Modern Physics Course Ph-207

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CHAPTER 1

RELATIVITY I

At the end of the 19th century, scientists believed that they had learned most of what there was to know about physics. Newton's laws of motion and his universal theory of gravitation, Maxwell's theoretical work in unifying electricity and magnetism, and the laws of thermodynamics and kinetic theory employed mathematical methods that successfully explain a wide variety of phenomena.

However, at the turn of the 20th century, a major revolution shook the world of physics. In 1900, Planck provided the basic ideas that led to the quantum theory, and in 1905, Einstein formulated his special theory of relativity.

1.1 Special Relativity

Light waves and other forms of electromagnetic radiation travel through free space at the speed $C = 3.00 * 10^8$ m/s. The speed of light sets an upper limit for the speeds of particles, waves, and the transmission of information.

Most of our everyday experiences deal with objects that move at speeds much less than that of light. Newtonian mechanics and early ideas on space and time were formulated to describe the motion of such objects, and this formalism is very successful in describing a wide range of phenomena. Although Newtonian mechanics works very well at low speeds, it fails when applied to particles whose speeds approach that of light.

In 1905, at the age of 26, Albert Einstein published his special theory of relativity. Regarding the theory, Einstein wrote. The relativity theory arose from necessity, from serious and deep contradictions in the old theory from which there seemed no escape. The strength of the new theory lies in the consistency and simplicity with which it solves all these difficulties, using only a few very convincing assumptions. Although Einstein made many important contributions to science, the theory of relativity alone represents one of the greatest intellectual achievements of the 20th century. With this theory, one can correctly predict experimental observations over the range of speeds from rest to speeds approaching the speed of light. Newtonian mechanics, which was accepted for over 200 years, is in fact a limiting case of Einstein's special theory of relativity.

The relativity deals with the analysis of physical events from coordinate systems moving with constant speed in straight lines with respect to one another. Also describes physical events from coordinate systems undergoing general or accelerated motion with respect to each other.

In general, the special theory of relativity follows from two basic postulates:

1. The laws of physics are the same in all reference systems that move uniformly with respect to one another. That is, basic laws such as F=dp/dt have the same mathematical form for all observers moving at constant velocity with respect to one another.
2. The speed of light in vacuum always measured to be $3*10^8$ m/s, and the measured value is independent of the motion of the observer or of the motion of the source of light. That is, the speed of light is the same for all observers moving at constant velocities.

1.2 The Principle of Relativity

In order to describe a physical event, it is necessary to establish a frame of reference, such as one that is fixed in the laboratory.

According to the principle of Newtonian relativity, the laws of mechanics must be the same in all inertial frames of reference. For example, if you perform an experiment while at rest in a laboratory, and an observer in a passing truck moving with constant velocity performs the same experiment, Newton's laws may be applied to both sets of observations. Specifically, in the laboratory or in the truck a ball thrown up rises and returns to the thrower's hand. Moreover, both events are measured to take the same time in the truck or in the laboratory, and Newton's second law may be used in both frames to compute this time. Although these experiments look different to different observers and the observers measure different values of position and velocity for the ball at the same times, both observers agree on the validity of Newton's laws and principles such as conservation of energy and conservation of momentum. The only thing that can be detected is the relative motion of one frame with respect to the other. That is, the notion of absolute motion through space is meaningless, as is the notion of a single, preferred reference frame. Indeed, one of the firm philosophical principles of modern science is that all observers are equivalent and that the laws of nature must take the same mathematical form for all observers. Laws of physics that exhibit the same mathematical form for observers with different motions at different locations are said to be covariant.

In order to show the underlying equivalence of measurements made in different reference frames and hence the equivalence of different frames for doing physics, we need a mathematical formula that systematically related measurements made in one reference frame to those in another. Such a relation is called a transformation, and the one satisfying Newtonian relativity is the so called Galilean transformation, which owes its origin to Galileo. It can be derived as follows.

1.2.1 Galilean Transformation of Coordinates

Consider two inertial systems or frames S and $S^/$. The frame $S^/$ moves with a constant velocity v along the $xx^/$ axes, where v is measured relative to the frame S. Clocks in S and $S^/$ are synchronized, and the origins of S and $S^/$ coincide at $t = t^/ = 0$. We assume that a point event, a physical phenomenon such as a light bulb flash, occurs at the point P.

An observer in the system S would describe the event with space-time coordinates (x, y, z, t), whereas an observer in $S^{/}$ would use ($x^{/}$, $y^{/}$, $z^{/}$, $t^{/}$) to describe the same event. As we can see from Figure 1.1, these coordinates are related by the equations.

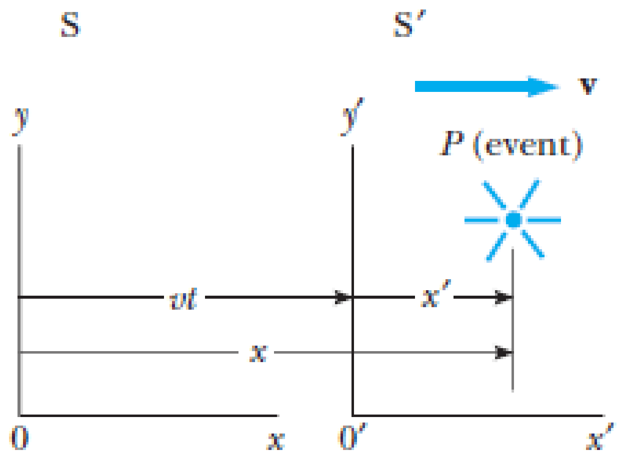


Figure 1.1 an event occurs at a point P. The event is observed by two observers in inertial frames S and $S^{/}$, in which $S^{/}$ moves with a velocity v relative to S.

$$
\begin{aligned}
x' &= x - vt \\
y' &= y \\
z' &= z \\
t' &= t
\end{aligned}
\qquad (1.1)
$$

These equations constitute what is known as a Galilean transformation of coordinates. Note that the fourth coordinate, time, is assumed the same in both inertial frames. That is, in classical mechanics, all clocks run at the same rate regardless of their velocity, so that the time at which an event occurs for an observer in S is the same as the time for the same event in $S^{/}$. Consequently, the time interval between two successive events should be the same for both observers. Although this assumption may seem obvious, it turns out to be incorrect when treating situations in which v is comparable to the speed of light.

In fact, this point represents one of the most profound differences between Newtonian concepts and the ideas contained in Einstein's theory of relativity.

An immediate and important consequence of the invariance of the distance between two points under the Galilean transformation is the invariance of force. For example if $F = \frac{kqQ}{(x_2 - x_1)^2}$ gives the electric force between two charges q, Q located at x_1 and x_2 on the x-axis in frame S, F', the force measured in S', is given by $F' = \frac{kqQ}{(x'_2 - x'_1)^2}$ = F since, $x'_2 - x'_1 = x_2 - x_1$. In fact, any force would be invariant under the Galilean transformation as long as it involved only the relative positions of interacting particles. Now suppose two events are separated by a distance dx and a time interval dt as measured by an observer in S. It follows from Equation 1.1 that the corresponding displacement dx' measured by an observer in S' is given by $dx' = dx - vdt$, where dx is the displacement measured by an observer in S. Because $dt = dt'$, we find that;

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v$$

$$u'_x = u_x - v \qquad (1.2)$$

where u_x and u'_x are the instantaneous velocities of the object relative to S and S', respectively. The result that called the Galilean addition law for velocities (or Galilean velocity transformation), are used in everyday observations and is consistent with our intuitive notions of time and space. To obtain the relation between the accelerations measured by observers in S andS', we take a derivative of Equation 1.2 with respect to time and use the results that $dt = dt'$ and v is constant:

$$\frac{du'_x}{dt'} = a'_x = a_x \qquad (1.3)$$

Thus, observers in different inertial frames measure the same acceleration for an accelerating object. The mathematical terminology is to say that lengths (ΔX), time intervals, and accelerations are invariant under a Galilean transformation. Transformation equations, in addition to converting measurements made in one inertial frame to those in another, may be used to show the variance of physical laws.

Exercise1: Assume that Newton's law $F_x = ma_x$ has been shown to hold by an observer in an inertial frame S. Show that Newton's law also holds for an observer in $S^/$ or is covariant under the Galilean transformation, that is, has the form $F_X^/ = m^/ a_X^/$. Note that inertial mass is an invariant quantity in Newtonian dynamics.

Exercise2: conservation of linear momentum is covariant under the Galilean transformation. Assume that two masses $m_1^/$ and $m_2^/$ are moving in the positive x direction with velocities $v_1^/$ and $v_2^/$ as measured by an observer in $S^/$before a collision. After the collision, the two masses stick together and move with a velocity $v^/$ in$S^/$. Show that if an observer in $S^/$ finds momentum to be conserved, so does an observer in S.

1.2.2 The Speed of Light

The speed of light was C only with respect to the ether or a frame fixed in the ether called the ether frame, where C;

$$C = (u_o\, \varepsilon_o)^{-1/2} = 3*10^8 \text{ m/s}$$

Permeability of vacuum μ_o =12.566 e^{-7} ($kg.m.s^{-2}.A^{-2}$)

Permittivity of vacuum ε_o =8.854 e^{-12} ($kg^{-1}.m^{-3}.s^4.A^2$)

In any other frame moving at speed v relative to the ether frame, the Galilean addition law expected to hold. Thus, the speed of light in this other frame was expected to be c – v for light travelling in the same direction as the frame, c + v for light travelling opposite to the frame, and in between these two values for light moving in an arbitrary direction with respect to the moving frame. Because the existence of the ether and a preferred ether frame would show that light was similar to other classical waves (in requiring a medium). Scientists realizing that the Earth moved rapidly around the Sun at 30 km/s, they decided to use the Earth itself as the moving frame in an attempt to improve their are a small changes in light velocity.

From our point of view of observers fixed on Earth, we may say that we are stationary and that the special ether frame moves past us with speed$v$. Determining the speed of light under these circumstances is just like determining the speed of an aircraft in a moving air current or wind, and consequently we speak of an “ether wind” blowing through our apparatus fixed to the Earth.

If v is the velocity of the ether relative to the Earth, then the speed of light should have its maximum value, c +v, when propagating downwind, as shown in Figure 1.2a. Likewise, the speed of light should have its minimum value, c - v, when propagating upwind, as in Figure 1.2b, and an intermediate value, $(c^2 - v^2)^{1/2}$, in the direction perpendicular to the ether wind, as in Figure 1.2c. If the Sun is assumed to be at rest in the ether, then the velocity of the ether wind would be equal to the orbital velocity of the Earth around the Sun, which has a magnitude of about $3 * 10^4$ m/s compared to $C = 3 * 10^8$ m/s. Thus, the change in the speed of light would be about 1 part in 10^4 for measurements in the upwind or downwind directions, and changes of this size should be detectable.

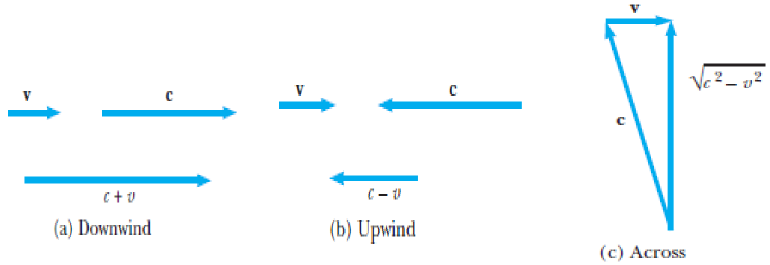


Figure 1.2 If the velocity of the ether wind relative to the Earth is **v**, and **c** is the velocity of light relative to the ether, the speed of light relative to the Earth is (a) $c + v$ in the downwind direction, (b) $c - v$ in the upwind direction, and (c) $(c^2 - v^2)^{1/2}$ in the direction perpendicular to the wind.

1.3 The Michelson Experiment

American physicist Albert A. Michelson and Edward W. Morley performed the famous experiment designed to detect small changes in the speed of light with motion of an observer through the ether in 1887.

The highly accurate experimental tool perfected by these pioneers to measure small changes in light speed was the Michelson interferometer, shown in Figure 1.3. One of the arms of the interferometer was aligned along the direction of the motion of the Earth through the ether.

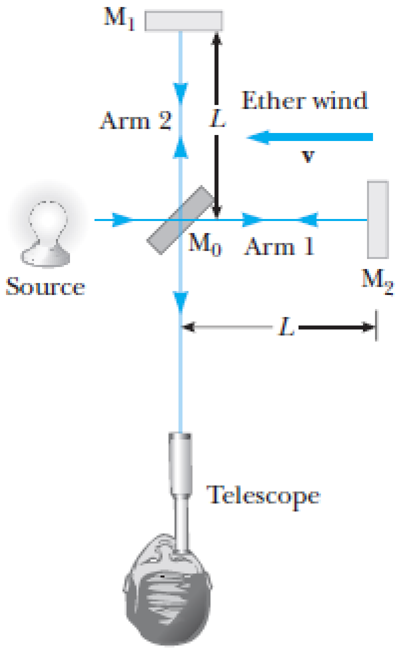


Figure 1.3 Diagram of the Michelson interferometer. According to the ether wind concept, the speed of light should be $c - v$ as the beam approaches mirror M_2 and $c + v$ after reflection.

The Earth moving through the ether would be equivalent to the ether flowing past the Earth in the opposite direction with speed *v*, as shown in Figure 1.3. This ether wind blowing in the opposite direction should cause the speed of light measured in the Earth's frame of reference to be $(c - v)$ as it approaches the mirror M2 in Figure 1.3 and $(c + v)$ after reflection.

The speed v is the speed of the Earth through space, and hence the speed of the ether wind and c is the speed of light in the ether frame. The two beams of light reflected from M1 and M2 would recombine, and an interference pattern consisting of alternating dark and bright bands, or fringes, would be formed.

During the experiment, the interference pattern was observed while the interferometer was rotated through an angle of 90°. This rotation would change the speed of the ether wind along the direction of the arms of the interferometer. The effect of this rotation should have been to cause the fringe pattern to shift slightly but measurably. Measurements failed to show any change in the interference pattern! Other researchers under various conditions repeated the Michelson–Morley experiment and at different times of the year when the ether wind was expected to have changed direction and magnitude, but the results were always the same: No fringe shift of the magnitude required was ever observed!

- The negative results of the Michelson experiment not only meant that the speed of light does not depend on the direction of light propagation but also contradicted the ether hypothesis.
- The negative results also meant that it was impossible to measure the absolute velocity of the Earth with respect to the ether frame.

Example: - Two boats move from the same point A, first one (1) pass across the river to the opposite coast and return back to same point A, the second boat (2) moves in same direction of river current and return back to point A.

- Which boat need more time to return to the starting point?

Assume the speed of river current$v$, the river width d.

Answer:-

- For the first boat in order to return to the starting point A, it must move against the river current direction with speed V.

$$V^2 = V'^2 + v^2\ ,\quad V'^2 = V^2 - v^2,\ V' = V\sqrt{1 - \frac{v^2}{V^2}}$$

In general S = d / t

$T_1 = 2d \ / \ V' = \frac{2d}{V\sqrt{1-\frac{v^2}{V^2}}}$ total period for first boat travel (departure and return back).

- The second boat, the period of departure $\frac{d}{(V+v)}$, why

the period of return $\frac{d}{(V-v)}$, why

$T_2 = \frac{d}{(V+v)} + \frac{d}{(V-v)}$, $T_2 = \frac{2d}{V(1-\frac{v^2}{V^2})}$

So: $T1 > T_2$

1.4 Postulates of Special Relativity

In the previous section, we noted the impossibility of measuring the speed of the ether with respect to the Earth and the failure of the Galilean velocity transformation in the case of light.

Albert Einstein proposed a theory that boldly removed these difficulties and at the same time completely altered our notion of space and time. Einstein based his special theory of relativity on two postulates.

- **The Principle of Relativity:** All the laws of physics have the same form in all inertial reference frames.
- **The Constancy of the Speed of Light:** The speed of light in vacuum has the same value, $c = 3 * 10^8$ m/s, in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

1.5 Consequences of Special Relativity

As mentioned before in old wave theory that considered the ether is the constant universal frame of reference, the speed of light is $C = 3*10^8$ m/s. So almost the moving must be fallow the Particular reference, maybe (way, earth surface, Sun or any other centre). However, must be selecting the suitable reference frame for each case. If the ether is widespread in all the space, then will we can attribute all the movements to the ether. The absence of ether means that there is no special universal reference frame, for that all the movement belong the observer.

❖ ***Summarise;-***

Relativity: - describing and analysis of the physical phenomena are arising from the absence of a distinct universal reference frame.

Special theory of relativity: - it is dealing with problems including inertial frames of reference moving uniformly for each other's.

General theory of relativity: -it is dealing with problems including accelerate frames of reference for are others.

Galilean Transformation and Newton's Laws:-

1- Inertial Law; for particle moving uniformly (U_x) respect for S system, the movement obey the first law of Newton. The XYZ is inertial coordinate, the speed of particle for observer in $S^/$ system is.

$$U_x^/ = U_x - v$$

2- Force or Accelerate Law, mass is absolute value and independent of observer position and the movement,

$$m = m^/, \; a = a^/$$

then

$$F = ma$$

$$F = F^{/} = ma = m^{/}a^{/}$$

3- Action and Reaction Law, the interaction between two particles A, B in S system can describe it in third law

$$F_{AB} = -F_{BA}$$

As mentioned in Galilean transformation (force is absolute quantity)

$$F^{/} = F$$

$$F^{/}_{AB} = -F^{/}_{BA}$$

Example:-

Two riders passed in front of fixe observer; first rider moving uniformly with speed 20km/h, second rider moving also uniformly in speed 10Km/h, find

1- The position of riders for observer at passing point after 0.1h.
2- The position of first rider to respect of second rider after 0.3h from passing point.

Answer;

1-

$$x_1 = x_o + v_1 t$$

$$x_1 = 0 + 20 * 0.1 = 2Km$$

$$x_2 = x_o + v_2 t = 0 + 10 * 0.1 = 1Km$$

2-

A basic premise of Newtonian mechanics is that a universal time scale exists that is the same for all observers. In fact, Newton wrote that "Absolute, true and mathematical time, of itself, and from its own nature, flows equably without relation to anything external. Thus, Newton and his followers simply took simultaneity for granted. In his special theory of relativity, Einstein abandoned this assumption. According to Einstein, a time interval measurement depends on the reference frame in which the measurement that made.

Two events that are simultaneous in one frame are in general not simultaneous in a second frame moving with respect to the first. That is, simultaneity is not an absolute concept, but one that depends on the state of motion of the observer.

Exercise;

Prove the liner momentum and kinetin energy are conservative at inertial frame S and motional inertial frame $S^{/}$ for two masses suffering from elastic collusion, assume the masses are m_1, m_2 and their peed before and after collusion are u_1, u_2 also U_1, U_2 respectively for S. Write done the equations' form at $S^{/}$ frame.

1.6 The Lorentz Transformation

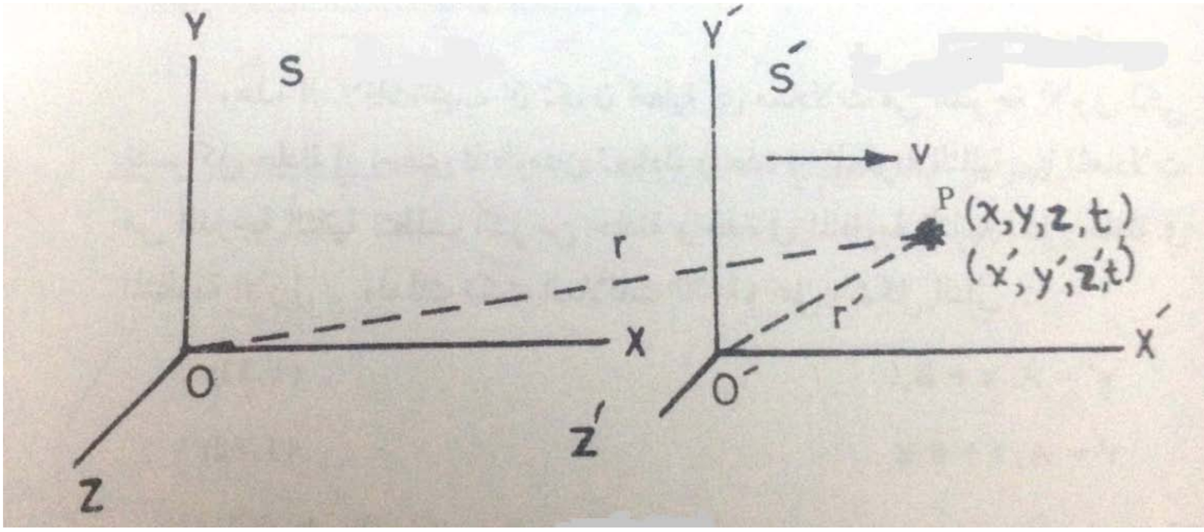
We have seen that the Galilean transformation is not valid when v approaches the speed of light, so that we need derive the correct coordinate and velocity transformation equations that apply for all speeds in the range of ($0 \leq v \geq c$). The Lorentz coordinate transformation is a set of formulas that relates the space and time coordinates of two inertial observers moving with a relative speed v.

The Lorentz velocity transformation is the set of formulas that relate the velocity components u_x, u_y, u_z of an object moving in frame S to the velocity components $u^{/}_x$, $u^{/}_y$, $u^{/}_z$ of the same object measured in frame $S^{/}$, which is moving with a speed v relative to S.

For simplicity, consider the standard frames, S and $S^{/}$, with $S^{/}$ moving at a speed v along the +x direction. The origins of the two frames coincide at t= t= 0. A reasonable guess about the dependence of $x^{/}$ on x and t is

$$x^{/} = x - vt, \; x = x^{/} + vt^{/}$$

If pulse emit in the moment, the speed of light is c independent of observer speed, the pulse will reach to the point p in the space after the period.



r=ct

r$^{/}$=ct$^{/}$

$$r^2 = x^2 + y^2 + z^2 = c^2t^2$$

$$r^{/2} = x^{/2} + y^{/2} + z^{/2} = c^{/2}t^{/2}$$

Then

$$y^{/} = y$$

$$z^{/} = z$$

$$x^{/} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\,(x - vt) = k(x - vt)$$

$$t^{/} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(t - \frac{v}{c^2}x\right) = k(t - \frac{v}{c^2}x)$$

This equations refer to Lorentz transformation from S frame to the S$^{/}$.

1.5.2 Time Dilation:

The fact, that observers in different inertial frames always measure different time intervals between a pair of events.

Consider the two events has been happened in one location X_o in reference inertial frame S; first event at time t_1 and the second event at time t_2, or the event happened in location X_o at system S at time t_1 and continued to the t_2, then the time interval between two events for observer have seating in the same frame is:

$$T_o = t_2 - t_1$$

The time interval for observer seating in inertial frame $S^/$ that moving uniformly with speed v respect to the frame S, in direction $XX^/$ is:

$$T^/ = t^/_2 - t^/_1$$

From Lorentz transformation

$$t^/ = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(t - \frac{v}{c^2}x\right) = k(t - \frac{v}{c^2}x)$$

$$t^/_1 = k(t_1 - \frac{v}{c^2}x_1)$$

$$t^/_2 = k(t_2 - \frac{v}{c^2}x_2)$$

Then ($T^/ = t^/_2 - t^/_1$)

$$T^/ = k(t_2 - t_1) - \frac{v}{c^2}k(x_2 - x_1)$$

When the observer has seating in $S^/$ frame measuring the time interval at the same location then:

$$X_1 = X_2 = X_o$$

$$T^{/} = kT_o = \frac{T_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

From the last equation:

First, if $0 \leq v \leq 0.01c$ then $T^{/} = T_o$

Second, if v=c then T=∞

Third, if $0.01c < v < c$ then $T^{/} > T_o$

That mean the period for event happened in S frame that are measuring by observer moving uniformly in speed v has been appear longer than the period that measuring by other observer seating in the event frame, and vice versa.

EXAMPLE: - What Is the Period of the Pendulum?

The period of a pendulum is measured to be 3.0 s in the rest frame of the pendulum. What is the period of the pendulum when measured by an observer moving at a speed of 0.95c with respect to the pendulum?

Solution: In this case, the proper time is equal to 3.0 s. From the point of view of the observer, the pendulum is moving at 0.95c past her. Hence, the pendulum is an example of a moving clock. Because a moving clock runs slower than a stationary clock by k,

$$T^{/} = kT = \frac{T}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$T^{/} = \frac{3.0s}{\sqrt{1 - \frac{(0.95c)^2}{c^2}}} = (3.2)(3.0\text{s}) = 9.6\text{s}$$

That is, a moving pendulum slows down and takes longer to complete one period.

1.5.3 Length Contraction

The measured distance between two points depends on the frame of reference. The proper length of an object is defined as the length of the object measured by someone who is at rest with respect to the object.

<u>The length of an object measured by someone in a reference frame that is moving relative to the object is always less than the proper length. This effect is known as length contraction.</u>

To understand length contraction quantitatively, consider a spaceship travelling with a speed v from one star to another and two observers, one on Earth and the other in the spaceship. The observer at rest on Earth (assumed to be at rest with respect to the two stars) measures the distance between the stars to be L, where L is the proper length.

According to this observer, the time it takes the spaceship to complete the voyage is T=L/v.

What does an observer in the moving spaceship measure for the distance between the stars?

Because of time dilation, the space traveller measures a smaller time of travel:

$$T^{/} = kT$$

The space traveller claims to be at rest and sees the destination star as moving toward the spaceship with speed v. Because the space traveller reaches the star in the shorter time $T^{/}$. He or she concludes that the distance, $L^{/}$ between the stars is shorter than L. This distance measured by the space traveller is giving by:

$$L^{/} = v\,T^{/} = vT/k$$

Because L = v T, we see that $L^{/}$= L/ k or:

$$L = \frac{L^{/}}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{or} \quad L^{/} = L\sqrt{1-\frac{v^2}{c^2}}$$

Where $\sqrt{1-\frac{v^2}{c^2}}$ is a factor less than 1.

This result may be interpreted as follows: If an object has a proper length L when it is measured by an observer at rest with respect to the object, when it moves with speed v in a direction parallel to its length, its length $L^{/}$ is measured to be shorter according to $L^{/}= L\sqrt{1-\frac{v^2}{c^2}}$

Note that the length contraction takes place only along the direction of motion.

<u>EXAMPLE: - </u>The Contraction of a Spaceship. A spaceship is measured to be 100 m long while it is at rest with respect to an observer. If this spaceship now flies by the observer with a speed of 0.99c, what length will the observer find for the spaceship?

Solution: - The proper length of the ship is 100 m. The length measured as the spaceship flies by is

$$L^{/}= L\sqrt{1-\frac{v^2}{c^2}}$$

$$L^{/}= 100\text{m}\sqrt{1-\frac{(0.99c)^2}{c^2}} = 14\text{m}$$

Exercise: - An observer on Earth sees a spaceship at an altitude of 435 m moving downward toward the Earth at 0.970c. What is the altitude of the spaceship as measured by an observer in the spaceship? **Homework.**

Chapter 2

RELATIVITY II

2.1 Relativistic Speed

Assume an objective moving with speed **u** to respect of **S** observer and with speed **u**$^{/}$ for **S**$^{/}$ observer that moving uniformly in v speed to the S system, so the speed component can take be the form.

$$u_x = \frac{dx}{dt}\ ,\ u_y = \frac{dy}{dt}\ ,\ u_z = \frac{dz}{dt}$$

In addition, the speed components for S$^{/}$can are taking the form.

$$u_x^{/} = \frac{dx^{/}}{dt}\ ,\ u_y^{/} = \frac{dy^{/}}{dt}\ \ ,\ u_z^{/} = \frac{dz^{/}}{dt}$$

In order to find the relation between u and u$^{/}$, we have to use the Lorentz transformation.

From equations below;

$$dx^{/} = k\ (dx - vdt)$$

$$dy^{/} = dy$$

$$dz^{/} = dz$$

$$dt^{/} = k\ (dt - \frac{v}{c^2}dx)$$

After solve the above equations;

$$u_x^{/} = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}$$

$$u_y^{/} = \frac{u_y}{k(1 - \frac{vu_x}{c^2})}$$

$$u_z^{/} = \frac{u_z}{k(1 - \frac{vu_x}{c^2})}$$

The reverse transformation;

$$u_x = \frac{u_x^{/} + v}{1 + \frac{vu_x^{/}}{c^2}}$$

$$u_y = \frac{u_y^{/}}{k(1 + \frac{vu_x^{/}}{c^2})}$$

$$u_z = \frac{u_z^{/}}{k(1 + \frac{vu_x^{/}}{c^2})}$$

As one can see the speed$u_y^{/}$, $u_z^{/}$ andu_y, u_z depend u_x or $u_x^{/}$ in less the transformations are not including x, because the different in time for both observer in systems S and $S^{/}$.

Also the Lorentz transformation for speed can be changing to the Galilean transformations if the speed amounts v and u_x very small with respect for light speed.

From the above we can conclude that:

1- The speed of light is independent on relativistic motion for both the source and observer.
2- The speed of objective cannot be more than the speed of light.
3- Galilean transformations for speed are correct just if the speed of objective is very small compare to the speed of light, also are not correct for the objectives moving with high speed close to the light speed.

Example:-

Two rockets' are moving with speed 0.3c for respect to the observer site on earth; at opposite direction for each other, find the rocket's speed Relative to other. V= 0.3c $u^{/}$=0.3c

$\longrightarrow$ $\longleftarrow$

B A

We can imagine the A rocket is fixed and B rocket flying in relativistic velocity $u^{/}$, then we can calculate the speed u;

$$u = \frac{u^{/} + v}{1 + \frac{vu^{/}}{c^2}}$$

$$u = \frac{0.3c + 0.3c}{1 + \frac{0.3c * \ 0.3c}{c^2}}$$

$$u = 0.55c$$

2.2 Relativistic Mass

The classical mechanics are considering that the all of objectives are fixed, but for special relativistic theory the mass of body are changing to the respect of their speed.

$$m = \frac{m_o}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Where $m_o$the rest mass, m is the relativistic mass of objective have u speed with respect to fixed observer.

For that, first in special relativistic m_o is the rest mass not the m, second m_o=m when the speed of body near the zero.

Example:-

What is the velocity that is required for particle has relativistic mass that are double of rest mass.

$$m = 2m_o$$

$$m = \frac{m_o}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Then

$$\sqrt{1 - \frac{u^2}{c^2}} = \frac{m_o}{m} = \frac{1}{2}$$

$$1 - \frac{u^2}{c^2} = \frac{1}{4}$$

$$u = 0.866c$$

2.3 Relativistic momentum:

At classical mechanics the momentum P_c can be define for particle have a mass m and speed u,

$$p_c = mu = m_o u$$

Relativistic theory considering the mass changing to respect of their speed, then the momentum can be writ as showing below.

$$P = mu = \frac{m_o u}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Where p is the relativistic momentum, also m is the relativistic mass.

Exercise: An electron, which has a mass of $9.11*10^{31}$ kg, moves with a speed of 0.750c, find its relativistic momentum and compare this with the momentum calculated from the classical expression p_c.

2.4 Relativistic Energy

The kinetic energy in classical form can write to the respect of rest mass m_o and speed u;

$$E_{kc} = \frac{1}{2} m_o u^2$$

But in relativistic form

$$E_k = m_o c^2 \left[\frac{1}{\sqrt{1-\frac{u^2}{c^2}}} - 1\right] = k\, m_o c^2 - m_o c^2$$

$$E_k = \frac{1}{2} m_o u^2 = E_{kc}$$

The constant term $m_o c^2$, which is independent of the speed, called the rest energy of the particle.

The term E, which depends on the particle speed, is therefore the sum of the kinetic and rest energies. We define it is the total energy $E = k m_o c^2$, that is;

$$E = E_k + m_o c^2$$

In many situations, the momentum or energy of a particle is measured rather than its speed. It is therefore useful to have an expression relating the total energy E to the relativistic momentum p. This is accomplished using E and p. By squaring these equations and subtracting, we can eliminate u. The result, after some algebra, is;

$$E^2 = p^2 c^2 + (m_o c^2)^2$$

Example: An electron has a speed u=0.850c. Find its total energy and kinetic energy in electron volts, if the rest energy of the electron is 0.511MeV.

Solution:

The total energy is;

$$E = k\, m_o c^2$$

$$E = \frac{m_o c^2}{\sqrt{1 - \frac{u^2}{c^2}}} = \frac{0.511MeV}{\sqrt{1 - \frac{(0.85c)^2}{c^2}}} = 0.97\text{Mev}$$

The kinetic energy is;

$$E_k = E - m_o c^2 = 0.459\text{MeV}$$

Exercise: The total energy of a proton is three times its rest energy. Where the mass of proton is $m_p = 1.67 * 10^{27}$kg

a- Find the proton's rest energy in electron volts.

b- With what speed is the proton moving?

c- Determine the kinetic energy of the proton in electron volts.

d- What is the proton's momentum?

2.5 Relativistic Force

The force in classical mechanics is defined as a time rate of change momentum.

$$F_c = \frac{dp_c}{dt} = \frac{d(m_o u)}{dt} = m_o \frac{du}{dt}$$

However, the mass are not fixed in relativistic theory, for that the force are affecting on the relativistic mass and were defined as a time rate of change relativistic momentum.

$$F = \frac{dp}{dt} = \frac{d(mu)}{dt} = m\frac{du}{dt} + u\frac{dm}{dt}$$

Special cases

$u \ll c$ then m ------ m_o and $\frac{dm}{dt}$ ----- 0

2.6 Relativistic Synchronization (instantaneous)

Assume there are two events have been happened in two locations x_1, x_2 at S system, stationary observer in this frame recorded these events at one time (instantaneously) $t_1=t_2$. So are these events appear instantaneously for observer site in $S^/$ that moving uniformly in speed v to respect of S, is $t_1^/ = t_2^/$ or not.

$$t_1^/ = k\ (t_1 - \frac{v}{c^2}x_1)$$

$$t_2^/ = k\ (t_2 - \frac{v}{c^2}x_2)$$

After subtracting the above equations;

$$t_2^/ - t_1^/ = k\ (t_2 - t_1) - \frac{v}{c^2}(x_2 - x_1)$$

As we know $t_1=t_2$

Then $$t_2^/ - t_1^/ = -\frac{v}{c^2}(x_2 - x_1)$$

Also $\frac{v}{c^2}(x_2 - x_1) \neq 0$ then $t_2^/ \neq t_1^/$

CHAPTER 3

The Electrical, Optical Properties and band Theory

3.1 Determination of Electric Charge

Electric charge is the physical property of matter that causes it to experience a force when placed in an electromagnetic field. There are two types of electric charges: positive and negative. Like charges repel and unlike attract.

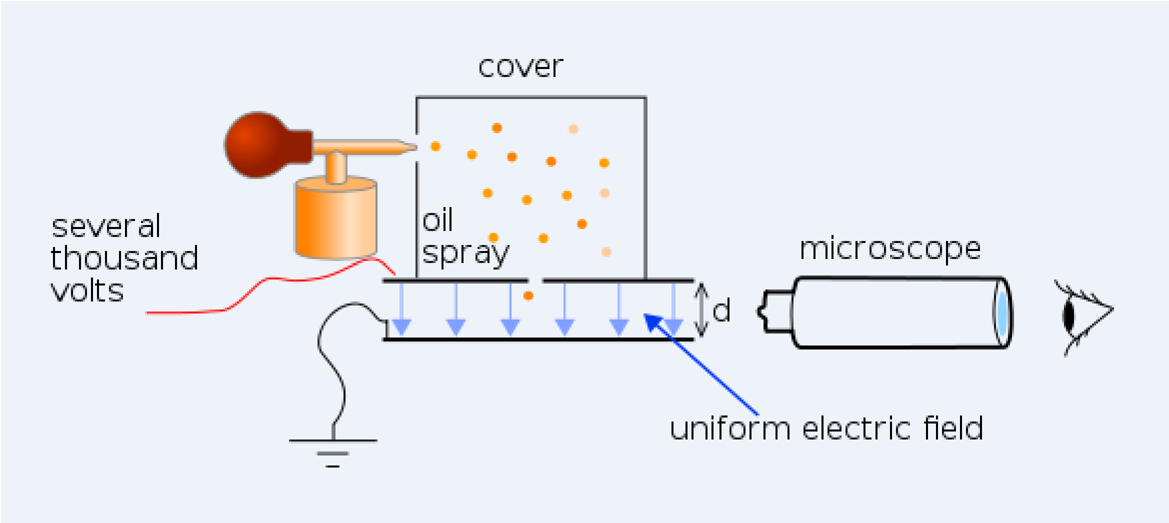
The electric charge, usually denoted as e or sometimes q, is the electric charge carried by a single proton, or equivalently, the magnitude of the electric charge carried by a single electron, which has charge $^{-}$e. This charge is a fundamental physical constant. This charge has a measured value of approximately 1.6×10^{-19} coulombs.

3.2 Millikan Method of Drop Model

The magnitude of the elementary charge was first measured in Robert A. Millikan's noted oil drop experiment in 1909.

The experiment entailed observing tiny charged droplets of oil between two horizontal metal electrodes. First, with zero applied electric field, the terminal velocity of a droplet was measured. Then an adjustable voltage was applied between the plates to induce an electric field, and the voltage was adjusted until the drops were suspended in mechanical equilibrium, indicating that the electrical force and the gravitational force were balanced.

Now using the known electric field, Millikan could determine the charge on the oil droplet. By repeating the experiment for many droplets, they confirmed that the charges were all small integer multiples of a certain base value, which was found to be 1.5926×10^{-19} C, within 1% of the currently accepted value of 1.602×x10−19 C. They proposed that this was the (negative of the) charge of a single electron.



Millikan and Fletcher's apparatus incorporated a parallel pair of horizontal metal plates. By applying a potential difference across the plates, a uniform electric field was created in the space between them. A ring of insulating material was used to hold the plates apart. Four holes were cut into the ring, three for illumination by a bright light, and another to allow viewing through a microscope.

A fine mist of oil droplets was sprayed into a chamber above the plates. The oil was of a type usually used in vacuum apparatus and was chosen because it had an extremely low vapour pressure. Some oil drops became electrically charged through friction with the nozzle as they were sprayed. Alternatively, charging could be brought about by including an ionising radiation source (such as an X-ray tube). The droplets entered the space between the plates and, because they were charged, could be made to rise and fall by changing the voltage across the plates.

Initially the oil drops are allowed to fall between the plates with the electric field turned off. They very quickly reach a terminal velocity because of friction with the air in the chamber. The field is then turned on and, if it is large enough, some of the drops (the charged ones) will start to rise. This is because the upwards electric force F_E is greater for them than the downwards gravitational force F_g. A likely looking drop is collected and kept in the middle of the field of view by alternately switching off the voltage until all the other drops have fallen. The experiment is then continuing with this one drop. The drop is allowed to fall and its terminal velocity v1 in the absence of an electric field is calculated. The drag force acting on the drop can then be calculated.

$$F_E = F_g$$

$$F_E = qE \ \& \ F_g = mg$$

$$qE = mg$$

$$E = \frac{V}{D}$$

Then

$$q = \frac{mgD}{V}$$

Where q is the charge, m the oil drop mass, D the distance between two plates and g the gravity.

Example; Oil drop suspended between two plates after applied different potential (8 Volts), find the mass of drop if the total charges is $3.2 * 10^{-15}$ C and the distance between the plates are 2cm, also what is the drop diameter if the oil density is 850 kg/m^3. Assume the gravity is 10 m/Sec^2

$$q = \frac{mgD}{V}$$

$$m = \frac{qV}{gD}$$

$$V_{(sphere)} = (4/3)\pi r^3$$

3.3 The Free Electron Theory

A model of a metal in which the free electrons, that is, those giving rise to the conductivity, are regarded as moving in a potential (due to the metal ions in the lattice and to all the remaining free electrons) which is approximated as constant everywhere inside the metal. Also known as Somerfield model; Somerfield theory.

The treatment of a metal as containing a gas of electrons completely free to move within it. The theory was originally proposed in 1900 to describe and correlate the electrical and thermal properties of metals. Quantum mechanics became the basis for the theory of most of the general properties of simple metals such as sodium, with one free electron per atom, magnesium with two, and aluminium with three. Transition metals, such as iron, have partially filled electronic d states and are not treated by the free-electron model.

For that,

Metals are good conductors (both electrical and thermal).

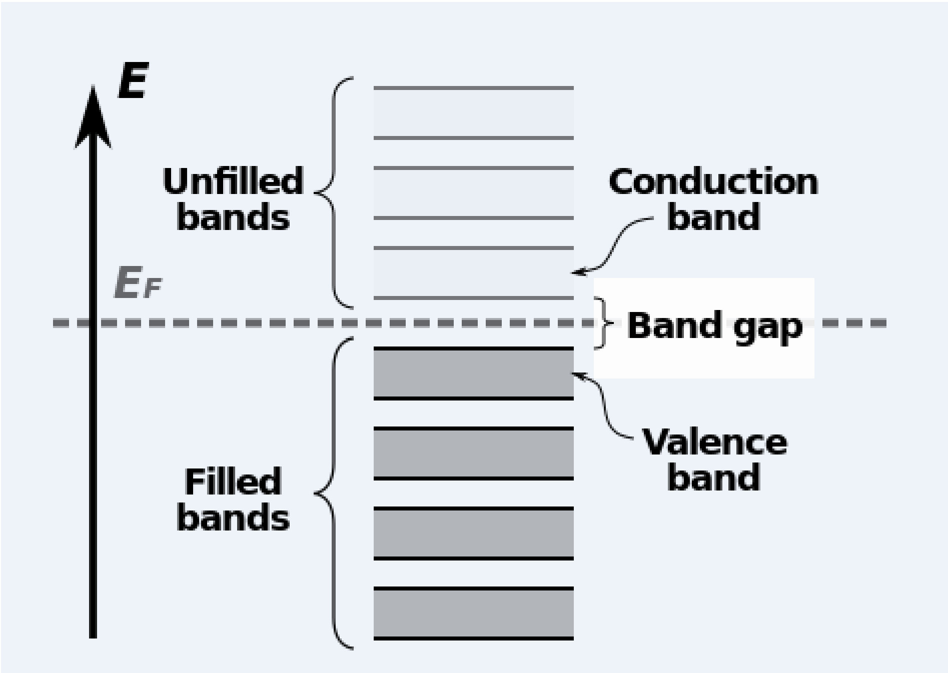
Electronic heat capacity has an additional (temperature dependent) contribution from the electrons.

3.4 Expression for Electrical Conductivity

The most important characteristic of a metal is its electrical conductivity; it is a measure of a material's ability to conduct an electric current also the specific conductance, is a measurement of the electrical conductance per unit distance in an electrolytic or aqueous solution. In metal, the electrically charged particles comprise ions and electrons. The ionic charge carriers comprise the cations, anions, and foreign ions (e.g. impurity ions, dopant ions and protons) and the electronic charge carriers are the electrons and electron holes.

Electrical conductivity or specific conductance is the reciprocal of electrical resistivity, and measures a material's ability to conduct an electric current. It is commonly represented by the Greek letter σ (sigma). Its SI unit is Siemens per metre (S/m).

In solid-state physics, the valence band and conduction band are the bands closest to the Fermi level and thus determine the electrical conductivity of the solid. The valence band is the highest range of electron energies in which electrons are normally present at absolute zero temperature, while the conduction band is the lowest range of vacant electronic states. On a graph of the electronic band structure of a material, the valence band is located below the Fermi level, while the conduction band is located above it. This distinction is meaningless in metals as the highest band is partially filled, taking on the properties of both the valence and conduction bands.



The formula for conductance should produce a quantity with dimensions independent of distance, like Ohm's Law for electrical resistance:

$$R = \frac{V}{I}$$

and conductance:

$$G = \frac{I}{V}$$

From the electrical formula: R$= \rho \frac{x}{A}$, where ρ is resistivity, x is length, and A is cross-sectional area, we have G = k$\frac{A}{x}$, where G is conductance, k is conductivity, x is length, and A is cross-sectional area.

3.5 Expression for Thermal Conductivity

Thermal conductivity is the property of a material to conduct heat; it's evaluated primarily in terms of Fourier's Law for heat conduction.

Heat transfer occurs at a lower rate across materials of low thermal conductivity than across materials of high thermal conductivity.

Correspondingly, materials of high thermal conductivity are widely used in heat sink applications and materials of low thermal conductivity are used as thermal insulation.

The thermal conductivity of a material may depend on temperature. The reciprocal of thermal conductivity is called thermal resistivity.

$$U = k\frac{A}{\Delta x}$$

Where U is the conductance and $\Delta x = x_2 - x_1$, Fourier's law of heat transfer is;

$$Q = U\Delta T$$

Where $\Delta T = T_2 - T_1$,

The reciprocal of conductance is resistance, R, given by:

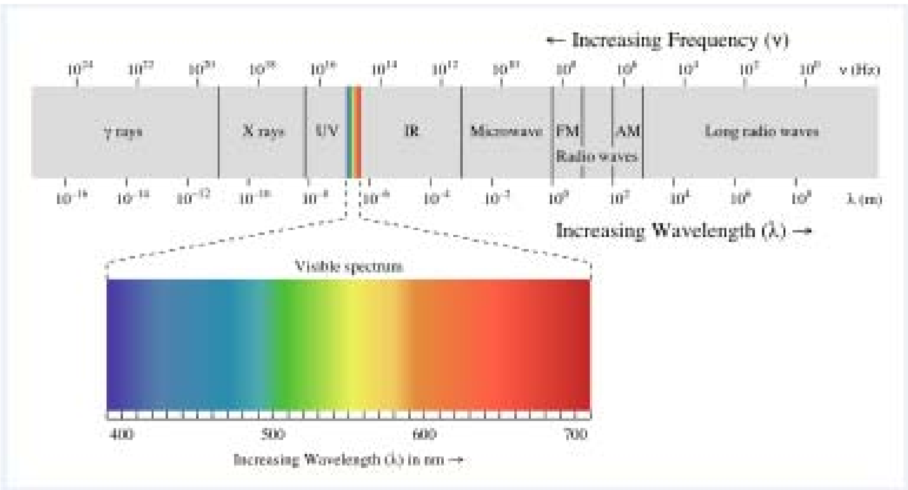
$$R = \frac{\Delta T}{Q}$$

3.6 Light

Light is electromagnetic radiation within a certain portion of the electromagnetic spectrum. The word usually refers to visible light, which is visible to the human eye and is responsible for the sense of sight. Visible light is usually defined as having wavelengths in the range of 400–700 nanometres (nm), or 4.00 × 10−7 to 7.00 × 10−7 m, between the infrared (with longer wavelengths) and the ultraviolet (with shorter wavelengths).

Electromagnetic spectrum and visible light;

Generally, EM radiation, or EMR (the designation "radiation" excludes static electric and magnetic and near fields), is classified by wavelength into radio, microwave, infrared, the visible region that we perceive as light, ultraviolet, X-rays and gamma rays.



The behaviour of EMR depends on its wavelength. Higher frequencies have shorter wavelengths, and lower frequencies have longer wavelengths. When EMR interacts with single atoms and molecules, its behaviour depends on the amount of energy per quantum it carries.

EMR in the visible light region consists of quanta (called photons) that are at the lower end of the energies that are capable of causing electronic excitation within molecules, which leads to changes in the bonding or chemistry of the molecule. At the lower end of the visible light spectrum, EMR becomes invisible to humans (infrared).

3.7 Optical properties

The importance of the optical properties of materials lies through the use in the identification of the components and materials in general, which is including the Absorbance, Luminosity, Photosensitivity, Reflectivity, Refractive index, Scattering, etc...

Absorption: In physics, absorption of electromagnetic radiation is the way in which the energy of a photon is taken up by matter, typically the electrons of an atom. Thus, the electromagnetic energy is transformed into internal energy of the absorber. The term absorption refers to the physical process of absorbing light, while absorbance does not always measure absorption: it measures attenuation (of transmitted radiant power). Absorption is one reason of attenuation.

Luminosity: luminosity is the total amount of energy emitted by a star, galaxy, or other astronomical object per unit time. It is relating to the brightness, which is the luminosity of an object in a given spectral region.

In SI, unit's luminosity is measured in joules per second or watts. Values for luminosity are often given in the terms of the luminosity of the Sun, which has a total power output of 3.846×10^{26} W.

Photosensitivity: Photosensitivity is the amount to which an object reacts upon receiving photons, especially visible light.

Reflectance: Reflectance of the surface of a material is its effectiveness in reflecting radiant energy. It is the fraction of incident electromagnetic power that is reflected at an interface. The reflectance spectrum of spectral reflectance curve is the plot of the reflectance as a function of wavelength.

Refractive index: In optics, the refractive index or index of refraction n of a material is a dimensionless number that describes how light propagates through that medium. It is defined as;

$$n = \frac{c}{v}$$

Where c is the speed of light in vacuum and v is the phase velocity of light in the medium. For example, the refractive index of water is 1.333; meaning that light travels 1.333 times faster in a vacuum than it does in water. When the light entering inside the material the refractive index is determined how much light bent or refracted.

Scattering: Scattering is a general physical process where some forms of radiation, such as light, sound, or moving particles, are forced to deviate from a straight trajectory by one or more paths due to localized non-uniformities in the medium through which they pass. In conventional use, this also includes deviation of reflected radiation from the angle predicted by the law of reflection. Reflections that undergo scattering are often called diffuse reflections and scattered reflections are called specular (mirror-like) reflections.

Scattering may also refer to particle-particle collisions between molecules, atoms, electrons, photons and other particles. Examples are cosmic rays scattering by the Earth's upper atmosphere; particle collisions inside particle accelerators; electron scattering by gas atoms in fluorescent lamps; and neutron scattering inside nuclear reactors.

Transmittance: Transmittance of the surface of a material is its effectiveness in transmitting radiant energy. **It is the fraction of incident electromagnetic power that is transmitted through a sample, in contrast to the transmission coefficient, which is the ratio of the transmitted to incident electric field.**

Internal transmittance refers to energy loss by absorption, whereas (total) transmittance is that due to absorption.

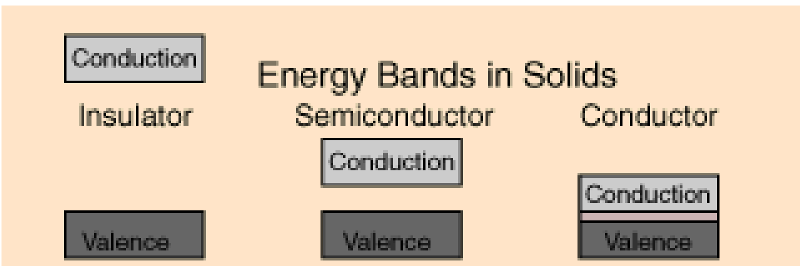
3.8 Energy Band

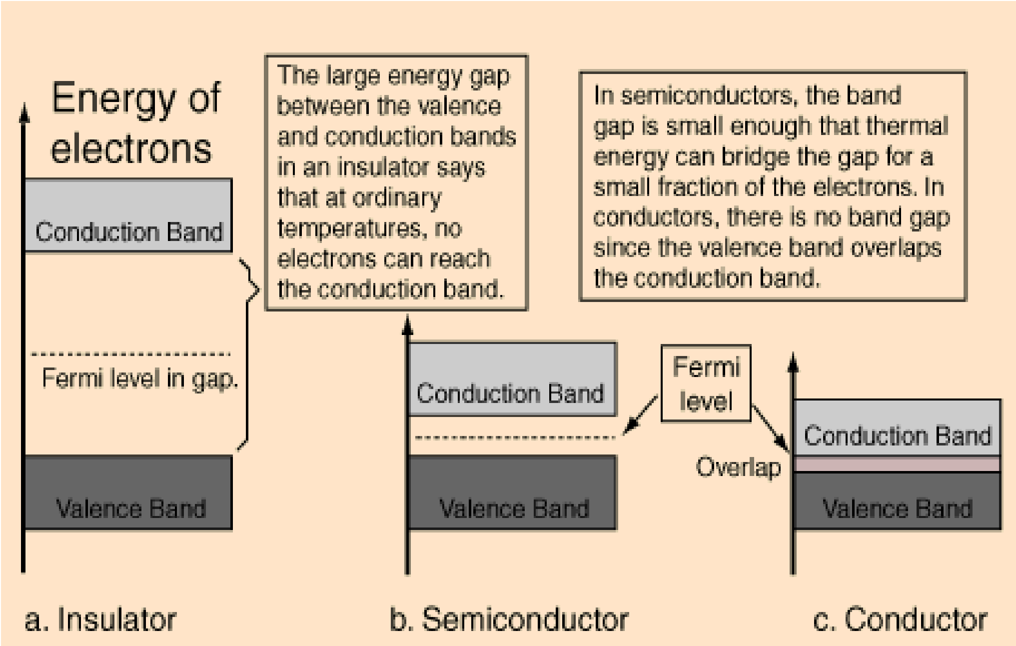
Energy bands consisting of a large number of closely spaced energy levels exist in crystalline materials. The bands can be thought of as the collection of the individual energy levels of electrons surrounding each atom. The wave functions of the individual electrons, however, overlap with those of electrons confined to neighbouring atoms.

The energy band model is crucial to any detailed treatment of semiconductor devices. It provides the framework needed to understand the concept of an energy band gap and that of conduction in an almost filled band as described by the empty states.

A useful way to visualize the difference between conductors, insulators and semiconductors is to plot the available energies for electrons in the materials. Instead of having discrete energies as in the case of free atoms, the available energy states form bands. Crucial to the conduction process is whether there are electrons in the conduction band. In insulators the electrons in the valence band are separated by a large gap from the conduction band, in conductors like metals the valence band overlaps the conduction band, and in semiconductors there is a small enough gap between the valence and conduction bands that thermal or other excitations can bridge the gap. With such a small gap, the presence of a small percentage of a doping material can increase conductivity dramatically.

An important parameter in the band theory is the Fermi level, the top of the available electron energy levels at low temperatures. The position of the Fermi level with the relation to the conduction band is a crucial factor in determining electrical properties.





Band gap, also called an energy gap or band gap, is an energy range in a solid where no electron states can exist. In above graphs of the electronic band structure of solids, the band gap generally refers to the energy difference (in electron volts) between the top of the valence band and the bottom of the conduction band in insulators and semiconductors.

Exercise:

Estimate the electric field strength required to produce conduction in diamond, an excellent insulator at room temperature. Assume a mean free path of ($5*10^{-8}$ m) and an energy gap of 7eV in diamond.

CHAPTER 4

Positive Ray and Particle Properties of Waves

4.1 Positive Rays and their Properties

Cathode rays consist of negatively charged particles called electrons. These electrons move away from cathode with very high speeds. These fast moving electrons collide with the molecules of the gas in the tube, split the molecule into atoms, and remove one or more electrons from the atoms. Thus, the atoms get converted into the positive ions due to loss of electrons. These positive ions get attracted by the negative electrode, and pass through the holes in the electrode plate to produce a glow only the glass wall of the discharge tube. A stream of these positively charged particles is called a positive ray (or anode ray).

Properties of positive rays

1. Positive rays consist of positively charged particles.
2. The nature of these rays depends on the gas used in the discharge tube.
3. These rays travel in straight lines.
4. These rays are deflected by an electrical field, and bend towards the negative plate. Thus, the deflection of the positive rays is in a direction opposite to that shown by the cathode rays.
5. These rays are also deflected by the magnetic fields in the direction opposite to that of the cathode rays.
6. These rays can produce mechanical as well as chemical effects.
7. The ratio of charge (e) to mass (m), i.e.,(e/m) for the particles in the positive rays is not the same for all gases.
8. The ratio e / m for the positive rays is very low as compared to the e / m value for cathode rays.

Then an anode ray (also positive ray or canal ray) is a beam of positive ions that is created by certain types of gas discharge tubes.

4.2 Electric Field and Magnetic Field

An electromagnetic field (also EM field) is a physical field produced by electrically charged objects. It affects the behaviour of charged objects in the vicinity of the field. The electromagnetic field extends indefinitely throughout space and describes the electromagnetic interaction. It is one of the four fundamental forces of nature (the others are gravitation, weak interaction and strong interaction).

The field can be viewed as the combination of an electric field and a magnetic field. The electric field is produced by stationary charges, and the magnetic field by moving charges (currents); these two are often described as the sources of the field.

4.3 Blackbody Radiation

A blackbody refers to an opaque object that emits thermal radiation. A perfect blackbody is one that absorbs all incoming light and does not reflect any. At room temperature, such an object would appear to be perfectly black (hence the term blackbody). However, if heated to a high temperature, a blackbody will begin to glow with thermal radiation.

Experimentally the total power per unit area emitted at all frequencies by a hot solid, e_{total} was proportional to the fourth power of its absolute temperature. Therefore, Stefan's law can write.

$$e_{\text{total}} = \int_0^\infty e_f \, df = \sigma T^4$$

Where e_{total} is the power per unit area emitted at the surface of the blackbody at all frequencies, e_f is the power per unit area per unit frequency emitted by the blackbody, T is the absolute temperature of the body and $\sigma = 5.67 * 10^{-8}$ W m^{-2} K^{-4} is the Stefan–Boltzmann constant given by that is not an ideal radiator will obey the same general law but with a coefficient, a, less than 1.

$$e_{\text{total}} = a\sigma T^4$$

Exercise: Estimate the surface temperature of the Sun from the following information. The Sun's radius is given by Rs = 7.0 * 10^8 m. The average Earth Sun distance is R =1.5 * 10^{11} m. The power per unit area (at all frequencies) from the Sun is measured at the Earth to be 1400 W/m2. Assume that the Sun is a blackbody.

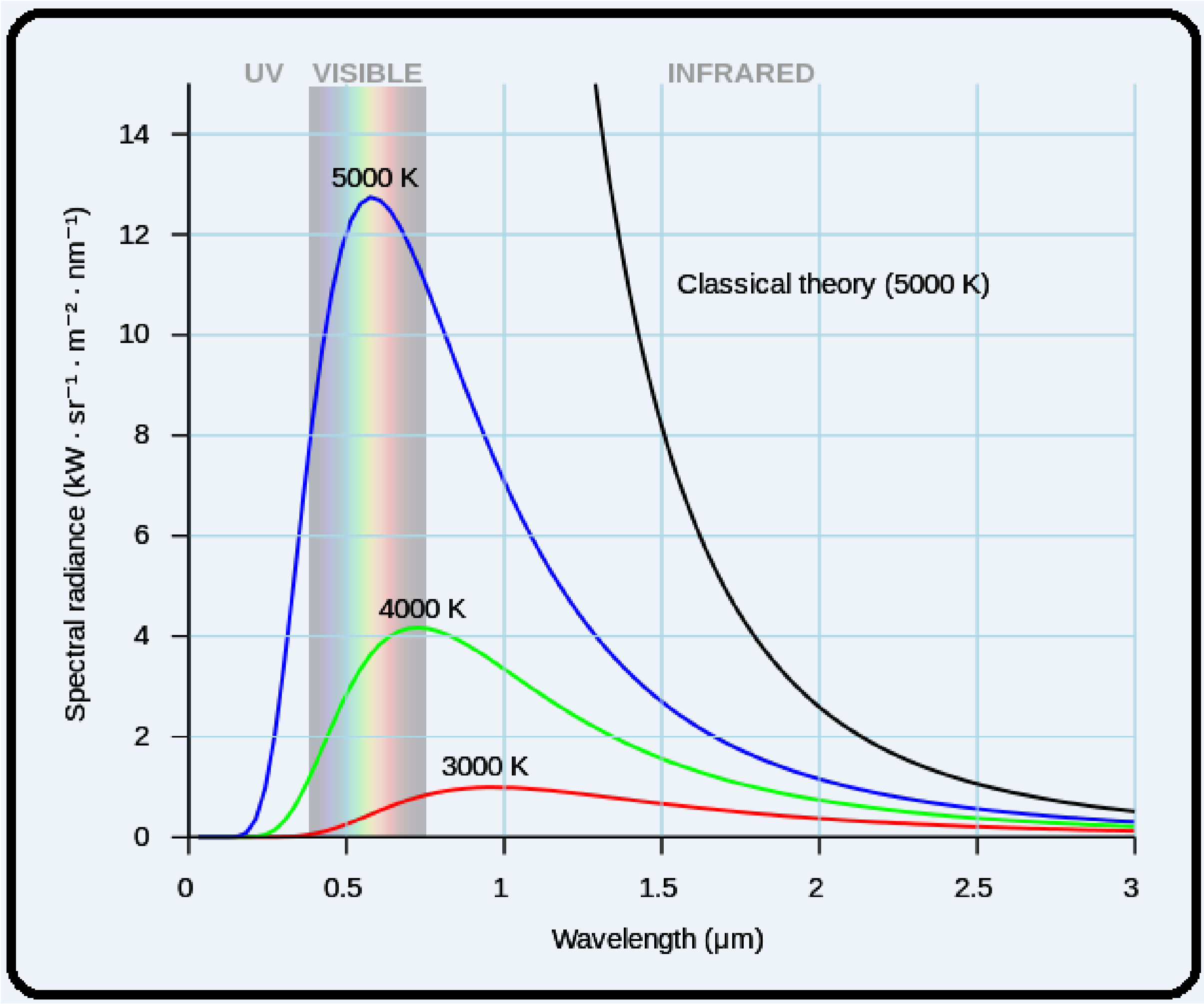
As known that glowing solids emit continuous spectra rather than the bands or lines emitted by heated gases. (See Figure below) In 1859, Gustav Kirchhoff proved a theorem as important as his circuit loop theorem when he showed by arguments based on **thermodynamics that for anybody in thermal equilibrium with radiation the emitted power is proportional to the power absorbed.** More specifically,

$$e_f = J(f, T)A_f$$

where e_f is the power emitted per unit area per unit frequency by a particular heated object, **A_f is the absorption power (fraction of the incident power absorbed per unit area per unit frequency by the heated object)**, and J(f, T) is a universal function (the same for all bodies) that depends only on f, the light frequency, and T, the absolute temperature of the body. **A blackbody is defined as an object that absorbs all the electromagnetic radiation falling on it and consequently appears black.** It has A_f =1 for all frequencies and so Kirchhoff's theorem for a blackbody becomes

$$e_f = J(f, T)$$

A perfect black body: the body in thermodynamic equilibrium absorbs all light that strikes it and radiates energy according to the radioactive emissive power for temperature T.



Emission from a glowing solid, note that the amount of radiation emitted (the area under the curve) increases rapidly with increasing temperature.

As can be seen in Figure above, the wavelength marking the maximum power emission of a blackbody, max, shifts toward shorter wavelengths as the blackbody gets hotter. This simple effect of $\lambda_{max} \propto T^{-1}$ was not definitely established, however, until about 20 years after Kirchhoff's seminal paper had started the search to find the form of the universal function J(f, T). In 1893, Wilhelm Wien proposed a general form for the blackbody distribution law J(f, T) that gave the correct experimental behaviour of max with temperature. This law is called Wien's displacement law and may be written Kirchhoff's law of thermal radiation refers to wavelength-specific radioactive emission and absorption by a material body in thermodynamic equilibrium, including radioactive exchange equilibrium.

$$\lambda_{max} T = 2.898 \times 10^{-3}\ \mathrm{m \cdot K}$$

Where λ_{max} max is the wavelength in meters corresponding to the blackbodies maximum intensity and T is the absolute temperature of the surface of the object emitting the radiation. The ratio of its emissive power to its dimensionless coefficient of absorption is equal to a universal function only of radioactive wavelength and temperature. That universal function describes the perfect blackbody emissive power.

Assuming that the peak sensitivity of the human eye (which occurs at about 500 nm—blue-green light) coincides with max for the Sun (a blackbody), we can check the consistency of Wien's displacement law with Stefan's law by recalculating the Sun's surface temperature:

$$T = \frac{2.898 \times 10^{-3}\ \mathrm{m \cdot K}}{500 \times 10^{-9}\ \mathrm{m}} = 5800\ \mathrm{K}$$

Kirchhoff's law states that: For a body of any arbitrary material emitting and absorbing thermal electromagnetic radiation at every wavelength in thermodynamic equilibrium.

A black body in thermal equilibrium has two notable properties:

- ❖ It is an ideal emitter: at every frequency, it emits as much energy as – or more energy than – any other body at the same temperature.
- ❖ It is a diffuse emitter: the energy is radiated isotropically, independent of direction.

Enter Planck

Planck's law: is describes the electromagnetic radiation emitted by a black body in thermal equilibrium at a definite temperature T.

The Planck constant (denoted h, also called Planck's constant) is a physical constant that is the quantum of action, central in quantum mechanics.

First recognized in 1900 by Max Planck, it was originally the proportionality constant between the minimal increment of energy, E, of a hypothetical electrically charged oscillator in a cavity that contained black body radiation, and the frequency, f, of its associated electromagnetic wave.

The light quantum behaved in some respects as an electrically neutral particle, as opposed to an electromagnetic wave. It was eventually called the photon.

The Planck–Einstein relation connects the particulate photon energy E with its associated wave frequency f:

$$E = hf$$

This energy is extremely small in terms of ordinarily perceived everyday objects.

Since the frequency f, wavelength λ, and speed of light C are related by f=c/ λ, the relation can also be expressed as;

$$E = \frac{hc}{\lambda}.$$

This leads to another relationship involving the Planck constant. With p denoting the linear momentum of a particle (not only a photon, but also other fine particles as well),

$$\lambda = \frac{h}{p}.$$

The spectral radiance of a body, B_v: describes the amount of energy it gives off as radiation of different frequencies. It is measured in terms of the power emitted per unit area of the body, per unit solid angle that the radiation is measured over, per unit frequency. Planck showed that the spectral radiance of a body at absolute temperature T is given by;

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{k_B T}} - 1}$$

Where k_B the Boltzmann constant, h the Planck constant and C the speed of light in the medium, whether material or vacuum. The spectral radiance can also be measured per unit wavelength instead of per unit frequency. In this case, it is given by

$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}.$$

The law may also be expressed in other terms, such as the number of photons emitted at a certain wavelength, or the energy density in a volume of radiation. The SI units of B_ν are $W \cdot sr^{-1} \cdot m^{-2} \cdot Hz^{-1}$, while those of B_λ are $W \cdot sr^{-1} \cdot m^{-3}$.

Every physical body spontaneously and continuously emits electromagnetic radiation. Near thermodynamic equilibrium, the emitted radiation is nearly described by Planck's law. Because of its dependence on temperature, Planck radiation is said to be thermal radiation. The higher the temperature of a body the more radiation it emits at every wavelength. Planck radiation has a maximum intensity at a specific wavelength that depends on the temperature. For example, at room temperature (~300 K), a body emits thermal radiation that is mostly infrared and invisible. At higher temperatures the amount of infrared radiation increases and can be felt as heat, and the body glows visibly red. At even higher temperatures, a body is dazzlingly bright yellow or blue-white and emits significant amounts of short wavelength radiation, including ultraviolet and even x-rays. The surface of the sun (~6000 K) emits large amounts of both infrared and ultraviolet radiation; its emission is peaked in the visible spectrum.

Planck radiation is the greatest amount of radiation that anybody at thermal equilibrium can emit from its surface, whatever its chemical composition or surface structure. The passage of radiation across an interface between media can be characterized by the emissivity of the interface (the ratio of the actual radiance to the theoretical Planck radiance), usually denoted by the symbol ε. It is in general dependent on chemical composition and physical structure, on temperature, on the wavelength, on the angle of passage, and on the polarization. The emissivity of a natural interface is always between ε = 0 and 1. In the limit of low frequencies (i.e. long wavelengths), Planck's law tends to the Rayleigh–Jeans law, while in the limit of high frequencies (i.e. small wavelengths) it tends to the Wien approximation.

Planck's law can be encountered in several forms depending on the conventions and preferences of different scientific fields. The various forms of the law for spectral radiance are summarized in the table below. Forms on the left are most often encountered in experimental fields, while those on the right are most often encountered in theoretical fields.

Planck's law expressed in terms of different spectral variables

with h		with $\hbar$	
variable	distribution	variable	distribution
Frequency ν	$B_\nu(\nu,T)=\frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/(k_B T)}-1}$	Angular frequency ω	$B_\omega(\omega,T)=\frac{\hbar\omega^3}{4\pi^3c^2}\frac{1}{e^{\hbar\omega/(k_B T)}-1}$
Wavelength λ	$B_\lambda(\lambda,T)=\frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/(\lambda k_B T)}-1}$	Angular wavelength y	$B_y(y,T)=\frac{\hbar c^2}{4\pi^3 y^5}\frac{1}{e^{\hbar c/(y k_B T)}-1}$
Wavenumber $\tilde{\nu}$	$B_{\tilde{\nu}}(\tilde{\nu},T)=2hc^2\tilde{\nu}^3\frac{1}{e^{hc\tilde{\nu}/(k_B T)}-1}$	Angular wavenumber k	$B_k(k,T)=\frac{\hbar c^2 k^3}{4\pi^3}\frac{1}{e^{\hbar ck/(k_B T)}-1}$

These distributions represent the spectral radiance of blackbodies the power emitted from the emitting surface, per unit projected area of emitting surface, per unit solid angle, per spectral unit (frequency, wavelength, wave number or their angular equivalents).

The Boltzmann constant (k_B or k), is a physical constant relating energy at the individual particle level with temperature. It is the gas constant R divided by the Avogadro constant $\mathbf{N_A}$:

$$k = \frac{R}{N_A}.$$

The Boltzmann constant has the dimension energy divided by temperature, the same as entropy. The accepted value in SI units is $1.38*10^{-23}$ J/K.

In chemistry and physics, the Avogadro constant is the number of constituent particles, usually atoms or molecules, that are contained the amount of substance given by one mole. Thus, the proportionality factor relates the molar mass of a compound to the mass of a sample. Avogadro's constant, often designated with the symbol N_A, has the value 6.022×10^{23} mol^{-1} in the International System of Units (SI).

Solid angle (symbol: Ω): is the two-dimensional angle in three-dimensional space that an object subtends at a point. It is a measure of how large the object appears to an observer looking from that point. In the International System of Units (SI), a solid angle is expressed in a dimensionless unit called a steradian (symbol: sr).

The steradian (symbol: sr): or square radian is the SI unit of solid angle. It is used in three-dimensional geometry, and is analogous to the radian that quantifies planar angles.

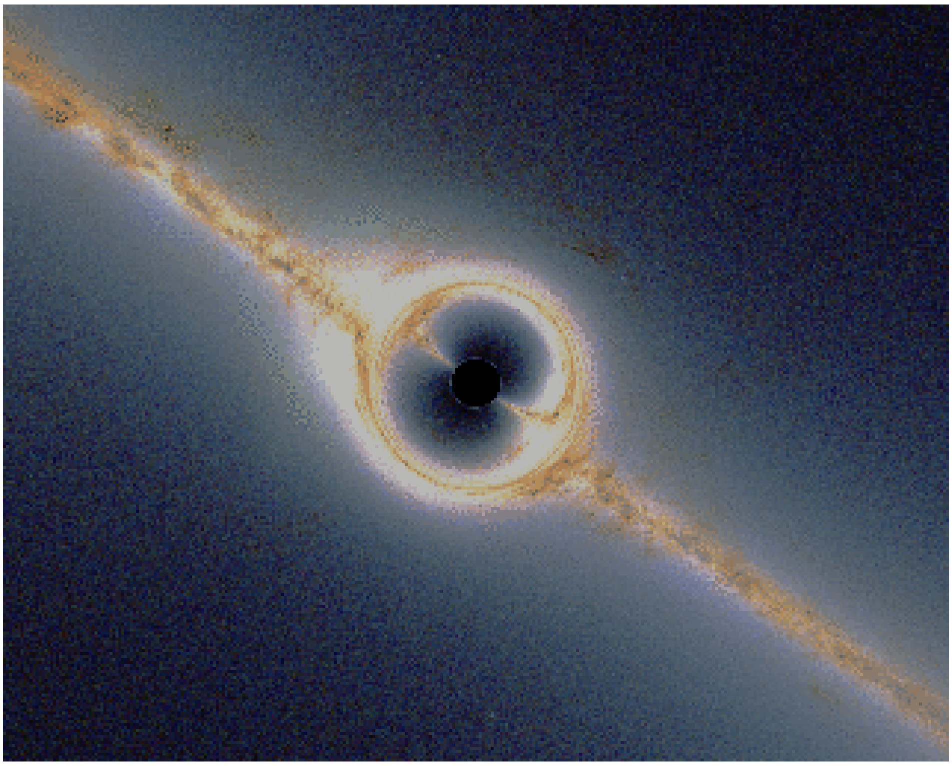
Black holes

A black hole is a region of space-time from which nothing escapes. Around a black hole there is a mathematically defined surface called an event horizon that marks the point of no return. It’s called "black" because it absorbs all the light that hits the horizon, reflecting nothing, making it almost an ideal black body (radiation with a wavelength equal to or larger than the radius of the hole may not be absorbed, so black holes are not perfect black bodies).

Physicists believe that to an outside observer, black holes have a non-zero temperature and emit radiation with a nearly perfect blackbody spectrum, ultimately evaporating. The mechanism for this emission is related to vacuum fluctuations in which a virtual pair of particles is separated by the gravity of the hole, one member being sucked into the hole, and the other being emitted. Planck's law with a temperature T describes the energy distribution of emission:

$$T = \frac{\hbar c^3}{8\pi G k_B M}$$

Where c is the speed of light, $\hbar$ is the reduced Planck constant, k_B is Boltzmann's constant, G is the gravitational constant and M is the mass of the black hole. These predictions have not yet been tested either observational or experimental.



4.4 The Quantum of Energy

After several experiments that plank did it, he was found very important formula, which describing and measuring the **spectral energy density**, he was found it proportional to T for long wavelengths or low frequency.

$$u(f, T) = \frac{8\pi hf^3}{c^3}\left(\frac{1}{e^{hf/k_B T} - 1}\right)$$

Planck's original theoretical justification of above equation is rather abstract because it involves arguments based on entropy, statistical mechanics, and several theorems proved earlier by Planck concerning matter and radiation in equilibrium.

Planck was convinced that blackbody radiation was produced by vibrating sub-microscopic electric charges, which he called resonators. He assumed that the walls of a glowing cavity were composed of literally billions of these resonators (whose exact nature was unknown at the time), all vibrating at different frequencies. Hence, according to Maxwell, each oscillator should emit radiation with a frequency corresponding to its vibration frequency.

Also according to classical Maxwellian theory, an oscillator of frequency f could have any value of energy and could change its amplitude continuously as it radiates any fraction of its energy.

Planck had to assume that the total energy of a resonator with mechanical Frequency f could only be an integral multiple of hf or,

$$E_{\text{resonator}} = nhf \qquad n = 1, 2, 3, \ldots$$

In addition, plank concluded that emission of radiation of frequency *f* occurred when a resonator dropped to the next lowest energy state. Thus, the resonator can change its energy only by the difference ΔE according to

$$\Delta E = hf$$

That is, it cannot lose just any amount of its total energy, but only a finite amount, hf, the so-called quantum of energy. Figure below shows the quantized energy levels and allowed transitions proposed by Planck.

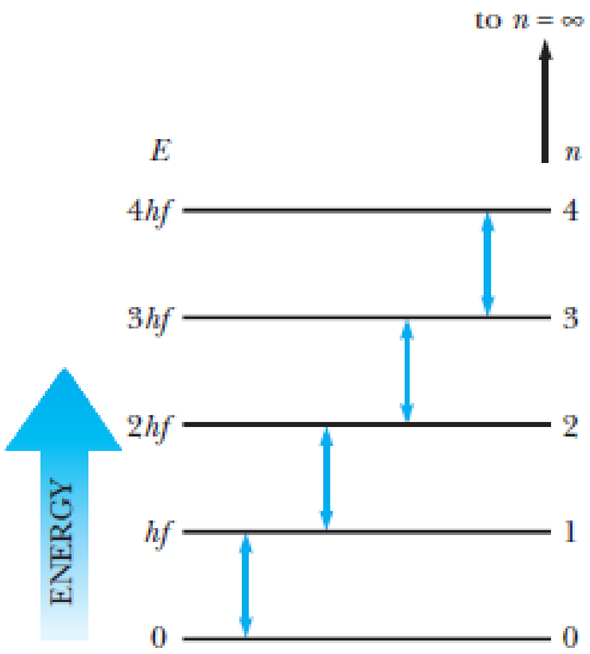


Figure. Allowed energy levels according to Planck's original hypothesis for an oscillator with frequency f. Allowed transitions are indicated by the double-headed arrows.

Exercise: How convenient that the Sun's emission peak is at the same wavelength as our eyes' sensitivity peak! Can you account for this?

Example: Consider the implications of Planck's conjecture that all oscillating systems of natural frequency f have discrete allowed energies E=nhf and that the smallest change in energy of the system is given by E =hf.

(a) First compare an atomic oscillator sending out 540-nm light (green) to one sending out 700-nm light (red) by calculating the minimum energy change of each. For the green quantum,

$$\Delta E_{\text{green}} = hf = \frac{hc}{\lambda}$$

$$= \frac{(6.63 \times 10^{-34}\,\text{J}\cdot\text{s})(3.00 \times 10^{8}\,\text{m/s})}{540 \times 10^{-9}\,\text{m}}$$

$$= 3.68 \times 10^{-9}\,\text{J}$$

Actually, the joule is much too large a unit of energy for describing atomic processes; so that the minimum energy change of an atomic oscillator sending out green light is;

$$\Delta E_{\text{green}} = \frac{3.68 \times 10^{-19}\,\text{J}}{1.602 \times 10^{-19}\,\text{J/eV}} = 2.30\ \text{eV}$$

For the red quantum the minimum energy change is

$$\Delta E_{\text{red}} = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\,\text{J}\cdot\text{s})(3.00 \times 10^{8}\,\text{m/s})}{700 \times 10^{-9}\,\text{m}}$$
$$= 2.84 \times 10^{-19}\,\text{J} = 1.77\ \text{eV}$$

(b) Now consider a pendulum undergoing small oscillations with length L=1 m. According to classical theory, if air friction is present, the amplitude of swing and consequently the energy decrease continuously with time. Actually, all systems vibrating with frequency *f* are quantized and lose energy in discrete packets or quanta, *hf*.

An energy change of one quantum corresponds to

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}} = 0.50\ \text{Hz}$$

Thus,

$$\Delta E = (6.63 \times 10^{-34}\,\text{J}\cdot\text{s})(0.50\ \text{s}^{-1})$$
$$= 3.3 \times 10^{-34}\,\text{J}$$
$$= 2.1 \times 10^{-15}\ \text{eV}$$

Exercise: Calculate the quantum number, n, for this pendulum with E = 1.5x 10^{-2} J. **Answer** 4.6x10^{31}

Exercise: An object of mass m on a spring of stiffness k oscillates with amplitude A about its equilibrium position. Suppose that m=300 g, k=10 N/m, and A=10 cm. (a) Find the total energy. (b) Find the mechanical frequency of vibration of the mass. (c) Calculate the change in amplitude when the system loses one quantum of energy.

CHAPTER 5

Structure of the Atom

5.1 The Atomic Nature of Matter

The discovery and proof of the graininess of the world seem especially fascinating for two reasons. First, because of the size of individual atoms, measurements of atomic properties are usually indirect and necessarily involve clever manipulations of large-scale measurements to infer properties of microscopic particles. Second, the historical evolution of ideas about atomicity shows clearly the real way in which science progresses. This progression is often nonlinear and involves an interdependence of physics, chemistry, and mathematics, and the convergence of many different lines of investigation. Democritus and Leucippus, who speculated that the unchanging substratum of the world was atoms in motion; the debonair French chemist Lavoisier and his wife, who established the conservation of matter in many careful chemical experiments; Dalton, who perceived the atomicity of nature in the law of multiple proportions of compounds; Avogadro, who in a most obscure and little-appreciated paper, postulated that all pure gases at the same temperature and pressure have the same number of molecules per unit volume; and Maxwell, who showed with his molecular-kinetic theory of gases how macroscopic quantities, such as pressure and temperature, could be derived from averages over distributions of molecular properties. The list could run on and who carried on very important theoretical and experimental work concerning Brownian motion, the zigzag movement of small-suspended particles caused by molecular impacts.

5.2 The Composition of Atoms

The question, “If matter is primarily composed of atoms, what are atoms composed of? Again, we can point to some primary discoveries that showed that atoms are composed of light, negatively charged particles orbiting a heavy, positively charged nucleus. These were;

- The discovery of the law of electrolysis in 1833 by Michael Faraday, through careful experimental work on electrolysis, Faraday showed that the mass of an element liberated at an electrode is directly proportional to the charge transferred and to the atomic weight of the liberated material but is inversely proportional to the valence of the freed material.
- The identification of cathode rays as electrons and the measurement of the charge-tomass ratio (e/me) of these particles by Joseph John (J. J.) Thomson in 1897. Thomson measured the properties of negative particles emitted from different metals and found that the value of e/me was always the same. He thus concluded that the electron is a constituent of all matter!
- The precise measurement of the electronic charge (e) by Robert Millikan in 1909. By combining his result for (e) with Thomson’s e/me value, Millikan showed unequivocally that particles about 1000 times less massive than the hydrogen atom exist.
- The establishment of the nuclear model of the atom by Ernest Rutherford and co-workers Hans Geiger and Ernest Marsden in 1913. By scattering fast-moving particles (charged nuclei of helium atoms emitted spontaneously in radioactive decay processes) from metal foil targets, Rutherford established that atoms consist of a compact positively charged nucleus (diameter=10^{-14} m) surrounded by a swarm of orbiting electrons (electron cloud diameter=10^{-10} m).

Let us describe these developments in more detail. We start with a brief example of Faraday's experiments, in particular the electrolysis of molten common salt (NaCl). Faraday found that if 96,500 C of charge (1 faraday) is passed through such a molten solution, 23.0 g of Na will deposit on the cathode and 35.5 g of chlorine gas will bubble off the anode. In this case, exactly 1-gram atomic weight or mole of each element is released because both are mono-valent. For divalent and trivalent elements, exactly and 1 of a mole, respectively, would be released. As expected, doubling the quantity of charge passed doubles the mass of the neutral element liberated. Faraday's results may be given in equation form as;

$$m = \frac{(q)\,(\text{molar mass})}{(96{,}500\ \text{C})\,(\text{valence})}$$

Where m is the mass of the liberated substance in grams, q is the total charge Passed in coulombs, the molar mass is in grams, and the valence is dimensionless.

Example; the Electrolysis of BaCl2

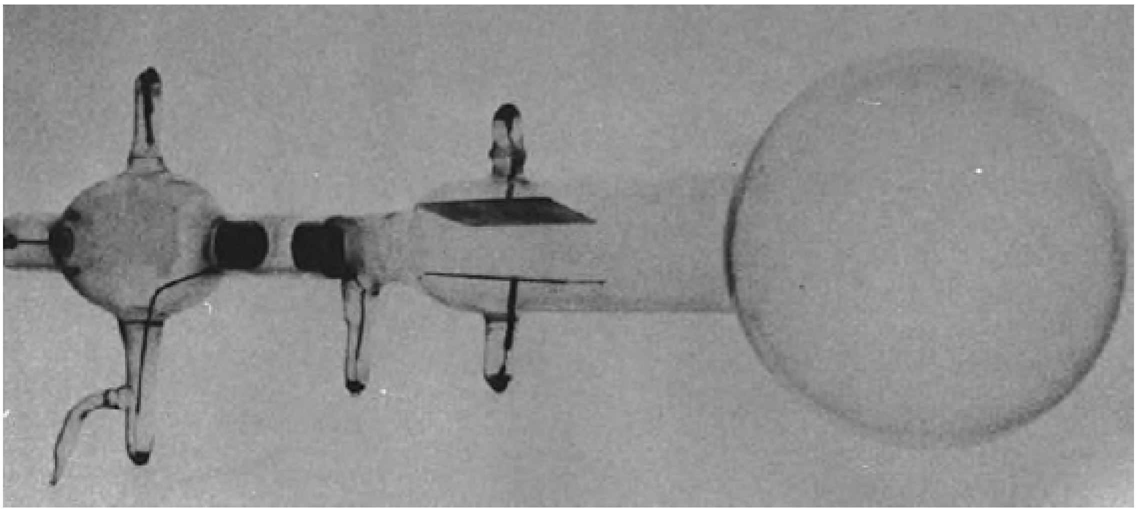
How many grams of barium and chlorine will you get if you pass a current of 10.0 A through molten $BaCl_2$ for 1 h? Barium has a molar weight of 137 g and a valence of 2. Chlorine has a molar weight of 35.5 g and a valence of 1.

Solution, using equation above and q? It, where I is the current and t is the time, we have,

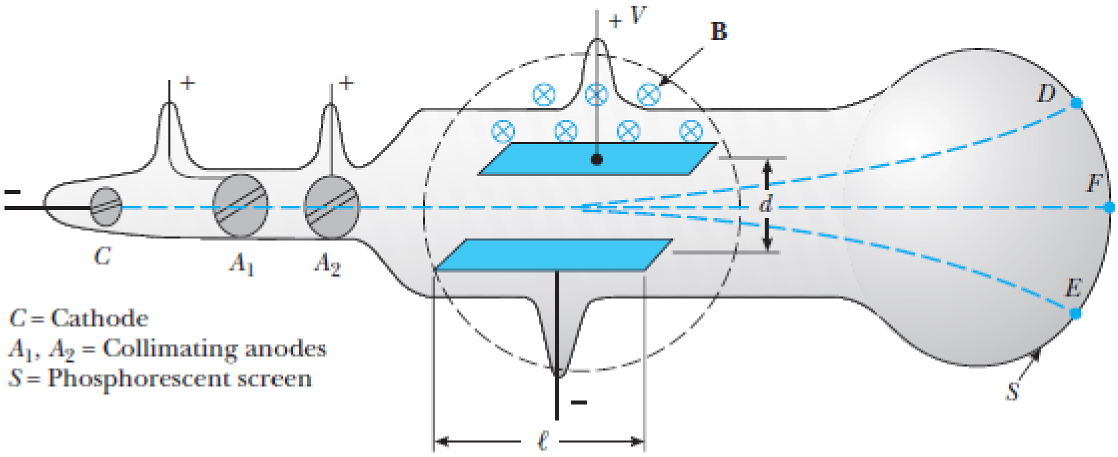
$$\begin{aligned} m_{\text{Ba}} &= \frac{(q)\,(\text{molar mass})}{(96{,}500\ \text{C})\,(\text{valence})} \\ &= \frac{(10.0\ \text{C/s})\,(3600\ \text{s})\,(137\ \text{g})}{(96{,}500\ \text{C})\,(2)} = 25.6\ \text{g} \\ m_{\text{Cl}} &= \frac{(10.0\ \text{C/s})\,(3600\ \text{s})\,(35.5\ \text{g})}{(96{,}500\ \text{C})\,(1)} = 13.2\ \text{g} \end{aligned}$$

Figure below shows the original vacuum tube used by Thomson in his e/me experiments. The various parts of the Thomson apparatus for easy reference.

Electrons are accelerated from the cathode to the anode, collimated by slits in the anodes, and then allowed to drift into a region of crossed (perpendicular) electric and magnetic fields. The simultaneously applied E and B fields are first adjusted to produce an un-deflected beam. If the B field is then turned off, the E field alone produces a measurable beam deflection on the phosphorescent screen. From the size of the deflection and the measured values of E and B, the charge-to-mass ratio, e/m_e, may be determined. The truly ingenious feature of this experiment is the manner in which Thomson measured v_x, the horizontal velocity component of the beam. He did this by balancing the magnetic and electric forces. In effect, he created a velocity selector, which could select out of the beam those particles having a velocity within a narrow range of values. This device was extensively used in the first quarter.

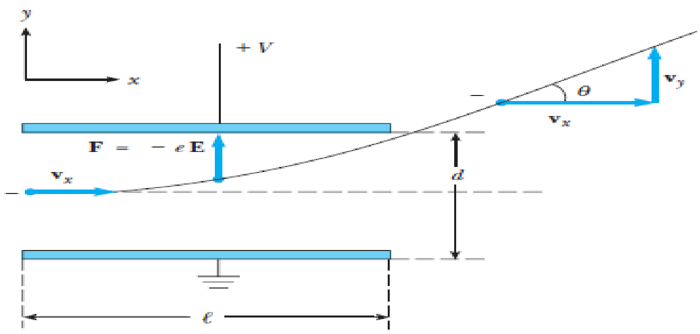


The original e/me tube used by J. J. Thomson.



A diagram of Thomson's e/me tube (patterned after J. J. Thomson, Philosophical Magazine. Electrons subjected to an electric field alone land at D, while those subjected to a magnetic field alone land at E. When both electric and magnetic fields are present and properly adjusted, the electrons experience no net deflection and land at F.

Of the 20th century in charge-to-mass measurements (q/m) on many particles and in early mass spectrometers. To gain a clearer picture of the Thomson experiment, let us analyze the electron's motion in his apparatus. Figure below shows the trajectory of a beam of negative particles entering the E and B field regions with horizontal velocity v_x . Consider first only an E field between the plates. For this case, v_x remains constant throughout the motion because there is no force acting in the x direction. The y component of velocity, v_y , is constant everywhere except between the plates, where the electron experiences a constant upward acceleration due to the electric force and follows a parabolic path. To solve for the deflection angle, θ, we must solve for v_x and v_y. Because v_y initially is zero,



Deflection of negative particles by an electric field

The electron leaves the plates with a y component of velocity given by,

$$v_y = a_y t$$

Because $a_y = F/m_e = Ee/m_e = Ve/m_e d$, and $t = \ell/v_x$, where d and ℓ are the dimensions of the region between the plates and V is the applied potential, we obtain

$$v_y = \frac{V\ell e}{m_e v_x d}$$

$$\tan\theta = v_y/v_x,$$

$$\tan\theta = \frac{V\ell}{v_x^2 d}\left(\frac{e}{m_e}\right)$$

Assuming small deflections, $\tan\theta \approx \theta$, so we have

$$\theta \approx \frac{V\ell}{v_x^2 d}\left(\frac{e}{m_e}\right)$$

Note that, the beam deflection, V, the voltage applied to the horizontal deflecting plates, and d is the spacing and length, respectively, of the horizontal deflecting plates can all be measured. Hence, one only needs to measure v_x to determine e/m_e. Thomson determined v_x by applying a B field and adjusting its magnitude to just balance the deflection of the still present E field. Equating the magnitudes of the electric and magnetic forces gives

$$qE = qv_x B$$

or

$$v_x = \frac{E}{B} = \frac{V}{Bd}$$

Substituting Equations immediately yields a formula for e/m_e entirely in terms of measurable quantities:

$$\frac{e}{m_e} = \frac{V\theta}{B^2\ell d}$$

The currently accepted value of e/m_e is $1.758803*10^{11}$ C/kg. Although Thomson's original value was only about $1.0*10^{11}$ C/kg, prior experiments on the electrolysis of hydrogen ions had given q/m values for hydrogen of about 10^8 C/kg. It was clear that Thomson had discovered a particle with a mass about 1000 times smaller than the smallest atom! In his observations, Thomson noted that the e/m_e ratio was independent of the discharge gas and the cathode metal. Furthermore, the particles emitted when electrical discharges were passed through different gases were found to be the same as those observed in the photoelectric effect. Based on these observations, Thomson concluded that these particles must be a universal constituent of all matter. Humanity had achieved its first glimpse into the subatomic world.

Example; *Deflection of an Electron Beam by E and B Fields*

Using the accepted e/m_e value, calculate the magnetic field required to produce a deflection of 0.20 rad in Thomson's experiment, assuming the values V=200 V, *l*=5.0 cm, and d=1.5 cm (the approximate values used by Thomson). Compare this value of B to the Earth's magnetic field.

Solution,

Because; $e/m_e = V\theta/B^2 l d$, solving for B gives

$$B = \sqrt{\frac{V\theta}{\ell d(e/m_e)}}$$

so

$$B = \left[\frac{(200\ \text{V})(0.20\ \text{rad})}{(0.050\ \text{m})(0.015\ \text{m})(1.76 \times 10^{11}\ \text{C/kg})}\right]^{1/2}$$
$$= [3.03 \times 10^{-7}\ \text{V}\cdot\text{kg/m}^2\ \text{C}]^{1/2}$$
$$= [3.03 \times 10^{-7}\ \text{N}^2/(\text{m/s})^2\ \text{C}^2]^{1/2}$$
$$= 5.5 \times 10^{-4}\ \text{N}/(\text{m/s})\cdot\text{C} = 5.5 \times 10^{-4}\ \text{T}$$

As the Earth's magnetic field has a magnitude of about 0.5×10^{-4} T, we require a field 11 times as strong as the Earth's field.

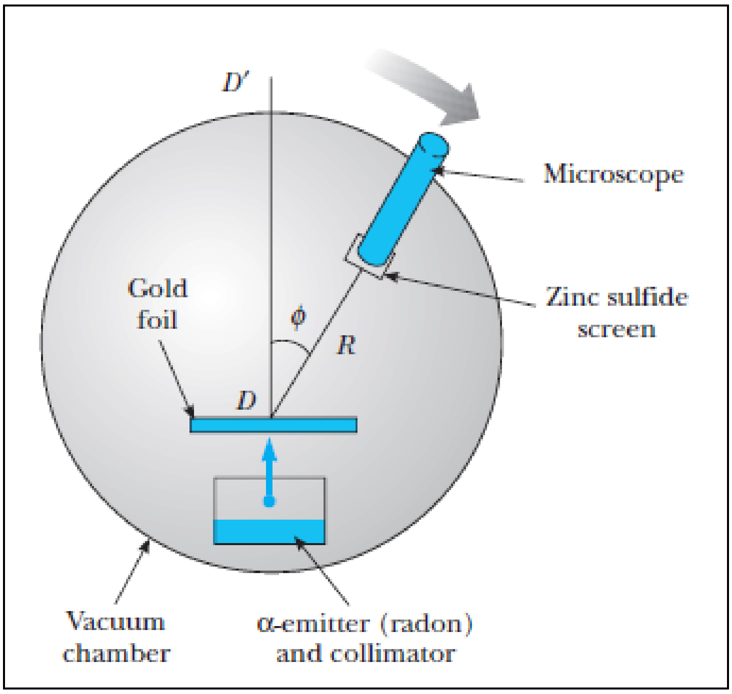
Exercise; Find the horizontal speed v_x for this case.

Rutherford Scattering Formula (atom model)

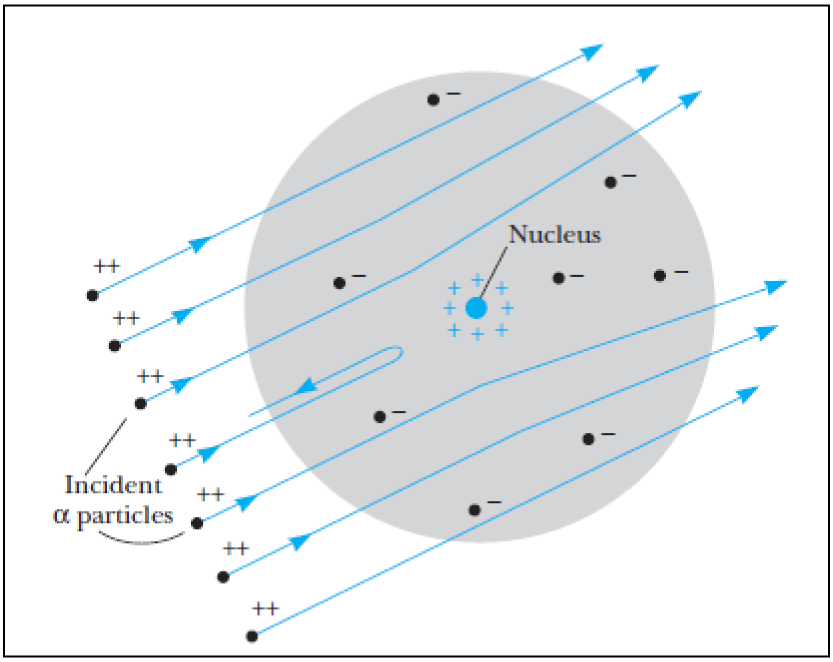
The atom as a homogeneous sphere of uniformly distributed mass and positive charge in which were embedded, like raisins in a plum pudding, negatively charged electrons, which just balanced the positive charge to produce electrically neutral atoms. Although such models possessed electrical stability against collapse or explosion of the atom, they failed to explain the rich line spectra of even the simplest atom, hydrogen.

The key to understanding the mysterious line spectra and the correct model of the atom were both furnished by Ernest Rutherford and his students Hans Geiger (1882–1945, German physicist) and Ernest Marsden (1899–1970, British physicist) through a series of experiments conducted from 1909 to 1914. Noticing that a beam of collimated α-particles broadened on passing through a metal foil yet easily penetrated the thin film of metal, they embarked on experiments to probe the distribution of mass within the atom by observing in detail the scattering of α-particles from foils. These experiments ultimately led Rutherford to the discovery that most of the atomic mass and all of the positive charge lie in a minute central nucleus of the atom. The accidental chain of events and the clever capitalization on the accidental discoveries leading up to Rutherford's monumental nuclear theory of the atom are nowhere better described than in Rutherford's own essay summarizing the development of the theory of atomic structure:

The essential experimental features of Rutherford's apparatus are shown in Figure below. A finely collimated beam of α-particles emitted with speeds of about $2 * 10^7$ m/s struck a thin gold foil several thousand atomic layers thick. Most of α's passed straight through the foil along the line $DD^/$ (again showing the porosity of the atom), but some were scattered at an angle φ. The number of scattered α's at each angle per unit detector area and per unit time was measured by counting the scintillations produced by scattered on the ZnS screen.



A schematic view of Rutherford's α scattering apparatus



Scattering of α-particles by a dense, positively charged nucleus

These scintillations were counted with the aid of the microscope. The distance from the point where α-particles strike the foil to the zinc sulphide screen is denoted R in Figure above.

Rutherford's basic insight was that because the mass and kinetic energy of α's are large, even a nearly head-on collision with a particle with the mass of a hydrogen atom would deflect the α-particle only slightly and knock the hydrogen atom straight ahead. Multiple scattering of the α-particles in the foil accounted for the small broadening (about 1^o) originally observed by Rutherford, but it could not account for the occasional large-scale deflections.

On the other hand, if all of the positive charge in an atom is assumed to be concentrated at a single central point and not spread out throughout the atom, the electric repulsion experienced by an incident α-particle in a head on collision becomes much greater. Because the charge and mass of the gold atom are concentrated at the nucleus, large deflections of the α-particle could be experienced in a single collision with the massive nucleus. This situation is shown in Figure above.

Example: (a) An α-particle of mass m_α and speed v_α strikes a stationary proton with mass m_p. If the collision is elastic and head-on, show that the speed of the proton after the collision, v_p, and the speed of the α-particle after the collision, $v_\alpha^{/}$ are given by

$$v_p = \left(\frac{2m_\alpha}{m_\alpha + m_p}\right) v_\alpha$$

And

$$v'_\alpha = \left(\frac{m_\alpha - m_p}{m_\alpha + m_p}\right) v_\alpha$$

(b) Calculate the percent change in velocity for an α-particle colliding with a proton.

Solution (a) Because the collision is elastic, the total kinetic energy is conserved; therefore,

$$\tfrac{1}{2} m_\alpha v_\alpha^2 = \tfrac{1}{2} m_\alpha v_\alpha'^2 + \tfrac{1}{2} m_p v_p^2 \qquad (1)$$

Conservation of momentum for this one-dimensional collision yields

$$m_\alpha v_\alpha = m_p v_p + m_\alpha v_\alpha' \qquad (2)$$

Solving Equation 1 for $(m_\alpha v_\alpha')^2$ yields

$$(m_\alpha v_\alpha')^2 = m_\alpha(m_\alpha v_\alpha^2 - m_p v_p^2) \qquad (3)$$

Solving Equation 2 for $(m_\alpha v_\alpha')^2$ and equating this to Equation 3 gives

$$(m_\alpha v_\alpha)^2 + (m_p v_p)^2 - 2 m_\alpha m_p v_\alpha v_p = (m_\alpha v_\alpha)^2 - m_\alpha m_p v_p^2$$

or

$$(m_p v_p)(m_p v_p - 2 m_\alpha v_\alpha + m_\alpha v_p) = 0$$

The solutions to this equation are

$$v_p = 0$$

and

$$v_p = \left(\frac{2 m_\alpha}{m_\alpha + m_p}\right) v_\alpha \qquad (4)$$

Because the proton must move when struck by the heavy α-particle, Equation 4 is the only physically reasonable solution for v_p. The solution for v_α' follows immediately from the substitution of Equation 4 into Equation 2.

Solution (b) Because an α particle consists of two protons and two neutrons, $m_\alpha = 4m_p$. Thus,

$$v_p = \left(\frac{2m_\alpha}{m_\alpha + m_p}\right) v_\alpha = \left(\frac{8m_p}{5m_p}\right) v_\alpha = 1.60 v_\alpha$$

$$v_\alpha' = \left(\frac{m_\alpha - m_p}{m_\alpha + m_p}\right) v_\alpha = \left(\frac{3m_p}{5m_p}\right) v_\alpha = 0.60 v_\alpha$$

The percent change in velocity of the α particle is

$$\%\text{ change in } v_\alpha = \left(\frac{v_\alpha' - v_\alpha}{v_\alpha}\right) \times 100\% = -40\%$$

Exercise: An α-particle with initial velocity v_α undergoes an elastic, head-on collision with an electron initially at rest. Using the fact that an electron's mass is about 1/2000 of the proton mass, calculate the final velocities of the electron and particle and the percent change in velocity of the α-particle.

In his analysis, Rutherford assumed that large-angle scattering is produced by a single nuclear collision and that the repulsive force between an α-particle and a nucleus separated by a distance r is given by Coulomb's law,

$$F = k\frac{(2e)(Ze)}{r^2}$$

where

2e is the charge on the, α + Ze is the nuclear charge, and k is the Coulomb constant. With this assumption, Rutherford was able to show that the number of α-particles entering the detector per unit time, Δ n, at an angle φ is given by;

$$\Delta n = \frac{k^2 Z^2 e^4 NnA}{4R^2(\frac{1}{2}m_\alpha v_\alpha^2)^2 \sin^4(\phi/2)}$$

Here R and φ are defined in Figure bove, N is the number of nuclei per unit area of the foil (and is thus proportional to the foil thickness), n is the total number of α-particles incident on the target per unit time, and A is the area of the detector. Geiger and Marsden confirmed the dependence of scattering on foil thickness, α-particle speed, and scattering angle experimentally.

Interactions of Photons with Matter

As we know the photons are electromagnetic radiation with zero mass, zero charge, and a velocity that is always c, the speed of light. Because they are electrically neutral, they do not steadily lose energy via columbic interactions with atomic electrons, as do charged particles. Photons travel some considerable distance before undergoing a more "catastrophic" interaction leading to partial or total transfer of the photon energy to electron energy. These electrons will ultimately deposit their energy in the medium. Photons are far more penetrating than charged particles of similar energy.

Energy Loss Mechanisms

The most interaction between photons and media that are including;

1- photoelectric effect
2- Compton scattering
3- pair production

Interaction Probability

• Linear attenuation coefficient, μ,

The probability of an interaction per unit distance travelled.

Dimensions of inverse length (cm^{-1}).

$$N = N_0 \, e^{-\mu x}$$

The coefficient μ depends on photon energy and on the material being traversed.

Mass Attenuation Coefficient (μ/ρ)

The probability of an interaction per g cm^{-2} of material traversed.

Mechanisms of Energy Loss: Photoelectric Effect

• In the photoelectric absorption process, a photon undergoes an interaction with an absorber atom in which the photon completely disappears.

• In its place, an energetic photoelectron is ejected from one of the bound shells of the atom.

• For gamma rays of sufficient energy, the most probable origin of the photoelectron is the most tightly bound or K shell of the atom.

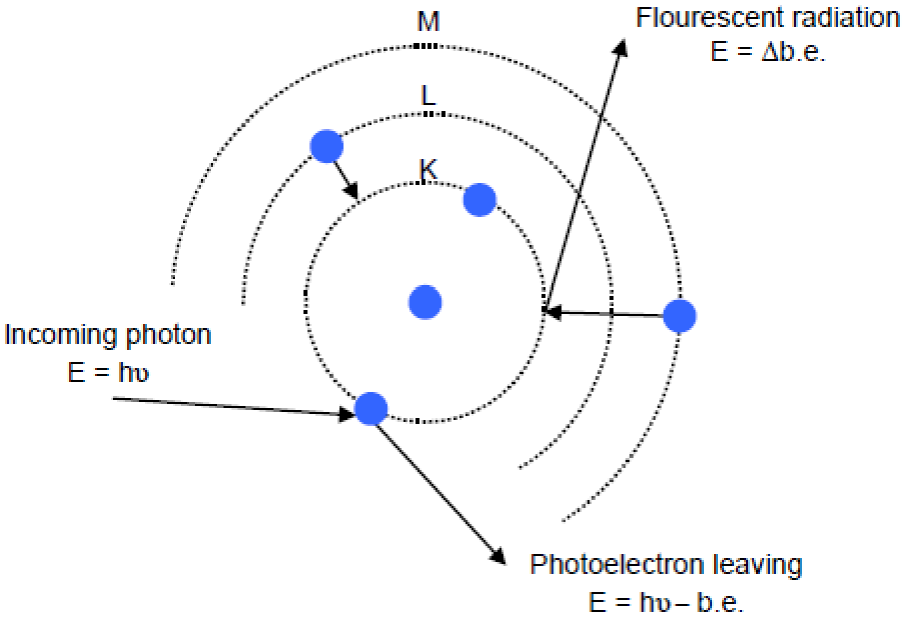
• The photoelectron appears with an energy given by

$$E_{e-} = h\nu - E_b$$

(E_b represents the binding energy of the photoelectron in its original shell)

Thus for gamma-ray energies of more than a few hundred keV, the photoelectron carries off the majority of the original photon energy.

Filling of the inner shell vacancy can produce fluorescence radiation, or x ray photon.



Events in the photoelectric scattering process

The photoelectric process is the predominant mode of photon interaction at

- a relatively low photon energies
- high atomic number Z

The probability of photoelectric absorption, symbolized τ (tau), is roughly proportional to

$$\tau \propto \frac{Z^n}{(h\nu)^3}$$

where the exponent n varies between 3 and 4 over the gamma-ray energy region of interest.

This severe dependence of the photoelectric absorption probability on the atomic number of the absorber is a primary reason for the preponderance of high-Z materials (such as lead) in gamma-ray shields.

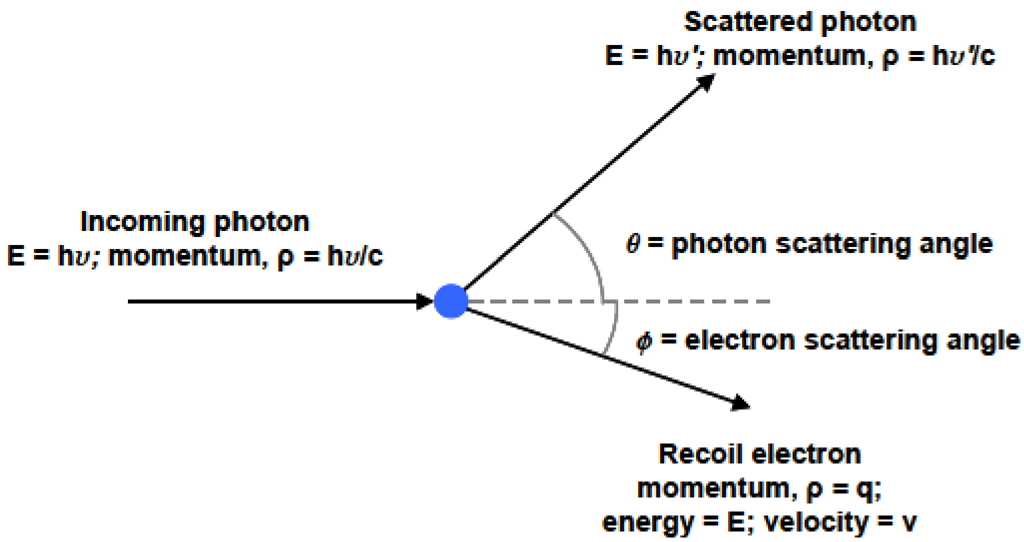
The photoelectric interaction is most likely to occur if the energy of the incident photon is just greater than the binding energy of the electron with which it interacts.

Compton Scattering

• Compton scattering takes place between the incident gamma-ray photon and an electron in the absorbing material.

• It is most often the predominant interaction mechanism for gamma-ray energies typical of radioisotope sources.

• It is the most dominant interaction mechanism in tissue.



Events in the Compton (incoherent scattering process)

In Compton scattering, the incoming gamma-ray photon is deflected through an angle θ with respect to its original direction.

The photon transfers a portion of its energy to the electron (assumed to be initially at rest), which is then known as a recoil electron, or a Compton electron.

• All angles of scattering are possible.

• The energy transferred to the electron can vary from zero to a large fraction of the gamma-ray energy.

• The Compton process is most important for energy absorption for soft tissues in the range from 100 keV to 10MeV.

The Compton scattering probability is is symbolized σ (sigma):

• Almost independent of atomic number Z;

• Decreases as the photon energy increases;

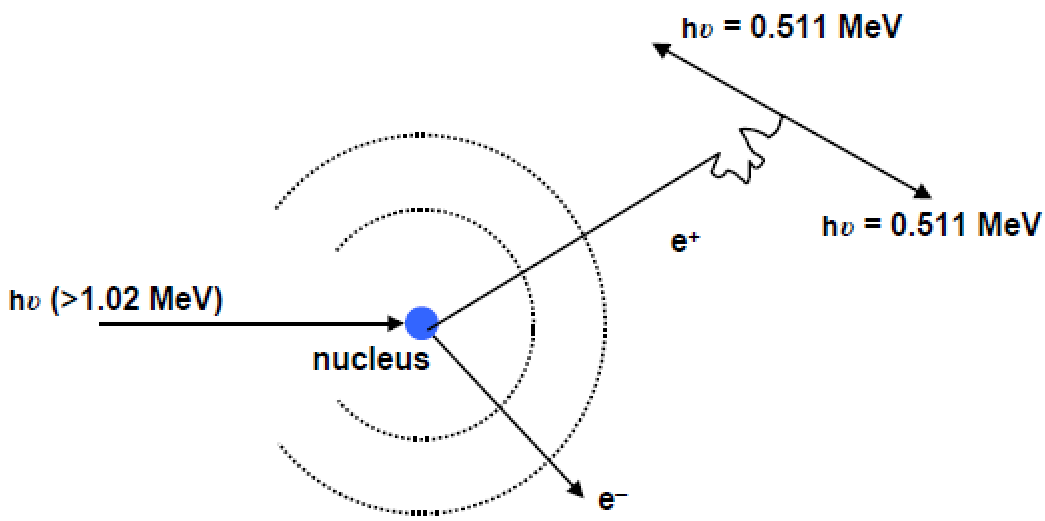
• Directly proportional to the number of electrons per gram, which only varies by 20% from the lightest to the heaviest elements (except for hydrogen).

Pair Production

If a photon enters matter with an energy in excess of 1.022 MeV, it may interact by a process called pair production.

The photon, passing near the nucleus of an atom, is subjected to strong field effects from the nucleus and may disappear as a photon and reappear as a positive and negative electron pair.

The two electrons produced, e- and e+, are not scattered orbital electrons, but are created, de novo, in the energy/mass conversion of the disappearing photon.



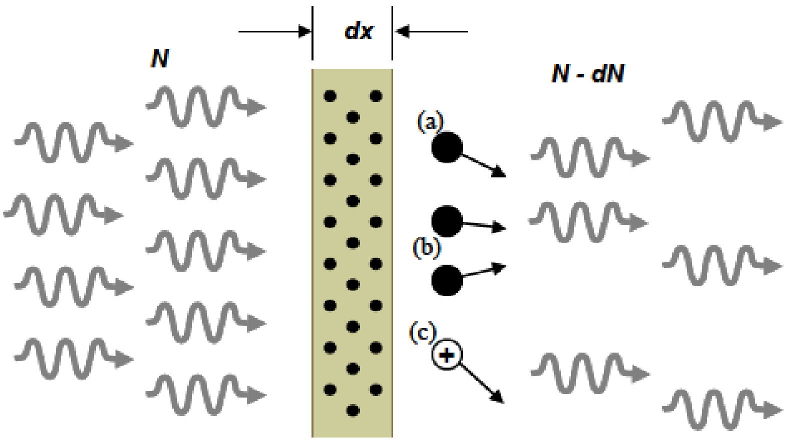
Pair Production Energetics

The kinetic energy of the electrons produced will be the difference between the energy of the incoming photon and the energy equivalent of two electron masses (2 x 0.511, or 1.022 MeV).

$$E_{e+} + E_{e-} = h\nu - 1.022 \ \ (\text{MeV})$$

Pair production probability, symbolized κ (kappa),

- Increases with increasing photon energy
- Increases with atomic number approximately as Z^2



Example; the researchers are determined that photoelectrons released from zinc by ultraviolet light were stopped by a voltage of 4.3 V. Find K_{max} and v_{max} for these electrons.

Solution

$$K_{\max} = eV_s = (1.6 \times 10^{-19}\ \text{C})(4.3\ \text{V}) = 6.9 \times 10^{-19}\ \text{J}$$

To find $v_{\max}$, we set the work done by the electric field equal to the change in the electron's kinetic energy, to obtain

$$\tfrac{1}{2}m_e v_{\max}^2 = eV_s$$

or

$$v_{\max} = \sqrt{\frac{2eV_s}{m_e}} = \sqrt{\frac{2(6.9 \times 10^{-19}\,\mathrm{J})}{9.11 \times 10^{-31}\,\mathrm{kg}}}$$
$$= 1.2 \times 10^6\ \mathrm{m/s}$$

Example; suppose that light of total intensity 1.0 *u*W/cm^2 falls on a clean iron sample 1.0 cm^2 in area. Assume that the iron sample reflects 96% of the light and that only 3.0% of the absorbed energy lies in the violet region of the spectrum above the threshold frequency.

- a- What intensity is actually available for the photoelectric effect?
- b- Assuming that all the photons in the violet region have an effective wavelength of 250 nm, how many electrons will be emitted per second
- c- Calculate the current in the phototube in amperes.

a- Because only 4.0% of the incident energy is absorbed, and only 3.0% of this energy is able to produce photoelectrons, the intensity available is

$$I = (0.030)(0.040)I_0 = (0.030)(0.040)(1.0\ \mu\mathrm{W/cm^2})$$
$$= 1.2\mathrm{nW/cm^2}$$

b-

For an efficiency of 100%, one photon of energy, *hf*, will produce one electron, so

Number of electrons/s

$$= \frac{1.2 \times 10^{-9}\ \mathrm{W}}{hf} = \frac{\lambda(1.2 \times 10^{-9})}{hc}$$
$$= \frac{(250 \times 10^{-9}\ \mathrm{m})(1.2 \times 10^{-9}\ \mathrm{J/s})}{(6.6 \times 10^{-34}\ \mathrm{J \cdot s})(3.0 \times 10^{8}\ \mathrm{m/s})}$$
$$= 1.5 \times 10^9$$

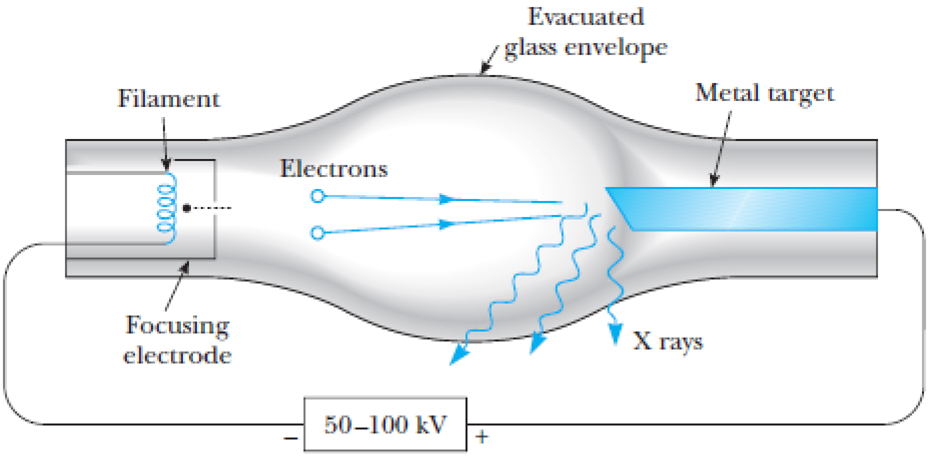
c-

$$i = (1.6 \times 10^{-19}\ \mathrm{C})(1.5 \times 10^9\ \mathrm{electrons/s})$$
$$= 2.4 \times 10^{-10}\ \mathrm{A}$$

A sensitive electrometer is needed to detect this small current.

X-Rays

The German physicist Wilhelm Roentgen discovered X-rays in 1895. He found that a beam of high-speed electrons striking a metal target produced a new and extremely penetrating type of radiation, see figure below.



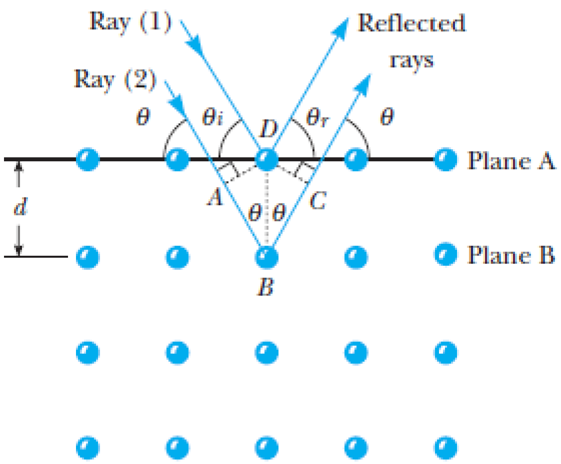
X-rays are produced by bombarding a metal target (copper, tungsten, and molybdenum are common) with energetic electrons having energies of 50 to 100 keV.

Within months of Roentgen's discovery the first medical x-ray pictures were taken, and within several years it became evident that x-rays were electromagnetic vibrations similar to light but with extremely short wavelengths and great penetrating power, see figure below.

One of the first images made by Roentgen using x-rays (December 22, 1895).

Rough estimates obtained from the diffraction of x rays by a narrow slit showed x-ray wavelengths to be about 10^{-10} m, which is of the same order of magnitude as the atomic spacing in crystals. Because the best artificially ruled gratings of the time had spacing's of 10^{-7} m, others researchers they suggested using single crystals such as calcite as natural three-dimensional gratings, the periodic atomic arrangement in the crystals constituting the grating rulings.

W. L. Bragg proposed a particularly simple method of analyzing the scattering of x-rays from parallel crystal planes in 1912. Consider two successive planes of atoms as shown in Figure below. Note that adjacent atoms in a single plane, A, will scatter constructively if the angle of incidence, θ_i, equals the angle of reflection, θ_r. Atoms in successive planes (A and B) will scatter constructively at an angle θ if the path length difference for rays (1) and (2) is a whole number of wavelengths, $n\lambda$. From the diagram, constructive interference will occur when;



Bragg scattering of x-rays from successive planes of atoms. Constructive interference occurs for ABC equal to an integral number of wavelengths.

$$AB + BC = n\lambda \qquad n = 1, 2, 3, \ldots$$

and because $AB = BC = d \sin\theta$, it follows that

$$n\lambda = 2d \sin\theta \qquad n = 1, 2, 3, \ldots$$

where n is the order of the intensity maximum, is the x-ray wavelength, d is the spacing between planes, and θ is the angle of the intensity maximum measured from plane A. Note that there are several maxima at different angles for a fixed d and corresponding to n = 1, 2, 3,.......

Example; X-rays of wavelength λ= 0.200 nm are aimed at a block of carbon. The scattered x-rays are observed at an angle of 45.0^{o} to the incident beam. Calculate the increased wavelength of the scattered x-rays at this angle.

Solution; The shift in wavelength of the scattered x-rays is given by Equation below Taking θ=45.0^{o}, we find from Compton scattering equation;

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$$

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

$$= \frac{6.63 \times 10^{-34}\,\mathrm{J\cdot s}}{(9.11 \times 10^{-31}\,\mathrm{kg})(3.00 \times 10^{8}\,\mathrm{m/s})}(1 - \cos 45.0^{\circ})$$

$$= 7.11 \times 10^{-13}\,\mathrm{m} = 0.00071\,\mathrm{nm}$$

Hence, the wavelength of the scattered x-ray at this angle is

$$\lambda = \Delta\lambda + \lambda_0 = 0.200711\,\mathrm{nm}$$

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