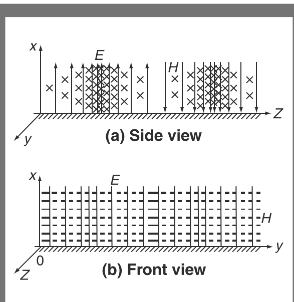
Chapter 2




# Ground Wave Propagation

Topics in this chapter include:

- Introduction
- Plane earth reflection
- Space and surface waves
- Transition between surface and space wave
- Tilt of surface waves
- Impact of imperfect earth
- Reduction factor and numerical distance
- Earth's behaviour at different frequencies
- Curved earth reflection

## 2–1 Introduction

The waves, which while traveling, glide over the earth's surface are called *ground waves*. Ground waves are always vertically polarized and induce charges in the earth. The number and polarity of these charges keep on changing with the intensity and location of the wave field. This variation causes the constitution of a current. In carrying this current, the earth behaves like a leaky capacitor. As the wave travels over the surface, it gets weakened due to absorption of some of its energy. This absorption, in fact, is the power loss in the earth's resistance due to the flow of current. This energy loss is partly replenished by the diffraction of energy, downward, from the portion of the wave present somewhat above the immediate surface of the earth. This process is shown in Fig. 2–1.

The energy propagated over paths near the earth's surface is considered to be made possible through ground waves. The earth's surface is normally considered to be a plane, provided the distance between the transmitter and the receiver does not cross a barrier $d$ which is given by

$$d = 50/(\mathrm{f_{MHz}})^{1/3} \text{ in miles} \tag{1}$$

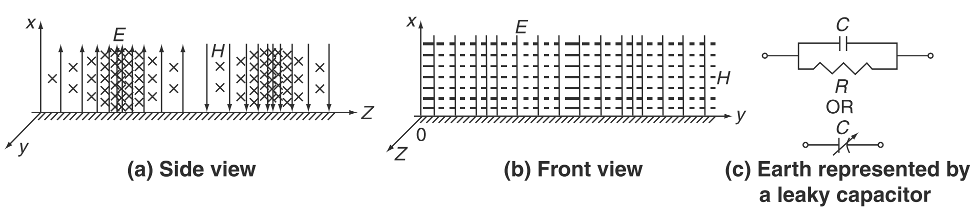



**Figure 2–1** Front and side view of the gliding wave and a leaky capacitor.

Beyond this distance, the effect of the curvature of the earth is to be accounted. Thus, the study of wave propagation can be divided into two parts, i.e., the waves that propagate over (i) the plane earth, and (ii) the spherical earth.

**EXAMPLE 2–1.1** Calculate the distance beyond which the earth's curvature is to be accounted at frequency of (a) 100 kHz, (b) 1 MHz, and (c) 10 MHz.

**Solution**

In view of (1), $d = 50/(f_{MHz})^{1/3}$ in miles

(a) 100 kHz = 0.1 MHz, $d=50/(0.1)^{1/3}=50/0.464=107.75$ miles
(b) 1 MHz, $d = 50/(1)^{1/3} = 50$ miles
(c) 10 MHz, $d = 50/(10)^{1/3} = 50/2.1544 =$23–21miles

## 2–2 Plane Earth Reflection

For elevated transmitting and receiving antennas within the line of sight of each other, the received resultant signal is a combination of the signal reaching the receiver through a direct path and that reaching after being reflected by the ground. These two paths are shown in Fig. 2–2.

For a smooth plane and finitely conducting earth, the magnitude and phase of the reflected wave differ from that of the incident wave. When the earth is rough, the reflected wave tends to be scattered and may be much reduced in amplitude compared with smooth earth reflection. The roughness is generally estimated by the Raleigh criterion given by the relation:

$$R = 4\pi\sigma \sin\theta/\lambda \tag{1}$$

where,
$\sigma$ is the standard deviation of the surface irregularities relative to the mean surface height,
$\theta$ is the angle of incidence measured from the normal angle, and
$\lambda$ is the wavelength.
If $R < 0.1$, the reflecting surface is considered as being smooth.
If $R > 10$, the reflecting surface is considered to be rough.

From the rough earth, the reflected wave tends to be scattered and may be much reduced in amplitude compared with that reflected from a smooth surface. Besides, a surface may be considered rough for waves incident at high angles (i.e., large $\theta$). It may approach to be smooth as the angle of incidence approaches the *grazing angle* (i.e., $\theta \to 0$). Also, when the incident wave is near grazing over a smooth earth, the reflection coefficient approaches minus one for both polarizations.

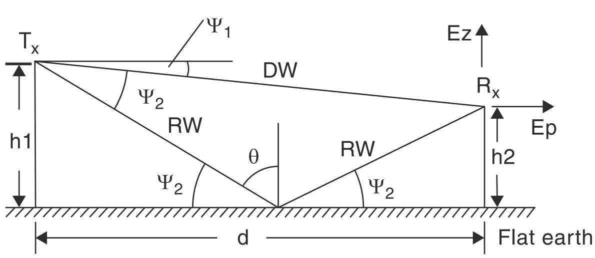



**Figure 2–2** Direct wave (DW) and reflected wave (RW) between $T_x$ and $R_x$.

The problem of reflection at the surface of a perfect dielectric results in the reflection factors for perpendicular and parallel polarizations. The earth, although not a good conductor like copper or silver, is also not a perfect dielectric. The relations for reflection factors for perpendicular and parallel polarizations obtained for perfect dielectric (4 and 5 below), therefore, need modification by accounting finite conductivity of the earth.

For a medium having dielectric constant $\varepsilon$ and conductivity $\sigma$, Maxwell's equations can be written as

$$\nabla \times H = J + \partial D/\partial t = \sigma E + \varepsilon \partial E/\partial t, \tag{2}$$

In view of sinusoidal time variation $[e^{j\omega t}]$, the above relation can be manipulated to yield

$$\nabla \times H = \varepsilon' \, \partial E/\partial t \tag{3}$$

where, $\varepsilon' = \{\varepsilon + (\sigma/j\omega)\}$ is a complex quantity

The expression for reflection coefficients $(E_r/E_I)$ for horizontal polarization $(R_H)$ and for vertical polarization $(R_V)$ are given as

$$R_H = \{\surd\varepsilon_1 \cos\theta - \surd(\varepsilon_2 - \varepsilon_1 \sin^2\theta)\}/\{\surd\varepsilon_1 \cos\theta + \surd(\varepsilon_2 - \varepsilon_1 \sin^2\theta)\} \tag{4}$$

$$R_V = [(\varepsilon_2/\varepsilon_1)\cos\theta - \surd\{(\varepsilon_2/\varepsilon_1) - \sin^2\theta\}]/[(\varepsilon_2/\varepsilon_1)\cos\theta + \surd\{(\varepsilon_2/\varepsilon_1) - \sin^2\theta\}] \tag{5}$$

If the medium 1 is free space with $\varepsilon_1 = \varepsilon_0$ and the medium 2 is the flat earth surface with $\varepsilon_2 = \varepsilon' = \{\varepsilon + (\sigma\ /\ j\omega)\}$, the above expressions can be modified as under.

$$R_H = \{\surd\varepsilon_0 \cos\theta - \surd[\{\varepsilon + (\sigma/j\omega)\} - \varepsilon_0 \sin^2\theta]\}/\{\surd\varepsilon_0 \cos\theta + \surd[\{\varepsilon + (\sigma/j\omega)\} - \varepsilon_0 \sin^2\theta]\} \tag{6}$$

For $\psi = \psi_2 = 90° - \theta$ or $\theta = 90° - \psi$, $\cos\theta = \sin\psi$ and $\sin\theta = \cos\psi$

$$\begin{aligned} R_H &= \{\surd\varepsilon_0 \cos(90° - \psi) - \surd[\{\varepsilon + (\sigma/j\omega)\} - \varepsilon_0 \sin^2(90° - \psi)]\}\{\surd\varepsilon_0 \cos(90° - \psi) \\ &\qquad + \surd[\{\varepsilon + (\sigma/j\omega)\} - \varepsilon_0 \sin^2(90° - \psi)]\} \\ &= \{\surd\varepsilon_0 \sin\psi - \surd[\{\varepsilon + (\sigma/j\omega)\} - (\varepsilon_0 \cos^2\psi)]\}/\{\surd\varepsilon_0 \sin\psi \\ &\qquad + \surd[\{\varepsilon + (\sigma/j\omega)\} - (\varepsilon_0 \cos^2\psi)]\} \\ &= \{\sin\psi - \surd[\{(\varepsilon/\varepsilon_0) + (\sigma/j\omega\varepsilon_0)\} - \cos^2\psi]\}/\{\sin\psi + \surd[\{(\varepsilon/\varepsilon_0) + (\sigma/j\omega\varepsilon_0)\} - \cos^2\psi]\} \end{aligned} \tag{7}$$

Putting $\varepsilon/\varepsilon_0 = \varepsilon_r$ and $\sigma/(j\omega\varepsilon_0) = x$ or $\mathrm{Re}|x| = \sigma/(\omega\varepsilon_0)$ or $x = (18\times10^9)\sigma/\mathrm{f}_{\mathrm{Hz}} = (18 \times 10^3)\sigma/\mathrm{f}_{\mathrm{MHz}}$

$$R_H = \{\sin\psi - \surd[(\varepsilon_r - jx) - \cos^2\psi]\}/\{\sin\psi + \surd[(\varepsilon_r - jx) - \cos^2\psi]\} \tag{8}$$

Similarly, the expression (5) of the reflection coefficient for parallel (vertical) polarization can be modified as under.

$$R_V = \{(\varepsilon_r - jx)\sin\psi - \surd[(\varepsilon_r - jx) - \cos^2\psi]\}/\{(\varepsilon_r - jx)\sin\psi + \surd[(\varepsilon_r - jx) - \cos^2\psi]\} \tag{9}$$

Since $R_H$ and $R_V$ are both complex quantities, these can be written as

$$R_H = |R_H|\angle R_H, \text{ and } R_V = |R_V|\angle R_V \tag{10}$$

where $|R_H|$ and $|R_V|$ are the amplitudes and $\angle R_H$ and $\angle R_V$ are the phase angles of $R_H$ and $R_V$ respectively.

From (8), (9) and (10), it is evident that the reflection factors are of complex nature and that the reflected wave will differ from the incident wave, both in magnitude and phase. The variation of these factors with angles of incidence, values of $x$ and frequencies $(f)$ is shown in Figs. 2–3 and 2–4. The parameter $x$ obtained for different relative dielectric constant $(\varepsilon_r)$, conductivity $(\sigma)$ of the earth over a range of frequencies (from 0.5 to 1000 MHz) may vary from 2 to more than 200. The relative dielectric constant is of the order of about 7 for a poor (low conductivity) earth, 15 for an average earth and about 30 for a good (high conductivity) earth. These curves for different relative values of $x$ and $f$ yield the following information:

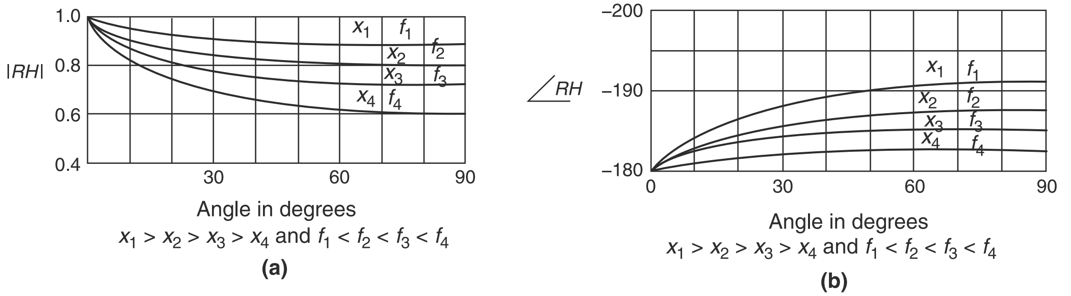



**Figure 2–3** Variation of $|R_h|$ and $\angle R_h$ with incident angle.

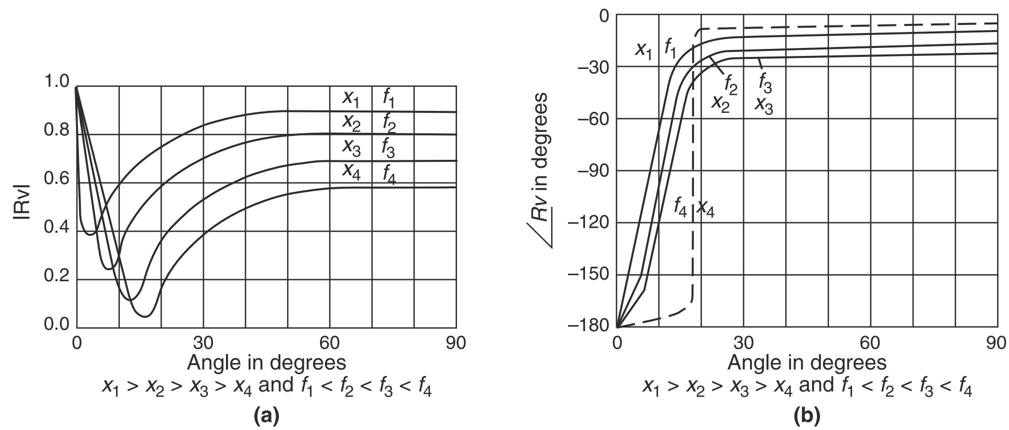



**Figure 2–4** Variation of $|R_V|$ and $\angle R_V$ with incident angle.

**When the incident wave is *horizontally polarized* (Fig. 2–3)**

- The phase of the reflected wave differs from that of the incident wave by nearly 180° for all angles of incidences.
- For angles of incidence near grazing ($\psi$ = 0), the reflected wave is equal in magnitude but 180° out of phase with the incident wave for all frequencies and for all ground conductivities.
- As the angle of incidence is increased, both the magnitude and phase of the reflection factor change, but not to a large extent. The change is greater for the higher frequencies and lower ground conductivities.

**When the incident wave is *vertically polarized* (Fig. 2–4)**

- At grazing incidence $E$, the reflected wave is equal to that of the incident wave and has an 180° phase reversal for all finite conductivities.
- As the angle increases from zero, the magnitude and phase of the reflected wave decrease rapidly. The magnitude reaches a minimum and the phase change goes through −90° at an angle known as *pseudo-Brewster angle* (or just *Brewster angle*) by the analogy of a perfect dielectric case. At angles of incidence above this critical angle, the magnitude increases again and the phase approaches zero.

- For very high frequencies and low conductivities, the Brewster angle has very nearly the same value as it has for a perfect dielectric. For $\varepsilon_r = 15$, the Brewster angle occurs at 14.5° for the perfect dielectric case.
- For lower frequencies and higher conductivities, the Brewster angle is less, approaching zero as $x$ becomes much larger than $\varepsilon_r$.
- When the incident wave is normal to the reflecting surface ($\psi = 90°$), it is evident that there is no difference between horizontal and vertical polarization. The reflection coefficients $R_V$ and $R_H$ should have the same value, as $E$ will be parallel to the reflecting surface in both cases. Comparison of these figures illustrate that $R_V$ and $R_H$ have the same magnitude but differ by 180° in phase. This is due to the different positive directions assigned for the reflected waves in two cases.
- For angles of incidence near grazing, a more accurate plot of reflection coefficient is often required. Such curves plotted on logarithmic scales are available.

Figures showing variation of the earth's constants (viz., conductivity and permittivity) in different regions of the globe are also available. The values of $\varepsilon_r$ and $\sigma$ for some commonly encountered terrain are given in Table 2–1.

The curves shown by Figs. 2–3 and 2–4 are labeled in terms of relative values of $x$ and $f$. The actual curves may be obtained by substituting desired values of $\sigma$, $\varepsilon_r$ and $f$ in the relevant equations. A curve obtained for a particular $x$ (say for good earth) will correspond to a particular frequency. This same curve may also apply for another frequency if the conductivity of the earth is changed to that of a very poor earth.

**EXAMPLE 2–2.1** Obtain the roughness factor at 3 MHz for an earth having $\sigma = 0.5$, with $\theta = 30°$. Calculate the ratio of roughness factors for the same earth and same $\theta$ if frequency is doubled.

**■ Solution**

In view of (1), $R = 4\,\pi\,\sigma\,\sin\theta\,/\,\lambda$

Since $\lambda$ at 3 MHz = 100 m

$R = 4\,\pi \times 0.5 \times \sin 30°/100 = \pi/100 = 0.031415927$

**EXAMPLE 2–2.2** Evaluate the roughness factors for the earth at 10 MHz if $\sigma = 5$ for $\theta$ equal to (a) 30°, (b) 45°, and (c) 60°.

**■ Solution**

In view of (4), $R = 4\,\pi\sigma\,\sin\theta/\lambda$

Since $\lambda$ at 10 MHz = 30 m

(*a*) $R = 4\,\pi \times 5 \times \sin 30°/30 = 10\pi/30 = \pi/3 = 1.0472$

(*b*) $R = 4\,\pi \times 5 \times \sin 45°/30 = 20\pi/30\sqrt{2} = \sqrt{2}\pi/3 = 1.481$

(*c*) $R = 4\,\pi \times 5 \times \sin 60°/30 = 10\pi\sqrt{3}/30 = \sqrt{3}\pi/3 = 1.8138$

**EXAMPLE 2–2.3** Estimate the values of parameter $x$ for flat earth with $\sigma = 4\times10^{-5}$at (a) $f$ = 300 kHz, (b) 1000 kHz, and (c) 3 MHz.

**Solution**

In view of the relation, $x = (18\times10^3)\sigma/\mathrm{f}_{\mathrm{MHz}}$

($a$) $f$ = 300 kHz = 0.3 MHz, $x = (18 \times 10^3) \times 4 \times 10^{-5}/0.3 = 72 \times 10^{-2}/0.3 = 2.4$

($b$) 1000 kHz = 1 MHz = $x = (18 \times 10^3)\times 4\times10^{-5}/1 = 72\times10^{-2}= 0.72$

($c$) 3 MHz = $x = (18\times10^3)\times 4\times10^{-5}/3 = 72\times10^{-2} / 3 = 0.24$

## 2–3 Space Wave and Surface Wave

According to Sommerfeld, the ground wave can be divided into two parts, a *space wave* and a *surface wave*. The space wave dominates at larger distances above the earth, whereas the surface wave is stronger nearer to the earth's surface. The expressions given by Norton for the electric field of an electric dipole above the surface of a finitely conducting plane earth clearly show the separation into space and surface waves.

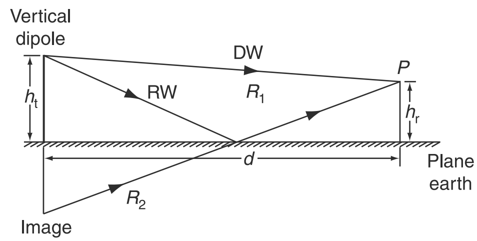



**Figure 2–5** Vertical dipole and its image.

At larger distances, the field expressions for the vertical dipole after neglecting the terms containing the higher orders of $1/R_1$ and $1/R_2$, reduces to

$$\begin{aligned} E_Z &= j30\beta Idl[\cos^2\psi(\{[\exp -j\beta R_1)]/R_1\} + R_V\{[\exp(-j\beta R_2)]/R_2\}) \\ &\quad +(1-R_V)(1-u^2+u^4\cos^2\psi)F\{[\exp -j\beta R_2)]/R_2\}] \end{aligned} \tag{1}$$

$$\begin{aligned} E_\rho &= -j30\beta Idl[\sin\psi\cos\psi(\{[\exp -j\beta R_1)]/R_1\} + R_V\{[\exp(-j\beta R_2)]/R_2\}) \\ &\quad -\cos\psi(1-R_V)u\{\surd(1-u^2\cos^2\psi)\}F\{[\exp(-j\beta R_2)]/R_2\}(1+0.5\sin^2\psi)] \end{aligned} \tag{2}$$

The dimensional parameters $h_t$, $h_r$, $R_1$, $R_2$ and $d$ are shown in Fig. 2–5.
$E_Z$ and $E_\rho$ are the $z$ and $\rho$ components of $E$ respectively. $R_1, R_2$ and $d$ are the respective distances from dipole and its image to the point $P$. $R_V$ is the reflection coefficient discussed earlier, and $F$ is the attenuation function that depends upon the earth's constants and the distance to the receiving point $P$.
$u^2 = 1/(\varepsilon_r - jx)$ ($\varepsilon_r$, $x$ and parameters involved in it are defined earlier)
Equations (1 and 2) may be combined and separated into the following two parts.
The field strengths for space and surface waves can be given as

$$\begin{aligned} E_{\text{total}}(\text{space}) &= \surd[E_Z^2(\text{space}) + E_\rho^2(\text{space})] \\ &= j30\beta Idl\cos\psi(\{[\exp(-j\beta R_1)]/R_1\} + R_V\{[\exp(-j\beta R_2)]/R_2\}) \end{aligned} \tag{3}$$

$$\begin{aligned} E_{\text{total}}(\text{surface}) &= \surd[E_Z^2(\text{surface}) + E_\rho^2(\text{surface})] \\ &= j30\beta Idl(1-R_V)F\{[\exp(-j\beta R_2)]/R_2\}] \\ &\quad \surd[1-2u^2+(\cos^2\psi)u^2(1+0.5\sin^2\psi)^2] \end{aligned} \tag{4}$$

In the above relations, $u^4$ and higher order terms are discarded.

A *surface wave* (also called *Norton surface wave*) contains the additional function $F$ representing the attenuation.

The expressions (3) and (4) represent the electric field of a vertical dipole above a finitely conducting plane earth. When the dipole is at the surface of the earth, the expression for the surface-wave part of this field reduces to

$$E_{\text{total}}(\text{surface}) = j30\beta Idl(1 - R_V)F\{[\exp(-j\beta R)]/R\}][a_z(1 - u^2) + a_\rho \cos\psi(1 + 0.5\sin^2\psi)\}]\,u\sqrt{(1 - u^2\cos^2\psi)}\} \quad (5)$$

In this expression $R$ ($R >> \lambda$) is the distance from the dipole to the point at which the field is being considered, $a_z$ and $a_\rho$ are the unit vectors respectively parallel to and perpendicular to the vertical dipole associated with $E_z$ and $E_\rho$. Also,

$$F = \{1 - j\sqrt{(\pi\omega)}e^{-\omega}[e_{rfc}(j\sqrt{\omega})]\} \quad (6)$$

$$\omega = \{-j\beta Ru^2(1 - u^2\cos^2\psi)/2\}[1 + \sin\psi/\{u\sqrt{(1 - u^2\cos^2\psi)}\}]^2 \quad (7)$$

$$e_{rfc}(j\sqrt{\omega}) = (2/\sqrt{\omega})\int_{i\sqrt{\omega}}^{\infty} e^{-v^2}dv \quad (8)$$

When $h_t$ is quite large, the wave is a plane wave and the space wave field is the total ground wave field. When $h_t$ is quite small, the incident wave will not be a plane wave. The expression for the total reflected field must contain terms in addition to those given by the space wave field. These additional terms are those which account for the surface wave.

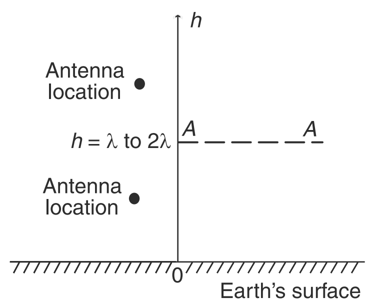



**Figure 2–6** Transition between surface and space waves.

## 2–4 Transition Between Surface and Space Wave

In case of vertical polarization if the antenna height is less than the barrier $A$-$A$ (Fig. 2–6), the surface wave dominates, $E$ is not a function of $h_t$ and $h_r$ and the ray action is not present. Above this barrier, the space wave dominates, ray action (*DR* and *RR*) comes into picture, $E$ is a function of frequency, conductivity and polarization and if $\sigma$ is finite, $h_t$ is less over the earth surface and large over the sea surface. In case of a horizontally polarized wave, $h_t = \lambda/10$ for much smaller $\sigma$ , even less than for good earth and sea water. Ground wave is almost negligible especially for $f > 30$ MHz.

## 2–5 Tilt of Wave Front due to Ground Losses

In Section 2–1, it was mentioned that the waves glide over the surface of the earth. Initially, $E$ (and hence the displacement current) originating from a vertical antenna can be considered to be entirely perpendicular to the earth. During the passage of travel, it gets weakened due to energy absorption by the earth. The farther it travels, the more energy is absorbed and weaker it becomes. The energy absorbed is the result of a current flow beneath the earth's surface up to a certain depth and the presence of earth resistance. As shown in Fig. 2–7, the wave front starts tilting in the forward direction as it progresses.The magnitude of tilt will depend upon the conductivity and permittivity of the earth. The forward tilt of $E$ results in a horizontal component of the current, and hence of the power '$P$' sufficient to furnish the power dissipated in earth over which the wave is passing. In general, the components of $E$ parallel and perpendicular to earth will neither be in phase nor will have equal magnitude and thus $E$ above the earth will be elliptically polarized.

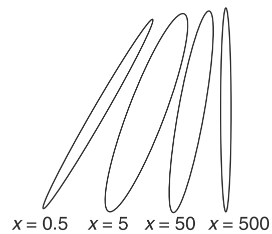



**Figure 2–7** Elliptic polarization and tilt of $E$ at the earth surface for $\varepsilon_r$ = 5 and for different values of $x$.

The illustration of Fig. 2–8 gives an idea about tilting of the wave during its travel. It shows the distribution and the alternation of the field ($E$) and charge ($Q$) just above the ground with the wave travel. It also shows the current flow inside the earth. The lengths and tips of arrows represent the magnitudes and direction of currents at different instants of time. The deeper the current penetrates, smaller is its magnitude. As long as the surface supporting the wave is a perfect conductor, $E$ and $Q$ distribution shall remain confined to the surface and $E$ will be entirely vertical.

The moment conductivity becomes finite, a horizontal component of the field $E$ comes into existence resulting in current flow inside the media. The more is the deviation, more will be the depth of penetration. Thus this distribution is true for any media having finite conductivity.

The surface wave impedance $Z_S$ of earth is given by

$$Z_S = \sqrt{}[\omega\mu/\sqrt{}(\sigma^2 + \omega^2\varepsilon_R)] \angle [(1/2)\tan^{-1}(\sigma/\omega\varepsilon_R)] \tag{1}$$

Also, the horizontal and vertical components of $E$ are

$$E_h = JsZ_s \quad \text{and} \quad E_v = \eta_v H \tag{2}$$

Thus,

$$E_h/E_v = Z_s/\eta_v = Z_s/377 \tag{3}$$

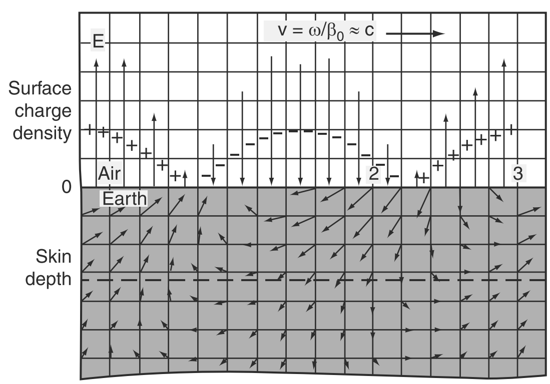



**Figure 2–8** Electric field, charge and current distribution.

**EXAMPLE 2–5.1** Evaluate the value of surface impedance if $\sigma = 5\times10^{-5}$, $\varepsilon_r = 15$, $\mu = \mu_0$ at (a) 5 kHz, (b) 50 kHz, and (c) 500 kHz.

**Solution**

In view of (1), $|Z_S| = \surd\,[\omega\mu\,/\surd\,(\sigma^2 + \omega^2\varepsilon_R)]$

$$\sigma^2 = 25 \times 10^{-10}$$

(a) $f = 5\,\text{kHz}, \omega = 2\pi f = 2\pi \times 5 \times 10^3 = \pi \times 10^4, \omega^2 = 10^9, \omega^2\varepsilon_R = 15 \times 10^9$

$$\begin{aligned}
\omega\mu &= \pi \times 10^4 \times 4\pi \times 10^{-7} = 4 \times 10^{-2}, \\
&\quad \omega\varepsilon_R = \pi \times 10^4 \times 15 \\
\sigma^2 + \omega^2\varepsilon_R &= 25 \times 10^{-10} + 15 \times 10^9 \cong 15 \times 10^9 \\
\surd(\sigma^2 + \omega^2\varepsilon_R) &= \surd(15 \times 10^9) = 3.873 \times 10^3 \\
\left[\omega\mu/\surd(\sigma^2 + \omega^2\varepsilon_R)\right] &= [0.04/3.873 \times 10^3] = 10.3 \times 10^{-6} \\
|Z_S| &= \surd[\omega\mu/\surd(\sigma^2 + \omega^2\varepsilon_R)] = \surd 10.3 \times 10^{-6} \\
&= 3.21 \times 10^{-3}
\end{aligned}$$

(b) $f = 50\text{ kHz}, \omega = 2\pi f = 2\pi \times 50\times10^3 = \pi\times10^5, \omega^2 = 10^{11}, \omega^2\varepsilon_R = 15\times10^{11}$

$$\begin{aligned}
\omega\mu &= \pi \times 10^5 \times 4\pi \times 10^{-7} = 0.4, \quad \omega\varepsilon_R = \pi \times 10^5 \times 15 \\
\sigma^2 + \omega^2\varepsilon_R &= 25 \times 10^{-10} + 15 \times 10^{11} \cong 15 \times 10^{11} \\
\surd(\sigma^2 + \omega^2\varepsilon_R) &= \surd(150 \times 10^{10}) = 12.247 \times 10^5 \\
\left[\omega\mu/\surd(\sigma^2 + \omega^2\varepsilon_R)\right] &= 0.4/12.247 \times 10^6 = 0.03266 \times 10^{-6} \\
|Z_S| &= \surd[\omega\mu/\surd(\sigma^2 + \omega^2\varepsilon_R)] = \surd(3.266 \times 10^{-8}) \\
&= 1.807 \times 10^{-4}
\end{aligned}$$

(c) $f = 500\text{ kHz}, \omega = 2\pi f = 2\pi \times 500\times10^3 = \pi\times10^6, \omega^2 = 10^{13}, \omega^2\varepsilon_R = 15\times10^{13}$

$$\begin{aligned}
\omega\mu &= \pi \times 10^6 \times 4\pi \times 10^{-7} = 4, \omega\varepsilon_R = \pi \times 10^6 \times 15 \\
\sigma^2 + \omega^2\varepsilon_R &= 25 \times 10^{-10} + 15 \times 10^{13} \cong 15 \times 10^{13} \\
\surd(\sigma^2 + \omega^2\varepsilon_R) &= \surd(15 \times 10^{13}) = 12.247 \times 10^6 \\
\left[\omega\mu/\surd(\sigma^2 + \omega^2\varepsilon_R)\right] &= 4/12.247 \times 10^6 = 0.3266 \times 10^{-6} \\
|Z_S| &= \surd[\omega\mu/\surd(\sigma^2 + \omega^2\varepsilon_R)] = \surd(0.3266 \times 10^{-6}) \\
&= 5.715 \times 10^{-4}
\end{aligned}$$

The depth of penetration of the current into the ground is the function of the ground constants and the frequency. Penetration of the order of 15 m occurs at broadcast frequencies, decreasing to one or two meters at the frequencies of short-wave communication. At low frequencies, the surface wave is dependent mainly on the conductivity, whereas at higher frequencies a high permittivity is important. Thus, over all frequencies, surface wave is best over sea and worst over dry land.

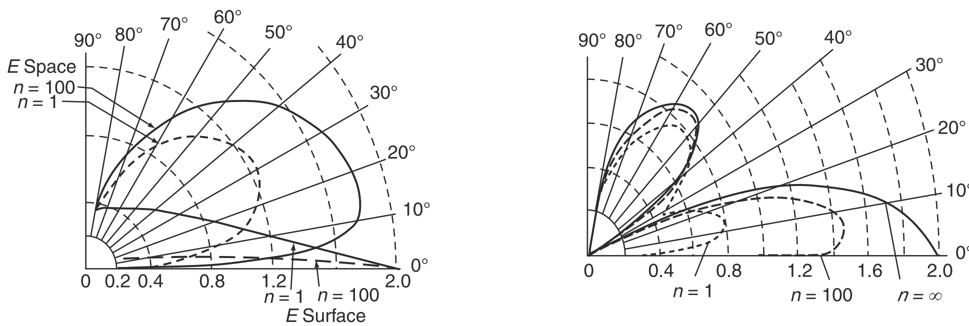
## 2–6 Impact of Imperfect Earth

Figure 2–9 illustrates *vertical radiation patterns* (VRPs) of a *vertical dipole* (VDP), and Fig. 2–10 illustrates vertical radiation patterns of a *horizontal dipole* (HDP) located at different heights above the earth's surface. Similar illustrations were included earlier in Chapter 15 in Figs. 15–5, 16–6 and 15–7. The parameter $n$ shown in the figures is computed by the relation '$n = x / \varepsilon_r$' for $\varepsilon_r$ of an average earth. From these figures, it can be noted that due to finite conductivity, the chief effect occurs at low angles where the space wave is much reduced from its value over that of a perfectly conducting earth. This is because the phase of $R_v$ changes rapidly for angles of incidences near the Brewster's angle. At Brewster's angle, the phase is nearly zero, whereas below this angle it is $-180°$.

## 2–7 Reduction Factor and Numerical Distance

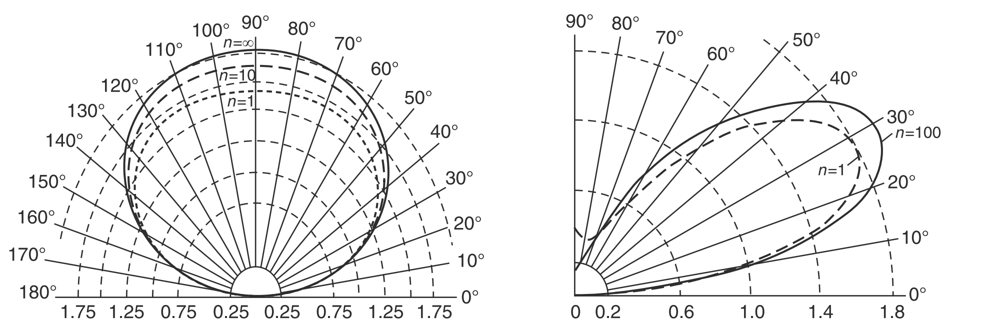
According to the Sommerfield analysis, the ground wave strength $E$ (for flat earth case) is given by the relation:

$$E = AE_0/d \tag{1}$$




**(a) VRP of a VDP at *h* = 0** **(b) VRP of a VDP at *h* = λ/2**

**Figure 2–9** Vertical radiation pattern of vertical dipole at $h = 0$ and $h = \lambda/2$.




**(a) VRP of a HDP at *h* = λ/4 in the plane perpendicular to the axis of the dipole** **(b) VRP of a HDP at *h* = λ/2 in the plane perpendicular to the axis of the dipole**

**Figure 2–10** Vertical radiation pattern of horizontal dipole at $h = \lambda/4$ and $h = \lambda/2$.

where $E_0$ is the field strength of the wave at the surface of the earth at a unit distance from the transmitter after neglecting the losses in this unit distance. It is a function of the transmitted power $P_t$ and the directivity of the antenna in the horizontal and vertical planes. Thus, if $P_t$ is 1 kW, $E_0$ is obtained to be 300 mW at 1 km distance.

The symbol $d$ stands for the distance from the transmitting antenna to the point at which $E$ is to be estimated, and $A$ is a factor called *reduction factor* which accounts for ground losses and is a function of the conductivity $\sigma$, permittivity $\varepsilon$, frequency $f$ and distance $d$ in terms of the wavelength $\lambda$. The factor $A$ can be expressed in terms of two auxiliary parameters $p$ and $b$, where the parameter $p$ is called the *numerical distance* and $b$, the *phase constant*. Parameter $b$ is a measure of the *power factor angle* of the earth. Both $p$ and $b$ are functions of $\sigma$, $f$ and characteristics of the earth taken as a conductor of radio frequency current or the power factor of the earth impedance.

For $p < 1$, $A$ slightly differs from unity, i.e., the loss in earth has little effect on ground wave field strength and $E$ is inversely proportional to the distance.

For $p > 1$, $A$ decreases rapidly.

For $p > 10$, $A$ varies as inversely proportional to the square of distance.

The values of $p$ and $b$ for vertically and horizontally polarized waves can be obtained from the following relations:

### (A) For Vertically Polarized Waves (VPW)

$$p = \frac{\pi}{x}\frac{d}{\lambda}\cos b \quad \text{and} \quad \tan b = \frac{\varepsilon + 1}{x} \tag{2}$$

Since ground waves are generally vertically polarized, the following approximation can be made in order to arrive at some simplified and meaningful results.

*(i) For conducting earth*

$$\frac{\varepsilon + 1}{x} < 0.3, \quad p = 1.75\frac{d}{c}\frac{f^2}{\sigma} \times 10^{-12} \tag{3}$$

*(ii) For dielectric earth*

$$\frac{\varepsilon + 1}{x} > 0.3, \quad p = \pi\frac{d}{c}\frac{f}{\varepsilon + 1} \tag{4}$$

### (B) For Horizontally Polarized Waves (HPW)

$$p \cong \pi\frac{d}{\lambda}\frac{x}{\cos b'} \quad \text{and} \quad \tan b' = \frac{\varepsilon - 1}{x} \tag{5}$$

In the above equations,

$b = 180^\circ - b'$, $\quad x = 1.80 \times 12^{12}\sigma/f$, $\quad d/\lambda$ is the distance in wavelength, $\sigma$ is the ground conductivity, $f$ is the frequency and $\varepsilon$ is the dielectric constant of the ground referred to air as unity, and $c$ is the velocity of light.

In view of the above, $A$ may be approximately expressed in terms of $p$ and $b$ by an empirical relation for $b \leq 90^\circ$.

$$A = \frac{2 + 0.3p}{2 + p + 0.6p^2} - \sqrt{(p/2)e^{-5p/8}}\sin b \tag{6}$$

When $b$ = 0 for a vertically polarized wave and 180° for a horizontally polarized wave, a resistive impedance is offered by the earth to the flow of RF current.

For $x >> \varepsilon_r$, at broadcast frequencies, $b \approx 0$

For $x << \varepsilon_r$, at HF and above, $b \approx 180°$
For $b = 90°$ earth often offers a capacitive impedance for either polarization

**EXAMPLE 2–7.1** A ground wave of 0.5 mV/m at 20 km distance is obtained from a transmitter operating at 2 MHz. The vertically polarized field produced is proportional to cos $\theta$, where $\theta$ is the angle of elevation. The other related parameters are
Antenna efficiency = 50%, $\sigma = 5\times10^{-5}$ and $\varepsilon_r = 15$. Estimate $E$ at the transmitting end.

**Solution**

$$x = (18 \times 10^3)\sigma/\mathrm{f_{MHz}} = (18 \times 10^3)5 \times 10^{-5}/2 = 0.45,\ \lambda \text{ at } 2\text{ MHz} = 150\,\text{m},$$
$$d = 2 \times 10^4\text{ m},\ E = E_1 \text{ at } d = 0.5 \times 10^{-3}\text{V/m}$$

For a vertically polarized wave,

$$\tan b = \frac{\varepsilon + 1}{x} = (15+1)/0.45 = 16/0.45 = 35.55,\ b = \tan^{-1}(35.55) = 88.38$$
$$\cos b = \cos(88.38) = 0.028$$
$$p = \frac{\pi}{x}\frac{d}{\lambda}\cos b = \frac{\pi}{0.45}\frac{2\times10^4}{150} \times 0.028 = \frac{6.2832 \times 10^3 \times 0.028}{6.75} = 26$$
$$A = \frac{2+0.3p}{2+p+0.6p^2} = \frac{2+0.3\times26}{2+26+0.6\times26\times26} = \frac{9.8}{433.8} = 0.0226$$

$E$ at the transmitting end = $E_1 \times d / A = (0.5 \times 10^{-3})\ 2\times10^4/\ 0.0226 = 442.48$ V/m
$P = (442.48/137.6)^2 = 10.34$ kW

## 2–8 Earth's Behavior at Different Frequencies

In view of Fig. 2–11, the following conclusions can be drawn.

### (i) At Broadcast and Lower Frequencies

- Ratio of capacitive reactance of the earth to the earth resistivity ($\rho = 1/\sigma$) is $>> 1$. Thus, the earth may be regarded as pure resistance.
- Values of $A$ and $p$ for a given physical distance is determined by $A$ which is a function of the term $(f^2/\sigma)$.

### (ii) At HF (10 MHz) and Above

(i) The impedance represented by the earth is primarily capacitive and $A$ is a function of the term $[f\ /\ (\varepsilon + 1)]$.

- The values of $\sigma$ and $\varepsilon$ that govern $A$ of the ground wave are suitably averaged values of the quantities for a distance below the

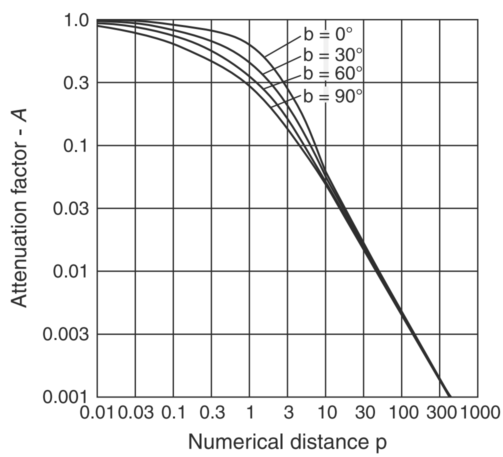



**Figure 2–11** Variation of attenuation factor $A$ with numerical distance $\rho$.

earth's surface. This distance referred is the depth to which there are ground currents of appreciable amplitudes and is called the *depth of penetration*.

- The depth of penetration depends on $f$, $\sigma$, $\varepsilon$ and ranges from a few feet at HF to 100's of feet at broadcast and lower frequencies.
- The earth's constants are not particularly sensitive to conditions existing at the actual surface of the ground, i.e., rain, etc.

## 2–9 Electrical Properties of the Earth

The conductivity $\sigma$ and permittivity $\varepsilon$ of earth widely vary for different types of soil. Values of a few typical soils and water are given in Table 2–1. It needs to be mentioned that hilly or mountainous regions normally have low conductivity ($10^{-3}$ to $5\times10^{-3}$ mhos/m), whereas flat regions have relatively high conductivity ($10\times10^{-3}$ to $30\times10^{-3}$ mhos/m). Also, the conductivity varies with temperature and salt content.

**Table 2–1** Typical ground constants

| Type of terrain | $\varepsilon_r$ | $\sigma$ (mhos/cm) |
|---|---|---|
| Sea water | 81 | 45000 |
| Fresh water | 80 | 100 |
| Pastoral, low hills, Rich soil, | 20 | 100 |
| Pastoral, medium hills, Forestation | 13 | 50 |
| Rocky soil, flat sandy | 10 | 20 |
| Cities, industrial areas | 5 | 10 |

## 2–10 Curved Earth Reflection

It was stated earlier that the effect of the curvature of the earth is entirely negligible up to a certain distance and all the relations obtained are valid up to this distance given by [$d$ = 50/ $(\mathrm{f_{MHz}})^{1/3}$]. When this distance gets doubled, the errors introduced in the estimation of various parameters remain small. For still greater distances, reduction in field strength below the free space value is much more. This enhanced reduction is mainly due to the curvature of the earth rather than due to losses in the ground. This is mainly because of the bulge of the earth which prevents surface waves from reaching the receiver by a straight-line path. The surface waves arrive at the receiver either through (a) diffraction around the earth, or (b) refraction in the lower atmosphere above the earth. The space-wave propagation too is affected by the earth's curvature. In this case, the wave from the ground is reflected from the curved surface instead of a flat surface. As a result, this wave will have a more diverged nature and hence will be weaker while reaching the receiver. As illustrated in Fig. 2–12, the effective antenna heights $h'_1$ and $h'_2$ are less than the actual antenna heights $h_1$ and $h_2$, and thus all equations obtained for flat earth are to be suitably modified.

At first glance, it appears that the problem of curved earth is easy and can be tackled by the application of Maxwell's equations in a simple manner as it was done in the case of flat earth. In Fig. 2–2, '$d$' is the distance between the transmitting and receiving antennas. In view of the curvature of the earth (Fig. 2–12), this distance elongates and exceeds $d$ of Fig. 2–2. The curvature also results in an increase of the reflection angle which is now greater than $\psi_2$ of Fig. 2–2. Since $\theta = 90° - \psi$ or $90° - \psi_2$, it will be different from that of flat earth. On the substitution of these new parameters in (7) and (8) of Sec. 2–2, altogether different values may result. Also, since $R_h$ and $R_v$ are complex quantities, $|R_h|$, $|R_v|$, $\angle R_h$ and $\angle R_v$ and the curves obtained therefrom will obviously differ. The estimation of field components $E_z$ and $E\rho$, etc., shall also be influenced by the change in $R_h$ and $R_v$. Apparently, the problem appears to be simple and straight forward. Jordon[14], however, has opined as under:

"The available solutions to this problem are much more involved than the plane earth solutions. One such solution is in the form of an infinite series of spherical harmonics with coefficients containing twelve Bessel functions. The convergence of the series is extremely slow, the main contribution being given by those terms

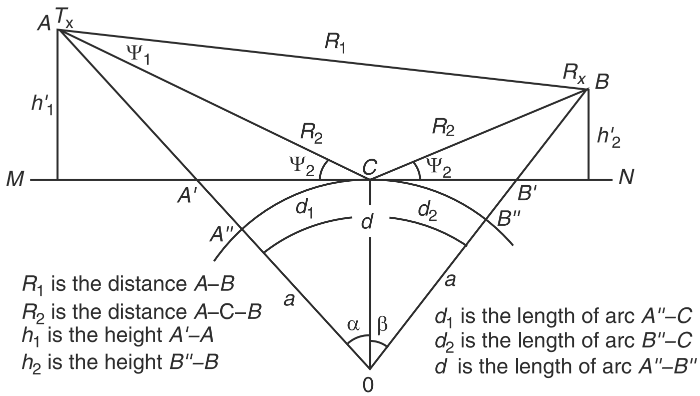



**Figure 2–12** Effective and actual antenna heights.

for which $n$ is of the order of the ratio $2\pi a/\lambda$ , where $a/\lambda$ is the radius of the earth in wavelengths. For commonly used radio frequencies, this ratio is of the order of $10^3$ to $10^8$."

To understand the further complexity of the problem, one may still refer to the paper of J. R. Wait[30] and the references cited therein, particularly from 42 to 61.

The problem of spherical earth basically revolves around the question whether transmitting and receiving antennas are within line-of-sight range or not. To address the problem, consider Fig. 2–13a which shows an elevated antenna $A$ and a point $C$ on the ground. The problem reduces to finding the distance to visible (optical) horizon. If the radius of the earth is $a$, antenna height is $h_1$ and the angle is $\alpha$ then from the right-angled triangle $OAC$,

$$\cos\alpha = \frac{a}{a+h_1} \cong 1 - \frac{h_1}{a} \tag{1}$$

$\alpha$ in all practical problems is small. Thus for small $\alpha$,

$$\cos\alpha \cong 1 - \frac{\alpha^2}{2} \tag{2}$$

From (1) and (2),

$$\alpha = d_1/a = \sqrt{2h_1/a} \tag{3}$$

Thus, the horizontal distance is

$$d_1 = \sqrt{2ah_1}\ \text{m} \tag{4}$$

Similarly, from Fig. 2–13b,

$$d_2 = \sqrt{2ah_2}\ \text{m} \tag{5}$$

If Figs. 2–13$a$ and 2–13$b$ are joined together by overlapping $OC$, it results in Fig. 2–13$c$ and the total horizontal distance $d$ can be given by

$$d_0 = d_1 + d_2 = \sqrt{2a}\left(\sqrt{h_1} + \sqrt{h_2}\right) \tag{6}$$

$$\begin{aligned} d_0 &= \sqrt{2 \times 6.37 \times 10^6}\left(\sqrt{h_1(m)} + \sqrt{h_2(m)}\right) \\ &= 3.57\left(\sqrt{h_1(m)} + \sqrt{h_2(m)}\right)\ \text{km} \end{aligned} \tag{7}$$

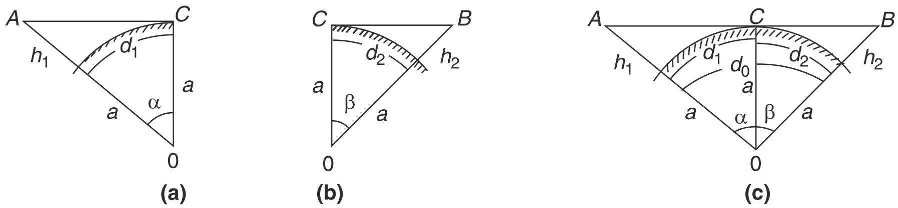



**Figure 2–13** Effective and actual antenna heights.

The distance $d_0$ can be termed as line-of-sight (LOS) distance/range. Let us confine our study to the case when the antennas are $A$ and $B$ and the distance $d < d_0$. As in case of flat earth, the total field at $R_x$ should be the sum of $DR$ ($AB$) and $RR$ ($ACB$). The curvature of the earth has the following effect on the wave propagation within the LOS range:

1. For fixed antenna heights, the path length difference between $DR$ and $RR$ will be different from that of flat earth case.
2. The reflection at the convex surface will result in divergence of the $RR$ path and hence will reduce the power received via $RR$.

To understand the process, consider Fig. 2–12. It illustrates a tangent plane $MN$ touching the earth at the point of reflection. The antenna heights can now be measured from this plane instead of the earth's surface. The heights $h_1'$ and $h_2'$ so obtained are the reduced heights and can be used for the actual heights $h_1$ and $h_2$ wherever they appear in equations. It needs to be mentioned that Fig. 2–12 does not represent parameters in true proportion as heights of antennas are much smaller than the radius of the earth. Practically, there is little difference between $h_1$ and $h_1'$ and $h_2$ and $h_2'$ and the deviations can be written as

$$h_1' = h_1 - \Delta h_1 \quad \text{and} \quad h_2' = h_2 - \Delta h_2 \tag{8}$$

In Fig. 2–12, $\Delta h_1$ and $\Delta h_2$ are shown as $A''A'$ and $B''B'$. Since $d_1$ and $d_2$ represent the LOS ranges at heights $h_1$ and $h_2$ respectively from (4) and (5)

$$\Delta h_1 = d_1^2/2a \quad \text{and} \quad \Delta h_2 = d_2^2/2a \tag{9}$$

From (8) and (9)

$$h_1' = h_1 - d_1^2/2a \quad \text{and} \quad h_2' = h_2 - d_2^2/2a \tag{10}$$

From triangles $OAC$ and $OBC$ shown in Fig. 2–12 with angles of incident and reflection being the same,

$$(a + h_1)\cos(\alpha + \psi_2) = a\cos\psi_2 \quad \text{and} \quad (a + h_2)\cos(\beta + \psi_2) = a\cos\psi_2 \tag{11}$$

Equation (11) is justified since $h_1$and $h_2 << a$, and $\psi$ ($\psi = \psi_2$)is the grazing angle $ACM$ and $BCN$. From the figure,

$$\tan\psi = \frac{\cos\alpha - \frac{a}{a+h_1}}{\sin\alpha} = \frac{\cos\beta - \frac{a}{a+h_2}}{\sin\beta} \tag{12}$$

In the derivation of (12), no assumptions were made. Therefore, the resulting expression is so rigorous that it cannot be solved analytically and requires graphical or some other approach for getting the solution. It may, however, be simplified since $h_1$and $h_2 << a$ and $\alpha$ and $\beta$ are also small. Thus, we may set

$$\frac{a}{a+h_1} \cong 1-\frac{h_1}{a} \quad \text{and} \quad \frac{a}{a+h_2} \cong 1-\frac{h_2}{a} \tag{13}$$

$$\cos\alpha \cong 1-\frac{\alpha^2}{2} \quad \text{and} \quad \cos\beta \cong 1-\frac{\beta^2}{2} \tag{14}$$

Thus,
$$\tan\psi = \frac{h_1/a-\alpha^2/2}{\alpha} = \frac{h_2/a-\beta^2/2}{\beta} \tag{15}$$

The above equation can also be expressed in terms of distances $d_1$ and $d_2$, $(d = d_1 + d_2)$ to get

$$\tan\psi = \frac{h_1/a-d_1^2/2}{d_1} = \frac{h_2/a-d_2^2/2}{d_2} \tag{16}$$

Since in almost all practical cases $h_1 > h_2 d_1$, (16) leads to

$$d_1 = \frac{d_2}{2} + 2\sqrt{\frac{d^2}{12}+\frac{a}{3}(h_1+h_2)} \times \cos\left[60^\circ + \frac{1}{3}\cos^{-1}\frac{ad(h_1-h_2)}{4\left\{\frac{d^2}{12}+\frac{a}{3}(h_1-h_2)\right\}^{3/2}}\right] \tag{17}$$

When $d < d_0$, the reflection point is located by the equations for flat earth which have the form

$$d_1 = \frac{h_1}{h_1+h_2}d \quad \text{and} \quad d_2 = \frac{h_2}{h_1+h_2}d \tag{18}$$

The expression for path difference (for flat earth case) given by (5) of Sec. 24–2 can be written in the modified form as below.

$$\Delta d = d_2 - d_1 \cong 2h_1h_2/d \tag{19}$$

This equation can be modified for spherical earth by replacing actual antenna heights $h_1$ and $h_2$ by the reduced heights $h_1'$ and $h_2'$. This results in

$$\Delta d \cong 2h_{1'}h_{2'}/d \tag{20}$$

Similarly, in case of flat earth

$$\tan\psi = \frac{h_1+h_2}{d} \quad \text{and for small angle} \quad \psi \cong \frac{h_1+h_2}{d} \tag{21}$$

This equation too gets modified in case of spherical earth and can be written as

$$\psi \cong \frac{h_1'+h_2'}{d} \tag{22}$$

## Problems

**2–1–1** **Distance beyond the earth's curvature.** Calculate the distance beyond which the earth's curvature is to be accounted at a frequency of (a) 30 kHz, (b) 3 MHz, and (c) 30 MHz.

**2–2–1** **Roughness factor.** Obtain the roughness factor at 10 MHz for an earth having $\sigma = 5\times10^{-5}$ mhos/cm for $\theta$ equal to (a) 5°, (b) 10°, and (c) 30°.

**2–2–2** **Roughness factor.** Evaluate the roughness factors for the earth if $\sigma = 5\times10^{-5}\theta = 15^\circ$ and $f$ equal to (a) 10 kHz, (b) 100 kHz, and (c) 1 MHz.

**2–2–3** **Parameter $x$.** Estimate the values of the parameter $x$ for flat earth with $\sigma = 3\times10^{-5}$at (a) $f = 100$ kHz, (b) 500 kHz, and (c) 3 MHz.

**2–5–1** **Surface impedance.** Evaluate the value of surface impedance if $\sigma = 3\times10^{-5}$, $\varepsilon_r = 10$, $\mu = \mu_0$ at (a) 5 kHz (b) 10 kHz, and (c) 20 kHz. Calculate the rate of change of $Z_s$ with doubling of frequency.

**2–7–1** **Transmitted power.** A ground wave of 0.5 mV/m at 20 km distance is obtained from a transmitter operating at 2 MHz. The horizontally polarized field produced is proportional to cos $\theta$ where $\theta$ is the angle of elevation. The other related parameters are
antenna efficiency = 30%, $\sigma = 5\times10^{-5}$ and $\varepsilon_r = 12$. Estimate the transmitted power.