suppose instead of using ① we had decided to rearrange the original problem as

$$x = \frac{x^2-3}{2} \quad ---- (2)$$

Now if we start with the initial guess $x_0 = 4$ and iterate $x_n = \frac{x_{n-1}^2 - 3}{2} = f(x_{n-1})$

$x_1 = \frac{16-3}{2} = 6.2$, $x_2 = 19.625$, $x_3 = 191.07$ etc

Note that the iterates are bigger and bigger $\Rightarrow$ the method is diverging. On the other hand if we start with $x_0 = 0$ and iterate using (2) we get $x_1 = -1.5$, $x_2 = -0.375$

$x_3 = -1.4297$, ... The iterates now converge (very slowly to $x = -1$ which is one of the roots of the problem.

Example: let $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = \cos x$. show that $|fx - fy| < |x-y|$ for every distinct x and y

solution: $|f(x) - f(y)| = \left|2 \sin\left(\frac{x-y}{2}\right) \sin\left(\frac{x+y}{2}\right)\right|$

$$= 2\left|\sin\left(\frac{x-y}{2}\right)\right| \left|\sin\left(\frac{x+y}{2}\right)\right|$$

$$\leq 2\left|\sin\left(\frac{x-y}{2}\right)\right| < 2\left|\frac{x-y}{2}\right| = |x-y|$$

since $|\sin w| < |w| \quad \forall\, w \neq 0$.

clearly f is not contraction but second iterate is a Contraction. ~~$\Rightarrow f$ has~~

Exercise: let $f: R \to R$ be given by

$f(x) = \sin x$.

show that $|f(x) - f(y)| < |x - y| \quad \forall\, x \neq y \in R$

Some application on fixed point theorem

We now return to the differential equation with boundary condition

$\frac{dy}{dx} = \phi(x, y) \quad , \; y(x_0) = y_0$

the initial value problem for the first order ordinary differential equation.

A solution to this system if it exists will be an element of the infinite dimential metric space $C([a, b])$ for some closed interval $[a, b]$ containing x_0. This system is equivalent to the integral equation

$$y(x) = y_0 + \int_{x_0}^{x} \phi(t, y(t))\, dt$$ [volterra integral equation].

We will now use Banach's theorem (fixed point Theorem) to that $\exists$ a unique solution clearly f is not contraction but second iterate is a Contraction.

Exercise: let $f: R \to R$ be given by

$f(x) = \sin x$.
show that $|f(x) - f(y)| < |x - y| \quad \forall\ x \neq y \in R$

Some application on fixed point theorem

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