Proof:((see Senddon)
Ex: page 55 and 57 H.W ((Senddon)).

Non-Linear P.D.Es of the first order:

Let $$F(x,y,z,\frac{\partial z}{\partial x},\frac{\partial z}{\partial y})=0 \qquad (5)$$

is a non-linear P.D.E of the first order ((in which F is not necessarily linear in $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$)) be derived from

$$g(x,y,z,a,b)=0 \qquad (6)$$

By eliminate the arbitrary constants a and b .then g is called a complete solution of non-linear P.D.E(5)
The general method for obtaining complete solution of (5) is called Charpit's method. Before considering a general solution by this method, we give special procedure for handling four types of equations,
Type I: if the P.D.E in the form $f(\frac{\partial z}{\partial x},\frac{\partial z}{\partial y})=0$, then $z=ax+h(a)y+c$, where $b=h(a)$
Type II: if the P.D.E in the form $z=ax+by+f(\frac{\partial z}{\partial x},\frac{\partial z}{\partial y})$, then $z=ax+by+f(a,b)$
Type III: if the P.D.E in the form $f(z,\frac{\partial z}{\partial x},\frac{\partial z}{\partial y})=0$ then $z=z(x+ay)$,such that $\frac{\partial z}{\partial x}=\frac{dz}{du}$ and $\frac{\partial z}{\partial y}=a\frac{dz}{du}$,where $u=x+ay$
Type IV: if the P.D.E in the form $f_1(x,\frac{\partial z}{\partial x})=f_2(y,\frac{\partial z}{\partial y})$, then $f_1(x,\frac{\partial z}{\partial x})=a$ and $f_2(y,\frac{\partial z}{\partial y})=a$ with $F_1(x,a)=\frac{\partial z}{\partial x}$ and $F_2(y,a)=\frac{\partial z}{\partial y}$ and $z=F_1(x,a)dx+F_2(y,a)dy$
Charpit's formulation is;

$$\frac{dp}{f_x+pf_z}=\frac{dq}{f_y+qf_z}=\frac{dx}{-f_p}=\frac{dy}{-f_q}=\frac{dz}{-(pf_p+qf_q)}=\frac{dF}{0}$$

We can solve these equations to obtained p and q associated with original P. D.E.
High order linear P.D.Es with constant and/or variable coefficients:
Consider the P.D.E

$$f(D,\overline{D})z=F(x,y) \qquad (7)$$

Where $f(D,\overline{D})$ denotes a differential operator of the type

$$f(D,\overline{D})=\sum_r\sum_s C_{rs}D^r\overline{D}^s$$

In which C_{rs} are constants , $D=\frac{\partial}{\partial x}$ and $\overline{D}=\frac{\partial}{\partial y}$. The general solution of equ.(7) may be written as; z=[C.F]+[P.I], complementary function(C.F) can be obtained from $f(D,\overline{D})z=0$, and particular integral (P.I) can be obtained from $P.I=\frac{1}{f(D,\overline{D})}F(x,y)$.

Theorem : if u is the complementary function and z_1 is the particular integral of a linear P.D.E ,then $u+z_1$ is a general solution of equation.
Proof: H.w
Theorem :if $u_1,u_2,...,u_n$ are the solutions of linear P.D.E $f(D,\overline{D})z=0$ then $\sum_{r=1}^{n}C_r u_r$,where $C's$, is also a solution.
Proof: using equation ( ) for each solution, then obtain the aims of proof due to each one is satisfies equ.( ).
We can classify linear differential operators $f(D,\overline{D})$ into two types (reducible and irreducible)
Definition: $f(D,\overline{D})$ is irreducible if it can be written as a product of linear factors of the form $aD+b\overline{D}+c$,while is irreducible, where a,b,c are constants.
Theorem: if the operator $f(D,\overline{D})$ is reducible, the order in which the linear factor occur is unimportant. The theorem will be proved if can show that $(a_rD+b_r\overline{D}+c_r)(a_sD+b_s\overline{D}+c_s)=(a_sD+b_s\overline{D}+c_s)(a_rD+b_r\overline{D}+c_r)$ for any variable operator can be written in the form $f(D,\overline{D})=\prod_{r=1}^{n}(a_rD+b_r\overline{D}+c_r)$.

Theorem:

a) if $a_rD+b_r\overline{D}+c_r$ is a factor of $f(D,\overline{D})$ and $\phi(\xi)$ is an arbitrary function of the single value ξ , then if $a_r\neq 0$, $u_r=e^{\frac{-c_r}{a_r}x}\phi(b_rx-a_ry)$ is a solution of $f(D,\overline{D})z=0$
b) if $a_rD+b_r\overline{D}+c_r$ is a factor of $f(D,\overline{D})$ and $\phi(\xi)$ is an arbitrary function of the single value ξ , then if $b_r\neq 0$, $u_r=e^{\frac{-c_r}{b_r}x}\phi(b_rx-a_ry)$ is a solution of $f(D,\overline{D})z=0$

Theorem: if $(a_rD+b_r\overline{D}+c_r)^n,(a_r\neq 0)$ is a factor of $f(D,\overline{D})$ and if the functions $\phi_{r_1},\cdots\phi_{r_n}$ are arbitrary functions, then $e^{\frac{-c_r}{a_r}x}\sum_{s=1}^{n}x^{s-1}\phi_{rs}(b_rx-a_ry)$ is a solution of $f(D,\overline{D})z=0$

EX: pages 70,73,105 H.W