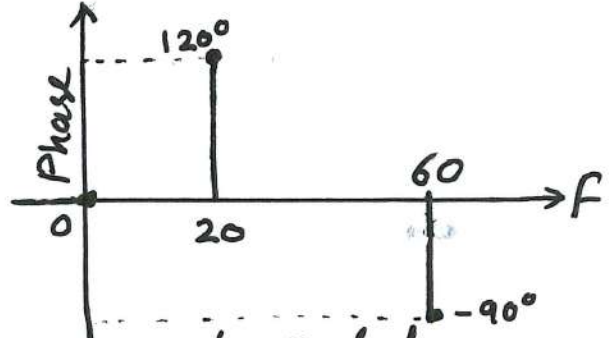
Sol.

$$x(t) = 7 \cos(2\pi \cdot 0 t + 0^\circ) + 10 \cos(40\pi t + 120^\circ) + 4 \cos(120\pi t - 90^\circ)$$

$$x(t) = 7 \cos(2\pi \cdot 0t + 0^\circ) + 10 \cos(2\pi \cdot 20t + 120^\circ) + 4 \cos(2\pi \cdot 60t - 90^\circ)$$



Single-sided Amp. Spectrum



single-sided phase spectrum

**B** Double-Sided Spectrum:

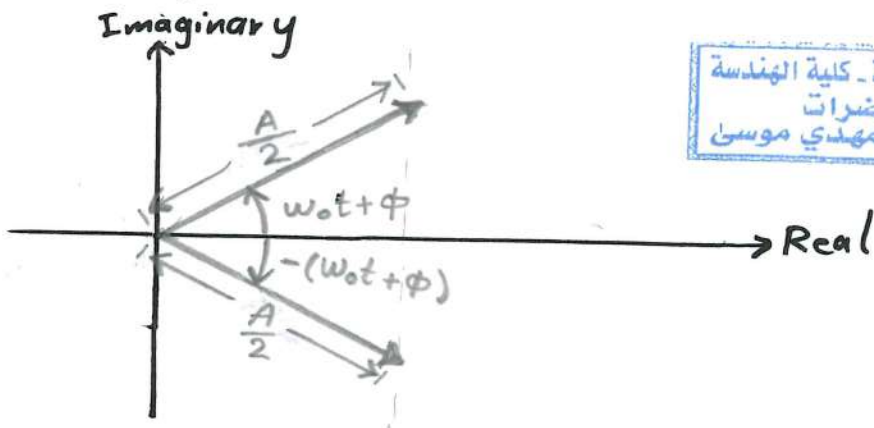
Since  $\cos \theta = \frac{1}{2} (e^{j\theta} + e^{-j\theta})$

$$\therefore A \cos(\omega_0 t + \phi) = \frac{A}{2} [e^{j(\omega_0 t + \phi)} + e^{-j(\omega_0 t + \phi)}]$$

$$A \cos(\omega_0 t + \phi) = \frac{A}{2} [e^{j\omega_0 t} e^{j\phi} + e^{-j\omega_0 t} e^{-j\phi}]$$

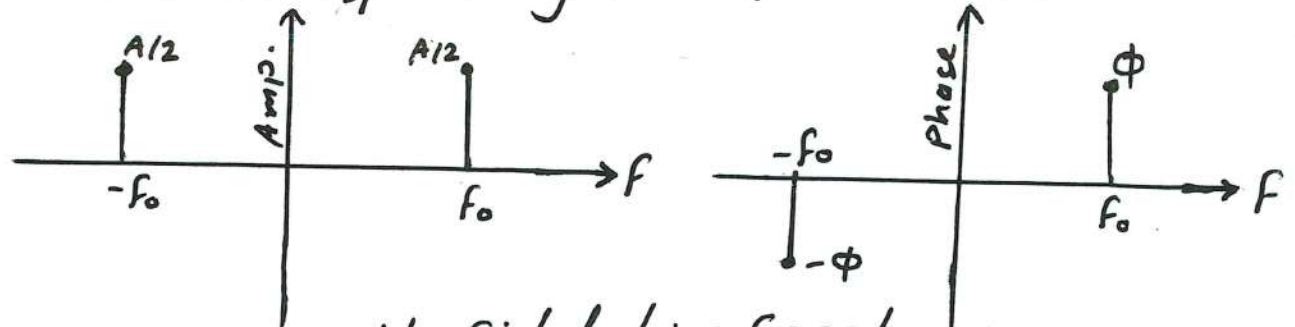
OR  $A \cos(\omega_0 t + \phi) = \frac{A}{2} e^{j\omega_0 t} e^{j\phi} + \frac{A}{2} e^{-j\omega_0 t} e^{-j\phi}$

The corresponding phasor diagram is:



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While the corresponding line spectrum is:



Double-Sided Line Spectrum

\* In double-sided spectrum; The Amplitude spectrum has even symmetry, while the phase has odd symmetry.

Example: Draw the double-sided line spectra of the following wave form:

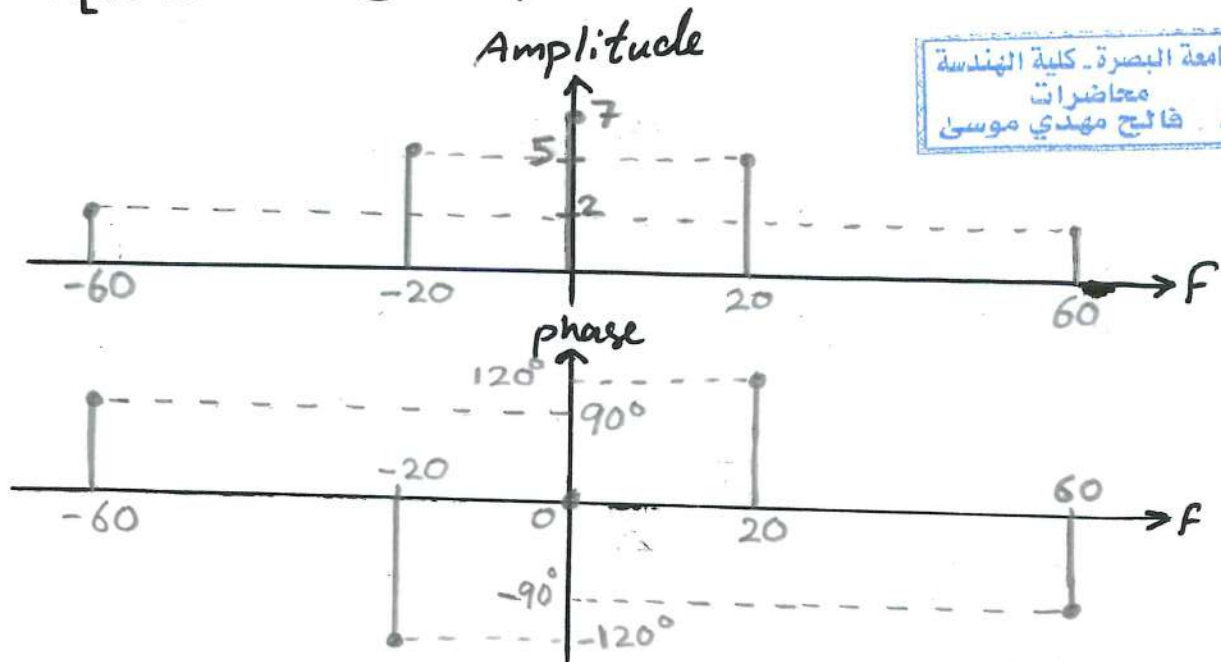
$$x(t) = 7 - 10 \cos(40\pi t - 60^\circ) + 4 \sin 120\pi t$$

Sol.

$$x(t) = 7 \cos(2\pi \cdot 0t + 0^\circ) + 10 \cos(2\pi \times 20t + 120^\circ) + 4 \cos(2\pi \times 60t - 90^\circ)$$

$$x(t) = 7 e^{j2\pi \cdot 0t} e^{j0} + \frac{10}{2} [e^{j2\pi \times 20t} e^{j120^\circ} + e^{-j2\pi \times 20t} e^{-j120^\circ}] + \frac{4}{2} [e^{j2\pi \times 60t} e^{-j90^\circ} + e^{-j2\pi \times 60t} e^{j90^\circ}]$$

$$x(t) = 7 e^{j2\pi \cdot 0t} e^{j0} + [5 e^{j2\pi \times 20t} e^{j120^\circ} + 5 e^{-j2\pi \times 20t} e^{-j120^\circ}] + [2 e^{j2\pi \times 60t} e^{-j90^\circ} + 2 e^{-j2\pi \times 60t} e^{j90^\circ}]$$



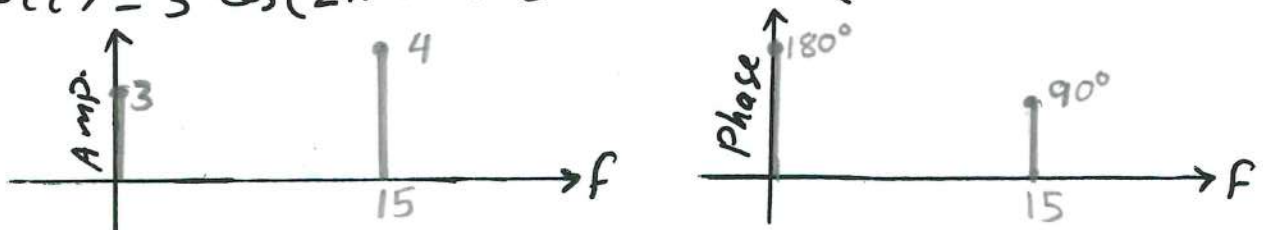
Example: Construct the single-sided and double-sided line spectra of:  $x(t) = -3 - 4 \sin 30\pi t$

Sol.

a) Single-sided line spectrum:

$$x(t) = -3 \cos(2\pi \cdot 0t + 0^\circ) - 4 \cos(2\pi \times 15t - 90^\circ)$$

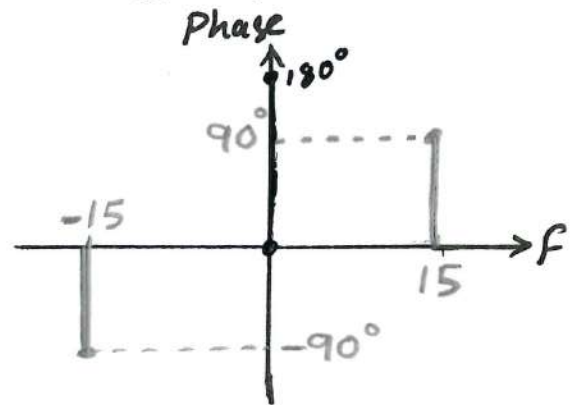
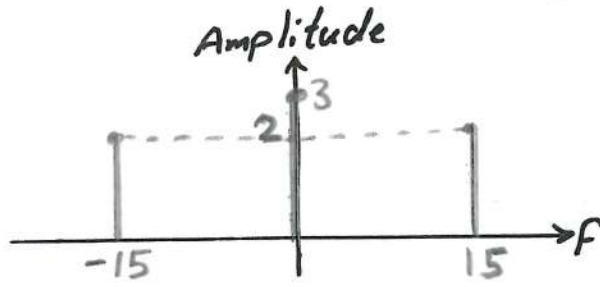
$$x(t) = 3 \cos(2\pi \cdot 0t + 180^\circ) + 4 \cos(2\pi \times 15t + 90^\circ)$$



b) Double-Sided Spectrum:

$$x(t) = 3 \cos(2\pi 0t + 180^\circ) + 4 \cos(2\pi \times 15t + 90^\circ)$$

$$x(t) = 3 e^{j2\pi 0t} e^{j180^\circ} + [2 e^{j2\pi \times 15t} e^{j90^\circ} + 2 e^{-j2\pi \times 15t} e^{-j90^\circ}]$$



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## 2. Fourier Series:

- \* Any periodic function of time  $x(t)$  having a fundamental period ( $T_0$ ) can be represented by an infinite sum of sinusoidal waveforms.
- \* The sinusoidal signal has only a single frequency so it is called "single tone" signal.
- \* This means that all the other periodic signals has an infinite frequencies not a single frequency.

## Complex (Exponential) Fourier Series:

$$x(t) = D_0 + \sum_{n=-\infty}^{\infty} D_n e^{j2\pi(nf_0)t}$$

where  $n$  is integer ( $n = \pm 1, \pm 2, \dots$ )

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$$D_0 = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) dt$$

$$D_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-j2\pi(nf_0)t} dt$$

and  $D_n$  is complex number  $\{ D_n = |D_n| e^{j\theta_n} \}$

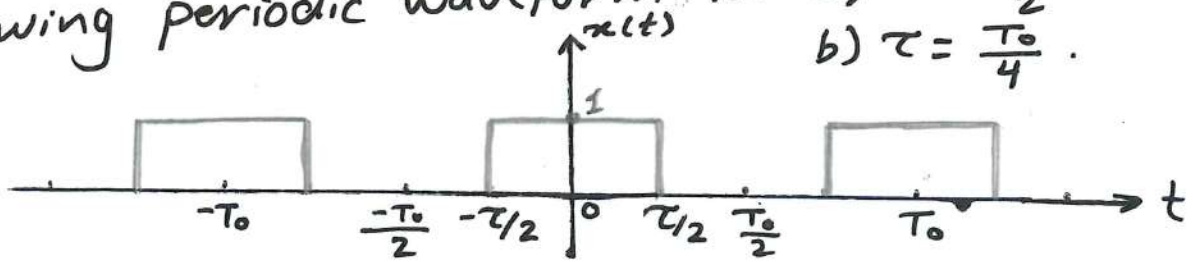
## Notes:

1. IF the periodic signal  $x(t)$  is real, then the amplitude spectrum has an even symmetry and the phase spectrum has an odd symmetry.
2. Since the index ( $n$ ) has only integer values, then the frequency spectra of the periodic signal exist at discrete frequencies ( $nf_0$ ).



3.  $D_0$  represents the DC component of  $x(t)$ , and it can be deduced from  $D_n$  by setting  $n=0$ . [2]

Example: Find the complex Fourier series of the following periodic waveform for a)  $\tau = \frac{T_0}{2}$ .  
b)  $\tau = \frac{T_0}{4}$ .



Sol.

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{j2\pi(nf_0)t}$$

$$D_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-j2\pi(nf_0)t} dt$$

where  $f_0 = \frac{1}{T_0}$

$$D_n = \frac{1}{T_0} \int_{-\tau/2}^{\tau/2} 1 \cdot e^{-j2\pi(nf_0)t} dt$$

$$D_n = \frac{1}{T_0} \times \frac{-1}{j2\pi n f_0} e^{-j2\pi(nf_0)t} \Big|_{-\tau/2}^{\tau/2}$$

, but  $f_0 = \frac{1}{T_0}$

$$\therefore D_n = \frac{-1}{j2\pi n} [e^{-j\pi n f_0 \tau} - e^{+j\pi n f_0 \tau}]$$

$$D_n = \frac{+1}{\pi n} \frac{[e^{j\pi n f_0 \tau} - e^{-j\pi n f_0 \tau}]}{j2}$$

$$D_n = \frac{\sin(\pi n f_0 \tau)}{\pi n} \times \frac{f_0 \tau}{f_0 \tau} = f_0 \tau \frac{\sin(\pi n f_0 \tau)}{\pi n f_0 \tau}$$

$$\therefore D_n = f_0 \tau \text{Sinc}(n f_0 \tau)$$

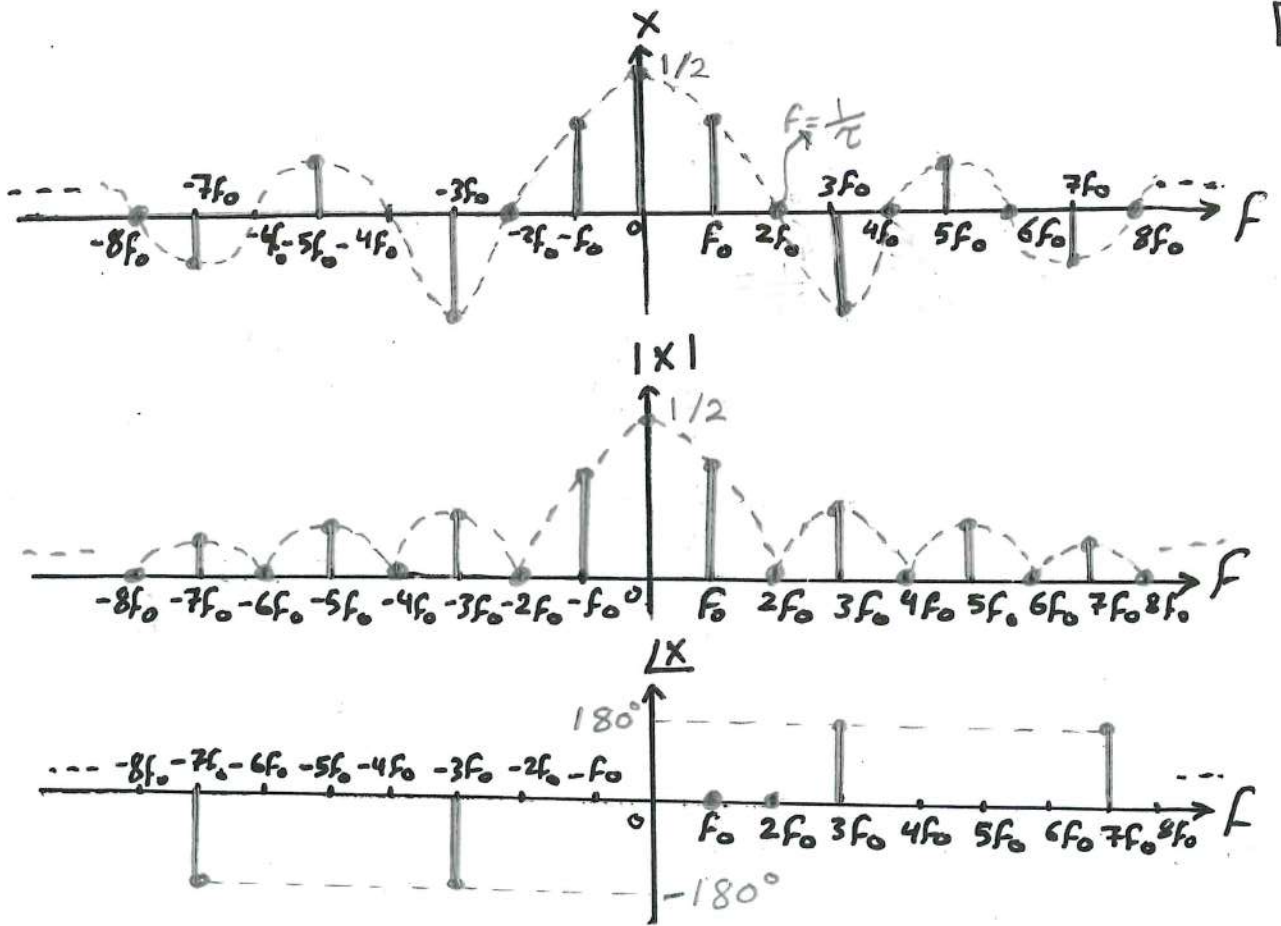
a) for  $\tau = \frac{T_0}{2}$ :

$$\therefore \frac{\tau}{T_0} = \frac{1}{2} = \tau f_0 \quad \text{where } \frac{\tau}{T_0} \text{ is the duty cycle.}$$

$$\therefore D_n = \frac{1}{2} \text{Sinc}\left(\frac{n}{2}\right)$$

$$\therefore x(t) = \sum_{n=-\infty}^{\infty} \left[ \frac{1}{2} \text{Sinc}\left(\frac{n}{2}\right) \right] e^{j2\pi(nf_0)t}$$

$$n = 0, \pm 1, \pm 2, \dots$$



b) For  $\tau = \frac{T_0}{4}$  :

$$\frac{\tau}{T_0} = \frac{1}{4} \Rightarrow \tau f_0 = \frac{1}{4}$$

$$D_n = \frac{1}{4} \text{sinc}\left(\frac{n}{4}\right) \Rightarrow x(t) = \sum_{n=-\infty}^{\infty} \left[ \frac{1}{4} \text{sinc}\left(\frac{n}{4}\right) \right] e^{j2\pi(n/f_0)t}$$

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