Contents

Le1, Le2

Chapter 1

Some special distributions

1.1 Normal distribution

A random variable of continuous type that has a p.d.f of the form

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}; \quad -\infty < x < \infty \tag{1.1}$$

is said to has a normal distribution, and any $f(x)$ of this form is called a normal p.d.f., where the parameters μ and σ satisfy $-\infty < \mu < \infty$ and $\sigma > 0$.

Remark 1: If X is a normal random variable with parameters μ and σ , we shall denote that by $X \sim n(\mu, \sigma^2)$.

Remark 2: In the normal distribution if $\mu = 0$ and $\sigma^2 = 1$ (i.e. $X \sim n(0,1)$). Then X is said to has the standard normal distribution. $\Rightarrow f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$

Theorem 1: If X is a normal random variable, then (Proof: see modual M216)

1. $E[X] = \mu$. $\quad E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$
2. $\mathrm{var}[X] = \sigma^2$. $\quad V(x) = E(x^2) - (E(x))^2$
3. $M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$. $\quad M_x(t) = E(e^{xt}) = \int_{-\infty}^{\infty} e^{xt} f(x)\,dx$

Theorem 2: If the normal random variable X is $n(\mu, \sigma^2)$; $\sigma > 0$. Then the normal random variable $Z = \frac{X-\mu}{\sigma}$ is $n(0,1)$ (Proof: see modual M216).

Example1: If $X \sim n(75, 100)$. Then
$\Pr(70 < X < 100) = \Pr(\frac{70-75}{10} < \frac{X-75}{10} < \frac{100-75}{10}) = \Pr(-0.5 < Z < 2.5)$
$= \Pr(Z < 2.5) - \Pr(Z < -0.5) = \Pr(Z < 2.5) + \Pr(Z < 0.5) - 1.$
$= 0.994 + 0.691 - 1 = 0.685.$

Theorem 3: If the normal random variable X is $n(\mu, \sigma^2)$; $\sigma > 0$. Then the normal random variable $V = \left(\frac{X-\mu}{\sigma}\right)^2$ is $\chi^2(1)$ (Proof: see modual M216).

Example 2: If $X \sim n(0, 4)$. Then

$$\Pr(X^2 < 15.36) = \Pr\left(\tfrac{(X-0)^2}{4} < \tfrac{15.36-0}{4}\right) = \Pr(V < 3.84) = 0.95.$$

1.2 Bivariate Normal distribution

Consider the following nonnegative function

$$f(x, y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} e^{-\frac{q}{2}} \quad ; \quad -\infty < x < \infty \quad -\infty < y < \infty \tag{1.2}$$

where, $\sigma_1 > 0,\ \sigma_2 > 0,\ |\rho| < 1$ and

$$q = \tfrac{1}{1-\rho^2}\left[\left(\tfrac{x-\mu_1}{\sigma_1}\right)^2 - 2\rho\left(\tfrac{x-\mu_1}{\sigma_1}\right)\left(\tfrac{y-\mu_2}{\sigma_2}\right) + \left(\tfrac{y-\mu_2}{\sigma_2}\right)^2\right]$$

Here, we shall show that

- $f(x, y)$ is a joint p.d.f.;
- $X \sim n(\mu_1, \sigma_1^2)$ and $Y \sim n(\mu_2, \sigma_2^2)$.

Define $f_1(x)$ by

$$f_1(x) = \int_{-\infty}^{\infty} f(x, y)\, dy = \int_{-\infty}^{\infty} \tfrac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} e^{-\frac{q}{2}}\, dy$$

Now

$$(1-\rho^2)q = \left[\left(\tfrac{x-\mu_1}{\sigma_1}\right)^2 - 2\rho\left(\tfrac{x-\mu_1}{\sigma_1}\right)\left(\tfrac{y-\mu_2}{\sigma_2}\right) + \left(\tfrac{y-\mu_2}{\sigma_2}\right)^2\right],$$

$$= \left[\left(\tfrac{y-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\tfrac{x-\mu_1}{\sigma_1}\right)\left(\tfrac{y-\mu_2}{\sigma_2}\right) + \rho^2\left(\tfrac{x-\mu_1}{\sigma_1}\right)^2 - \rho^2\left(\tfrac{x-\mu_1}{\sigma_1}\right)^2 + \left(\tfrac{x-\mu_1}{\sigma_1}\right)^2\right],$$

$$= \left[\left(\tfrac{y-\mu_2}{\sigma_2}\right) - \rho\left(\tfrac{x-\mu_1}{\sigma_1}\right)\right]^2 + (1-\rho^2)\left(\tfrac{x-\mu_1}{\sigma_1}\right)^2,$$

$$= \left(\tfrac{\sigma_1(y-\mu_2) - \rho\sigma_2(x-\mu_1)}{\sigma_1\sigma_2}\right)^2 + (1-\rho^2)\left(\tfrac{x-\mu_1}{\sigma_1}\right)^2,$$

$$= \left(\frac{(y-\mu_2) - \rho\frac{\sigma_2}{\sigma_1}(x-\mu_1)}{\sigma_2}\right)^2 + (1-\rho^2)\left(\tfrac{x-\mu_1}{\sigma_1}\right)^2,$$

$$= \left(\frac{y - (\mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x-\mu_1))}{\sigma_2}\right)^2 + (1-\rho^2)\left(\tfrac{x-\mu_1}{\sigma_1}\right)^2,$$

$$= \left(\tfrac{y-b}{\sigma_2}\right)^2 + (1-\rho^2)\left(\tfrac{x-\mu_1}{\sigma_1}\right)^2.$$

Where, $b = \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x-\mu_1)$.

1.3 Principal Theorems

Theorem 4: Let $X_1, X_2, ..., X_n$ be stochastically independent random variables having, respectively, the normal distributions $n(\mu_1, \sigma_1^2)$, $n(\mu_2, \sigma_2^2)$,..., and $n(\mu_n, \sigma_n^2)$.The random variable $Y = \sum_{i=1}^{n} k_i X_i$, where k_i are constants for all $i = 1, 2, ..., n$ is $n(\sum_{i=1}^{n} k_i\mu_i, \sum_{i=1}^{n} k_i^2\sigma_i^2)$

Theorem 5: Let $X_1, X_2, ..., X_n$ be stochastically independent random variables having, respectively, the chi-square distributions $\chi^2(r_1)$, $\chi^2(r_2)$,...,$\chi^2(r_n)$. Then $V = \sum_{i=1}^{n} X_i$ is $\chi^2(\sum_{i=1}^{n} r_i)$.

Theorem 6: Let $X_1, X_2, ..., X_n$ denote random sample of size n from a distribution which is $n(\mu, \sigma^2)$. The random variable $W = \sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma}\right)^2$ is $\chi^2(n)$.

Theorem 7: Let $X_1, X_2, ..., X_n$ denote random sample of size n from a distribution which is $n(\mu, \sigma^2)$.Then,

1. $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is $n(\mu, \frac{\sigma^2}{n})$.
2. $\frac{nS^2}{\sigma^2}$ is $\chi^2(n-1)$.

$\bar{X}$ is the mean of a random sample of size n

S^2 is the variance of a random sample of size n.

1.4 t-distribution

Let W denotes a random variable which is $X \sim n(0, 1)$, let V denotes a random variable which is $X \sim \chi^2(r)$, and let W and V be stochastically independent. Then the random variable

$$T = \frac{W}{\sqrt{\frac{V}{r}}} \tag{1.4}$$

is said to has a t-disribution with r degrees of freeedom.

Remark: If X has a t-disribution with r degrees of freeedom, we shall denote that by $X \sim t(r)$

1.4.1 F-distribution

Let U and V be stochastically independent chi-square random variables with r_1 and r_2 degrees of freedom, respectively. Then the random variable

$$F = \frac{\frac{U}{r_1}}{\frac{V}{r_2}} = \frac{r_2 U}{r_1 V} \tag{1.5}$$

1.3 Principal Theorems

Theorem 4: Let $X_1, X_2, ..., X_n$ be stochastically independent random variables having, respectively, the normal distributions $n(\mu_1, \sigma_1^2)$, $n(\mu_2, \sigma_2^2)$,..., and $n(\mu_n, \sigma_n^2)$.The random variable $Y = \sum_{i=1}^{n} k_i X_i$, where k_i are constants for all $i = 1, 2, ..., n$ is $n(\sum_{i=1}^{n} k_i\mu_i, \sum_{i=1}^{n} k_i^2\sigma_i^2)$

Theorem 5: Let $X_1, X_2, ..., X_n$ be stochastically independent random variables having, respectively, the chi-square distributions $\chi^2(r_1)$, $\chi^2(r_2)$,...,$\chi^2(r_n)$. Then $V = \sum_{i=1}^{n} X_i$ is $\chi^2(\sum_{i=1}^{n} r_i)$.

Theorem 6: Let $X_1, X_2, ..., X_n$ denote random sample of size n from a distribution which is $n(\mu, \sigma^2)$. The random variable $W = \sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma}\right)^2$ is $\chi^2(n)$.

Theorem 7: Let $X_1, X_2, ..., X_n$ denote random sample of size n from a distribution which is $n(\mu, \sigma^2)$.Then,

1. $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is $n(\mu, \frac{\sigma^2}{n})$.

2. $\frac{nS^2}{\sigma^2}$ is $\chi^2(n-1)$. $S^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

1.4 t-distribution

Definition: Let W denotes a random variable which is $X \sim n(0, 1)$, let V denotes a random variable which is $X \sim \chi^2(r)$, and let W and V be stochastically independent. Then the random variable

$$T = \frac{W}{\sqrt{\frac{V}{r}}} \tag{1.4}$$

is said to has a t-disribution with r degrees of frecedom, and p.d.f.

$$f(t) = \frac{\Gamma[\frac{(r+1)}{2}]}{\sqrt{r\pi}\,\Gamma(\frac{r}{2})\,(1+\frac{t^2}{r})^{\frac{(r+1)}{2}}}, \qquad -\infty < t < \infty \tag{1.5}$$

Remark: If X has a t-disribution with r degrees of freeedom, we shall denote that by $X \sim t(r)$

Example: Let X has a t-distribution with 10 degree of freedom, then

$$\begin{aligned}
\Pr(|T| > 2.228) &= 1 - \Pr(|T| \leq 2.228) = 1 - \Pr(-2.228 \leq T \leq 2.228) \\
&= 1 - [\Pr(T \leq 2.228) - \Pr(T \leq -2.228)] = 2 - 2\Pr(T \leq 2.228) \\
&= 2 - 2(0.975) = 2 - 1.950 = 0.05 \qquad (t_{0.975}(10) = 2.228)
\end{aligned}$$

1.4.1 F-distribution

Definition: Let U and V be stochastically independent chi-square random variables with r_1 and r_2 degrees of freedom, respectively. Then the random variable

$$F = \frac{\frac{U}{r_1}}{\frac{V}{r_2}} = \frac{r_2 U}{r_1 V} \tag{1.6}$$

is said to has a F-distribution with r_1 and r_2 degrees of freedom, and p.d.f.

$$g(f) = \begin{cases} \dfrac{\Gamma[\frac{(r_1+r_2)}{2}](\frac{r_1}{r_2})^{\frac{r_1}{2}}(f)^{\frac{r_1}{2}-1}}{\Gamma(\frac{r_1}{2})\Gamma(\frac{r_2}{2})(1+\frac{r_1 f}{r_2})^{\frac{(r_1+r_2)}{2}}} & 0 < f < \infty \\ 0 & elsewhere \end{cases} \tag{1.7}$$

Remark: If X has a F-disribution with r degrees of freeedom, we shall denote that by $X \sim F(r_1, r_2)$

Remark: If $X \sim F(r_1, r_2)$, then $\frac{1}{X} \sim F(r_2, r_1)$

Chapter 2

Interval Estimation تقدير

2.1 Random Interval

Definition1: A random interval is an iterval, at least one of whose end points is random variable.

Let X denote a random variable and consider the event (interval) $1 < X < 2$, this interval is equivalent to the random interval $X < 2 < 2X$. Thus, the interval $(X, 2X)$ is a random interval. Also in this case, $\Pr(1 < X < 2) \equiv \Pr(X < 2 < 2X) = p$.

Example1: Let $X \sim \chi^2(16)$. The probility that the random interval $(X, 3.3X)$ contains the point $X = 26.3$ is
$\Pr[26.3 \in (X, 3.3X)] = \Pr(X < 26.3 < 3.3X)$
Thus, the random interval $(X, 3.3X)$ include the point $X = 26.3$, is given by

$\Pr(X < 26.3 < 3.3X) = \Pr(7.97 < X < 26.3) = \Pr(X < 26.3) - \Pr(X < 7.97) = 0.95 - 0.05 = 0.9$
Suppose L is the length of the random interval $(X, 3.3X)$, then

$$L = 3.3X - X = 2.3X \Rightarrow E(L) = E(2.3X) = 2.3E(X) = 2.3 * 16 = 36.8$$

Example1: Let $X \sim n(0, \sigma^2)$, $\sigma^2 > 0$

1. Find $\Pr[\sigma \in (|X|\ ,\ |10X|)]$.
2. Expected length of the interval $(|X|\ .\ |10X|)$.

   Hint: $Z_{0.841} = 1$, $Z_{0.540} = 0.1$

1. $\Pr[\sigma \in (|X|\ ,\ |10X|)] = \Pr(|X| < \sigma < |10X|) = \Pr(\frac{\sigma}{10} < |X| < \sigma)$
   $= \Pr(-\sigma < X < -\frac{\sigma}{10}) + \Pr(\frac{\sigma}{10} < X < \sigma) = 2\Pr(\frac{\sigma}{10} < X < \sigma)$

Since $X \sim n(0, \sigma^2) \Rightarrow Z = \frac{X-\mu}{\sigma} = \frac{X}{\sigma} \sim n(0.1)$.

Therefore,

$\Pr(|X| < \sigma < |10X|) = 2\Pr(\frac{\sigma}{10} < X < \sigma) = 2\Pr(\frac{1}{10} < \frac{X}{\sigma} < 1) = 2\Pr(\frac{1}{10} < Z < 1)$

$= 2[\Pr(Z < 1) - \Pr(Z < \frac{1}{10})] = 2[0.841 - 0.540] = 2[0.301] =$ 0.602.

2. Suppose L is the length of the interval $(|X|, |10X|)$, then $L = |10X| - |X| = 9|X|$.
   Thus,
   $E(L) = E(9|X|) = 9E(|X|) = 9[\int_{-\infty}^{0}(-x)f(x)dx + \int_0^{\infty}(x)f(x)dx] = 18\int_0^{\infty}(x)f(x)dx$
   Hence,
   $18\int_0^{\infty}(x)f(x)dx = 18\int_0^{\infty}\frac{1}{\sigma\sqrt{2\pi}}x\,e^{-\frac{1}{2\sigma}x^2}dx = \frac{18}{\sigma\sqrt{2\pi}}\left[-\sigma e^{-\frac{1}{2\sigma}x^2}\right]_0^{\infty} = \frac{-18\sigma^2}{\sigma\sqrt{2\pi}}[0-1] =$
   $\frac{18\sigma}{\sqrt{2\pi}}$

2.2 Confidence Interval for Means

Let $X_1, X_2, ..., X_n$ be arandom sample of size n from a distribution which has $n(\mu, \sigma^2)$. Here, we want to find $(1-\alpha)\%$ confidence interval for μ.

1. Case I: where σ^2 is known// Since $X_1, X_2, ..., X_n$ is arandom sample of size n from a distribution which has $n(\mu, \sigma^2)$; σ^2 is known, then

   $\bar{X} \sim n(\mu, \frac{\sigma^2}{n}) \;\Rightarrow\; \mathbf{Z} = \frac{\sqrt{n}(\bar{X}-\mu)}{\sigma} \sim n(0,1)$.
   Thus, there is a tabulate value $z_{\frac{\alpha}{2}}$, such that

   $\Pr(-z_{\frac{\alpha}{2}} < Z < z_{\frac{\alpha}{2}}) = 1-\alpha \;\Rightarrow\; \Pr(-z_{\frac{\alpha}{2}} < \frac{\sqrt{n}(\bar{X}-\mu)}{\sigma} < z_{\frac{\alpha}{2}}) = 1-\alpha$

   And

   $\Pr(\bar{X} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}) = 1-\alpha.$

   Therefore, $(\bar{X} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}})$ is a $(1-\alpha)\%$ confidence interval for μ.

   Example: In the preceding discussion if $n = 40$, $\sigma^2 = 10$ and $\bar{x} = 7.164$.

   Here, we need to find a 80% confidence interval for μ.
   $1-\alpha = 0.80 \;\Rightarrow \alpha = 1 - 0.80 = 0.20$,
   and, $z_{\frac{\alpha}{2}} = z_{\frac{0.20}{2}} = z_{0.1} = 1.282$,

7. Sampling of normal distribution

مفاهيم

This section deals with the concepts and problems of hypotheses test about the parameters of normal distribution. The section is subdivided into four subsections; the first two are dealing with just one normal population and the last two are dealing with several normal populations.

7.1 Test on the mean (μ)

We shall assume that, we have a random sample of n observations (i.e. $x_1,\cdots,x_n \overset{iid}{\sim} n(\mu,\sigma^2)$). Consequently, we will be interested in testing hypotheses about the mean μ. Here, there are three of hypotheses about the mean μ, which can be formulated as

(a) $\begin{matrix} H_0: \mu = \mu_0 \\ H_1: \mu > \mu_0 \end{matrix}$,

(b) $\begin{matrix} H_0: \mu = \mu_0 \\ H_1: \mu < \mu_0 \end{matrix}$, ($\mu$ is constant)

(c) $\begin{matrix} H_0: \mu = \mu_0 \\ H_1: \mu \neq \mu_0 \end{matrix}$.

In this manner, there are two cases to consider depending on whether or not σ^2 is assumed known.

7.1.1 σ^2 is known

Suppose $x_1,\cdots,x_n \overset{iid}{\sim} n(\mu,\sigma^2)$, ($\sigma^2$ is assumed known), then

$$f(x,\mu) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x_i-\mu)^2} \quad ; -\infty < x < \infty$$

and,

$$L(x,\mu) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2} = (2\pi\sigma^2)^{-\frac{n}{2}} e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2} \quad .$$

Thus,

$$L_0(x,\mu) = (2\pi\sigma^2)^{-\frac{n}{2}} e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu_0)^2} \quad ,$$

And,

$$L_1(x,\mu)=(2\pi\sigma^2)^{-\frac{n}{2}}\, e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}$$

For $k>0$, then

$$\frac{L_0(x,\mu)}{L_1(x,\mu)}\le k \;\Rightarrow\; e^{-\frac{1}{2\sigma^2}\left[\sum_{i=1}^{n}(x_i-\mu_0)^2-\sum_{i=1}^{n}(x_i-\mu)^2\right]}\le k \;\Rightarrow\; -\sum_{i=1}^{n}(x_i-\mu_0)^2+\sum_{i=1}^{n}(x_i-\mu)^2\le k_1 ,$$

$$-\left(\sum x_i^2-2\mu_0\sum x_i+n\mu_0^2\right)+\left(\sum x_i^2-2\mu\sum x_i+n\mu^2\right)\le k$$

Then,

$$(\mu_0-\mu_1)\sum_{i=1}^{n}x_i\le k_2 \qquad (7.1)$$

$$(\mu_0-\mu)\sum x_i\le k$$

if

(a) If $H_1:\mu>\mu_0$, we have

$$C=\{x_1,\cdots,x_n:\bar{x}\ge c\}$$

Since $x_1,\cdots,x_n \overset{iid}{\sim} n(\mu,\sigma^2)$, then under H_o we have

$$x_1,\cdots,x_{25}\overset{iid}{\sim} n(\mu_o,\sigma^2)\Rightarrow \bar{x}\sim n(\mu_o,\frac{\sigma^2}{n}) \text{ and } \frac{\sqrt{n}(\bar{x}-\mu_o)}{\sigma}\sim n(0,1).$$

Then,

$$P_o(\bar{x}\ge c)=\alpha \;\Rightarrow\; P_o\left(\frac{\sqrt{n}(\bar{x}-\mu_o)}{\sigma}\gtrless\frac{\sqrt{n}(c-\mu_o)}{\sigma}\right)=\alpha .$$

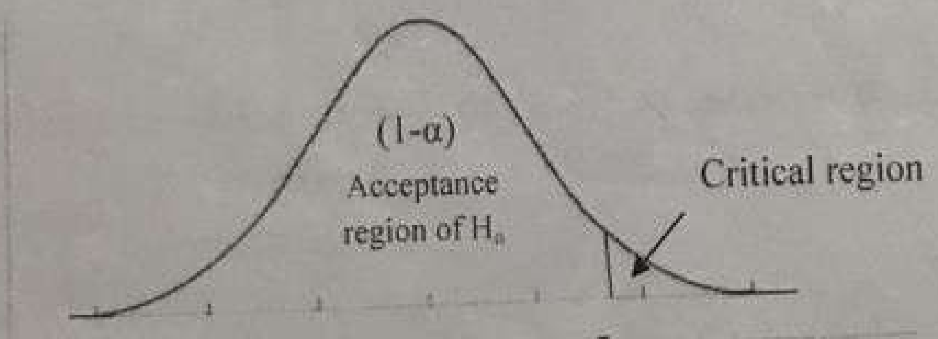
Then, the critical region can be written as:

$$\boxed{z\ge z_\alpha}$$

منطقة الرفض
نرفض فيها H_0
ونقبل H_1

Where,

$z=\dfrac{\sqrt{n}(\bar{x}-\mu_o)}{\sigma}$, and z_α is tabulate value (from normal distribution with significance level α).

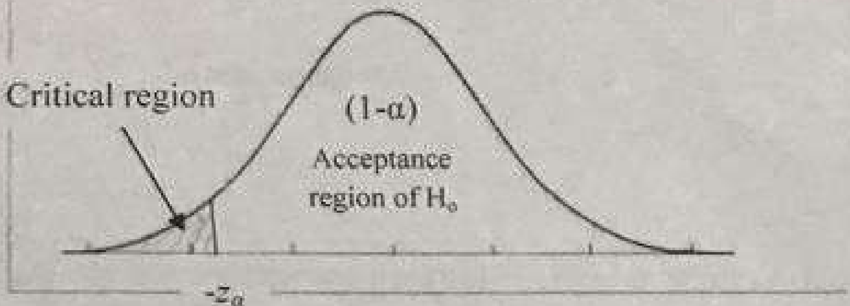


(b) If $H_1 : \mu < \mu_0$

Similarly, we can get the critical region in the form,

$$\boxed{z \leq -z_\alpha}$$

$z = \dfrac{\sqrt{n}(\bar{x} - \mu_0)}{\sigma}$, and z_α is tabulate value (from normal distribution with significance level α).

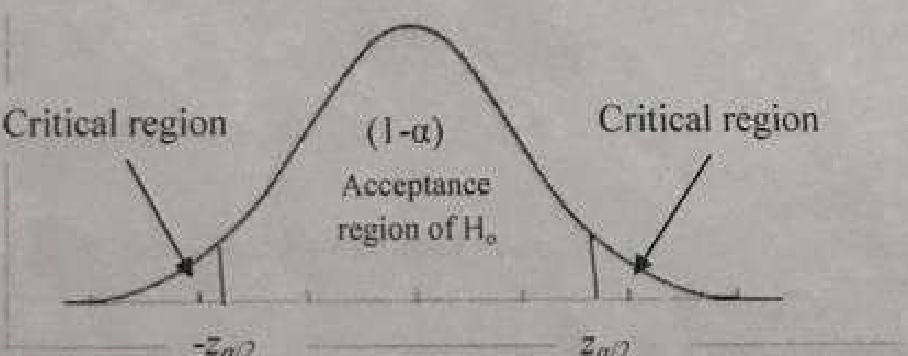


(c) If $H_1 : \mu \neq \mu_0$

In this case, the critical region can be written as:

$$\boxed{z > z_{\frac{\alpha}{2}} \quad \& \quad z < -z_{\frac{\alpha}{2}}}$$

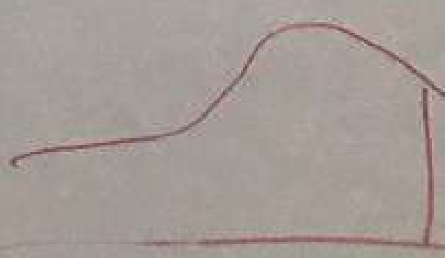
$z = \dfrac{\sqrt{n}(\bar{x} - \mu_0)}{\sigma}$, and $z_{\frac{\alpha}{2}}$ is tabulate value (from normal distribution with significance level α).



<u>Example (7.1)</u>

If in the preceding discussion n=40, $\sigma^2 = 10$, $\bar{x} = 7.164$ and $\alpha = 0.05$, then 1.6

(a) If $\begin{array}{l} H_0 : \mu = 2 \\ H_1 : \mu > 2 \end{array}$, the critical region is



$z \geq z_\alpha$

Where,

$$z=\frac{\sqrt{n}(\bar{x}-\mu_\circ)}{\sigma}=\frac{\sqrt{40}(7.164-2)}{\sqrt{10}}=10.328.$$

From the table of normal distribution we have $z_\alpha=z_{0.05}=1.645$.

Since $z=10.328$ falls in the critical region (acceptance region of H_1), then we accept H_1 (i.e. $\mu>2$).

(b) If $\begin{matrix} H_\circ:\mu=2 \\ H_1:\mu<2 \end{matrix}$, the critical region is

$$z\le -z_\alpha$$

Since $z=10.328$ falls in the rejection region of H_1, then we reject H_1 (i.e. $\mu=2$).

(c) If $\begin{matrix} H_\circ:\mu=2 \\ H_1:\mu\neq 2 \end{matrix}$, the critical region is

$$z\ge z_{\frac{\alpha}{2}} \quad \& \quad z<-z_{\frac{\alpha}{2}}$$

And,

$$z_{\frac{\alpha}{2}}=z_{\frac{0.05}{2}}=z_{0.025}=1.96.$$

Since $z=10.328$ falls in the critical region (acceptance region of H_1), then we accept H_1 (i.e. $\mu\neq 2$).

7.1.2 σ^2 is unknown

If $x_1,\cdots,x_n \overset{iid}{\sim} n(\mu,\sigma^2)$, ($\sigma^2$ is assumed unknown).

In this case, $H_\circ:\mu=\mu_0$, and the alternative hypothesis can be taken as:

(a) $H_1:\mu>\mu_0$,

(b) $H_1:\mu<\mu_0$,

(c) $H_1:\mu\neq\mu_0$.

By using (7.1), and

$$x_1,\cdots,x_{25} \overset{iid}{\sim} n(\mu_\circ,\sigma^2) \Rightarrow \bar{x} \sim n(\mu_\circ,\frac{\sigma^2}{n}) \text{ and } \frac{\sqrt{n}(\bar{x}-\mu_\circ)}{\sigma} \sim n(0,1),$$

And,

$$W = \frac{ns^2}{\sigma^2} \sim \chi^2(n-1).$$

Thus, from t-distribution we can define the test statistic as:

$$T = \frac{z}{\sqrt{\frac{W}{n-1}}} = \frac{\frac{\sqrt{n}(\bar{x}-\mu_\circ)}{\sigma}}{\sqrt{\frac{ns^2}{\sigma^2(n-1)}}} = \frac{\sqrt{n-1}(\bar{x}-\mu_\circ)}{s} \sim t(n-1) \tag{7.2}$$

Accordingly, the critical region of each case of H_1 can be written as:

(a) $H_1 : \mu > \mu_0$, the critical region is

$$t \geq t_\alpha$$

Where,

t is defined in (7.2) above, and t_α is tabulate value (from the table of t-distribution)

(b) $H_1 : \mu < \mu_0$, the critical region is

$$t \leq -t_\alpha$$

t is defined in (7.2) above, and t_α is tabulate value (from the table of t-distribution)

(c) $H_1 : \mu \neq \mu_0$, the critical region is

$$t > t_{\frac{\alpha}{2}} \quad \& \quad t < -t_{\frac{\alpha}{2}},$$

t is defined in (7.2) above, and $t_{\frac{\alpha}{2}}$ is tabulate value (from the table of t-distribution)

6. Test of hypotheses-Sampling from normal distribution

Definition (6.1): (A statistical hypothesis) is a statement about the value of a population parameter.
بيان تأكيد

Definition (6.2): (Null hypothesis) is usually the hypothesis for which the researchers wish to gather evidence to reject. H_0
الأدلة للرفض

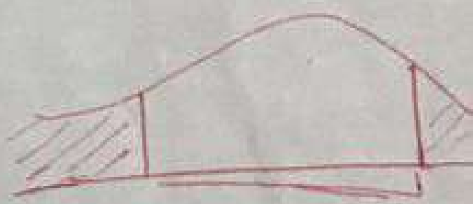
Definition (6.3): (Alternative hypothesis) is usually the hypothesis for which the researchers wish to gather evidence to support. H_1
اختبار الإحصائية لدعم

Definition (6.4): (Test statistic) the sample statistic one uses to either reject H_0 or not to reject H_0.
احصائية العينة

الحرجة القيم المنفصل
Definition (6.5): (Critical values) the values of the test statistic that separate the rejection and non-rejection regions.

منطقة الرفض
Definition (6.6): (Rejection region) the set of values for the test statistic that leads to rejection of H_0.

Definition (6.7): (Non-rejection region) the set of values not in the rejection region that leads to non-rejection of H_0.


critical

The steps of Test of hypotheses اهمية السكان

نوع المجتمع

1. Determine the type of population
2. Formulate the null and alternative hypotheses
3. Determine level of significance α
4. Find the statistic-value
5. Critical values and rejection region
6. Process of checking to see whether the test statistic falls in the rejection region

سواء
تعبير

The **Null hypothesis** H_0 is always expressed in the form of an equality.

i.e. $H_0: \mu = 5$ or $H_0: p = 9.7$

<u>Example (6.1)</u>

نقل مطالبات حوادث

The department of transportation claims the average number of accidents each day is 3.2. We believe the claimed average is too small. Here, the state of H_0 and H_1

$H_0: \mu = 3.2$ (μ is the average number of accidents)

$H_1: \mu > 3.2$

<u>Example (6.2)</u>

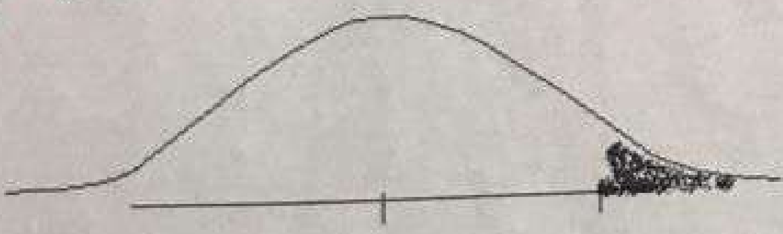
The department of mathematics claims the average of success is 55%. We believe believe the claimed average is incorrect, and then H_0 and H_1 can be formulated as :

$H_\circ : \mu = 0.55$ (μ is the average number of success)

$H_1 : \mu \neq 0.55$

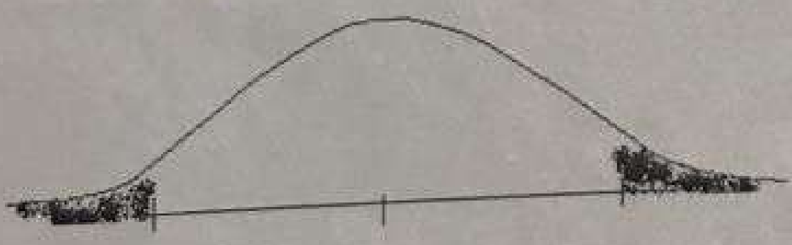
Definition (6.8): ( ) is one where H_1 is directional and includes < or > (see example 6.1)

one tailed test



or

Definition (6.9): (two-tailed test) is one where H_1 no directional is included and utilises $\neq$ (see example 6.2)



<u>Example (6.3)</u>

Suppose $X \sim n(\mu, \sigma^2)$, and we need to make the hypotheses about the mean μ, then we have three cases:

(a) $H_\circ : \mu = \mu_\circ$ **(One-tailed test)**
$H_1 : \mu > \mu_\circ$