Chapter-3
Wire Antennas

1. Introduction

In Chapter-2, we have just indicated that a short dipole antenna is not a good radiator of electromagnetic power because of its low radiation resistance and low radiation efficiency. We now examine the radiation characteristics of a center-fed thin straight antenna having a length comparable to a wavelength, as shown in Fig.5. Such an antenna is a ***linear dipole antenna.*** If the current distribution along the antenna is known, we can find its radiation field by integrating over the entire length of the antenna the radiation field due to an elemental dipole. The determination of the exact current distribution on such a seemingly simple geometrical configuration (a straight wire of a finite radius) is, however, a very difficult boundary-value problem even if the wire is assumed to be perfectly conducting. The current must be zero at the ends of the wire where charges are deposited, and the tangential electric field due to all currents and charges must vanish at every point on the wire surface. An analytical formulation of the problem leads to an integral equation in which the current distribution along the antenna is the unknown function under the integral. Unfortunately, an exact solution of the integral equation does not exist. Various approximate solutions have been attempted. With the advent of high-speed digital computers, numerical solutions for current distributions and input impedances can be obtained for linear antennas of specific lengths and thicknesses. The ratio of the voltage and the current at the feed points is the input impedance. Both the solution procedure and the numerical results are quite involved, and we shall not delve into them in this chapter. For our purposes the knowledge of the exact current distribution on the linear antenna is not of prime importance; a good estimate will give us considerable useful information on the radiation characteristics of the antenna. We

assume a sinusoidal current distribution on a very thin, straight dipole. Such a current distribution constitutes a kind of standing wave over the dipole and represents a good approximation.
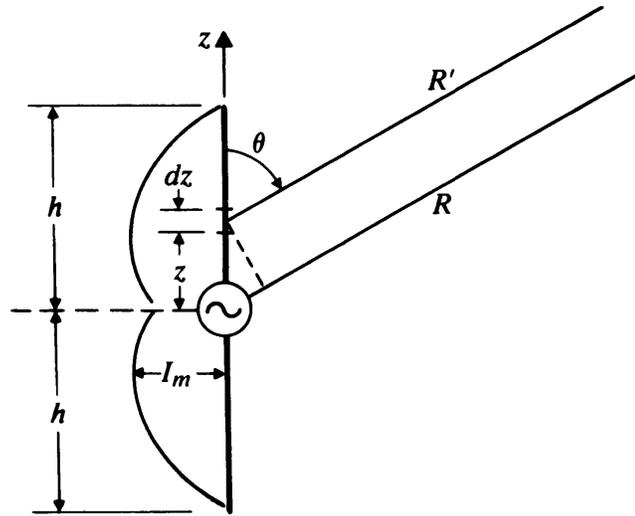

Fig.5: A center-fed linear dipole with sinusoidal current distribution

Since the dipole is center-driven, the currents on the two halves of the dipole are symmetrical and go to zero at the ends. We write the current phasor as

$$I(z) = I_m \sin \beta(h - |z|),$$
$$= \begin{cases} I_m \sin \beta(h - z), & z > 0, \\ I_m \sin \beta(h + z), & z < 0. \end{cases} \quad \dots (51)$$

We are interested only in the far-zone fields. The far-field contribution from the differential current element Idz is, from Eqs.(19a,b),

$$dE_\theta = \eta_0 \, dH_\phi = j \frac{I \, dz}{4\pi} \left(\frac{e^{-j\beta R'}}{R'} \right) \eta_0 \beta \, \sin \theta. \qquad \text{.... (52)}$$

Now R' in Eq.(52) is slightly different from R measured to the origin of the spherical coordinates, which coincides with the center of the dipole. In the far zone, $R \gg h$,

$$R' = (R^2 + z^2 - 2Rz \cos \theta)^{1/2} \cong R - z \cos \theta. \qquad \text{.... (53)}$$

The magnitude difference between $1/R'$ and $1/R$ is insignificant, but the approximate relation in Eq.(53) must be retained in the phase term. Using Eqs.(51) and (53) in Eq. (52) and integrating, we have

$$E_\theta = \eta_0 H_\phi$$

$$= j \frac{I_m \eta_0 \beta \, \sin \theta}{4\pi R} e^{-j\beta R} \int_{-h}^{h} \sin \beta(h - |z|) e^{j\beta z \cos \theta} \, dz. \qquad \text{.... (54)}$$

The integrand in Eq. (54) is the product of an even function of z, $\sin \beta(h - |z|)$, and

$$e^{j\beta z \cos \theta} = \cos (\beta z \cos \theta) + j \sin (\beta z \cos \theta),$$

where $\sin(\beta z \cos\theta)$ is an odd function of z. Integrating between symmetrical limits $-h$ and h, we find that only the part of the integrand containing the product $\sin \beta(h - |z|)\cos(\beta z \cos\theta)$ does not vanish. Equation (54) then reduces to

$$E_\theta = \eta_0 H_\phi = j \frac{I_m \eta_0 \beta \, \sin \theta}{2\pi R} e^{-j\beta R} \int_0^h \sin \beta(h - z) \cos (\beta z \cos \theta) \, dz$$

$$= \frac{j60 I_m}{R} e^{-j\beta R} F(\theta), \qquad \text{.... (55)}$$

where

$$F(\theta) = \frac{\cos (\beta h \cos \theta) - \cos \beta h}{\sin \theta}. \qquad \text{.... (56)}$$

The factor $|F(\theta)|$ is the E-plane ***pattern function*** of a linear dipole antenna. It describes the radiation pattern or the variation of the normalized far field, $|E_\theta|$, versus the angle θ. The exact shape of the radiation pattern represented by $|F(\theta)|$ in Eq. (56) depends on the value of $\beta h = 2\pi h/\lambda$ and can be quite different for different antenna lengths. The radiation pattern, however, is always symmetrical with respect to the $\theta = \pi/2$ plane. Figure-6 shows the E-plane patterns for four different dipole lengths measured in terms of wavelength: $2h/\lambda = 1/2, 1, 3/2$ and 2. The H-plane patterns are circles inasmuch as $F(\theta)$ is independent of ϕ. From the patterns in Fig.6, we see that the direction of maximum radiation tends to shift away from the $\theta = 90°$ plane when the dipole length approaches $3\lambda/2$. For $2h = 2\lambda$ there is no radiation in the $\theta = 90°$ plane.

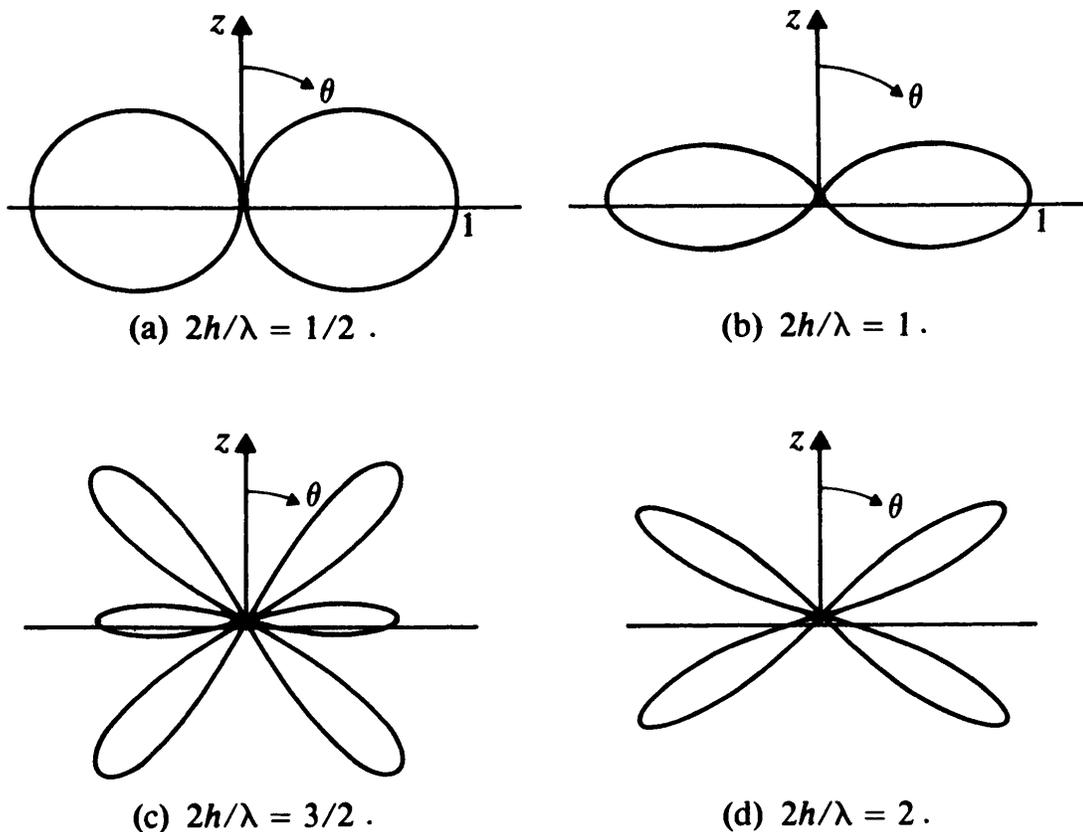(a) $2h/\lambda = 1/2$.

(b) $2h/\lambda = 1$.

(c) $2h/\lambda = 3/2$.

(d) $2h/\lambda = 2$.

Fig.6: E-plane radiation pattern for center-fed dipole antenna

2. The Half-Wave Dipole

The half-wave dipole having a length *2h = λ/2* is of particular practical importance because of its desirable pattern and impedance characteristics. We shall now examine its properties in more detail.

For a half-wave dipole, *βh = 2πh/λ = π/2*, the pattern function in Eq. (56) becomes

$$F(\theta) = \frac{\cos\left[(\pi/2)\cos\theta\right]}{\sin\theta}. \qquad \text{.... (57)}$$

This function has a maximum equal to unity at *θ = 90°* and has nulls at *θ = 0° and 180°*. The corresponding E-plane radiation pattern is sketched in Fig.6(a). The far-zone field phasors are, from Eq. (55),

$$E_\theta = \frac{j60I_m}{R} e^{-j\beta R} \left\{ \frac{\cos\left[(\pi/2)\cos\theta\right]}{\sin\theta} \right\} \qquad \text{.... (58)}$$

$$H_\phi = \frac{jI_m}{2\pi R} e^{-j\beta R} \left\{ \frac{\cos\left[(\pi/2)\cos\theta\right]}{\sin\theta} \right\}. \qquad \text{.... (59)}$$

The magnitude of the time-average Poynting vector is

$$\mathscr{P}_{av} = \frac{1}{2} E_\theta H_\phi^* = \frac{15I_m^2}{\pi R^2} \left\{ \frac{\cos\left[(\pi/2)\cos\theta\right]}{\sin\theta} \right\}^2. \qquad \text{.... (60)}$$

The total power radiated by a half-wave dipole is obtained by integrating over the surface of a great sphere:

$$P_r = \int_0^{2\pi} \int_0^{\pi} \mathscr{P}_{av} R^2 \sin\theta \, d\theta \, d\phi$$

$$= 30I_m^2 \int_0^{\pi} \frac{\cos^2\left[(\pi/2)\cos\theta\right]}{\sin\theta} \, d\theta. \qquad \text{.... (61)}$$

The integral in Eq. (61) can be evaluated numerically to give a value of *1.218*. Hence:

$$P_r = 36.54 I_m^2 \quad \text{(W)}, \qquad\qquad \text{.... (62)}$$

from which we obtain the radiation resistance of a free-standing half-wave dipole:

$$R_r = \frac{2P_r}{I_m^2} = 73.1 \quad (\Omega). \qquad\qquad \text{.... (63)}$$

Neglecting losses, we find that the input resistance of a thin half-wave dipole equals 73.1 (Ω) and that the input reactance is a small positive number that can be made to vanish when the dipole length is adjusted to be slightly shorter than $\lambda/2$. (As we have indicated before, the actual calculation of the input impedance is tedious and is beyond the scope of this chapter.)

The directivity of a half-wave dipole can be found by using Eq. (35). We have, from Eqs. (32) and (60),

$$U_{\text{max}} = R^2 \mathscr{P}_{av}(90°) = \frac{15}{\pi} I_m^2 \qquad\qquad \text{.... (64)}$$

and

$$D = \frac{4\pi U_{\text{max}}}{P_r} = \frac{60}{36.54} = 1.64, \qquad\qquad \text{.... (65)}$$

which corresponds to $[10\log_{10} 1.64]$ or $[2.15 \text{ (dB)}]$ referring to an omnidirectional radiator.

The <u>half-power beamwidth</u> of the radiation pattern is the angle between the two solutions of the equation

$$\frac{\cos\left[(\pi/2)\cos\theta\right]}{\sin\theta} = \frac{1}{\sqrt{2}}, \qquad 0 < \theta < \pi,$$

which can be solved either numerically or graphically to give a beamwidth of 78°. <u>Thus a half-wave dipole is only slightly more directive than a short Hertzian dipole that has a directivity of 1.76 (dB) and a beamwidth of 90°</u>.

3. Thin λ/4 Monopole over a conducting Ground

Since current is charge in motion, we can use the ***method of images*** and replace the conducting ground by the image of the vertical antenna. A little thought will convince us that the image of a vertical antenna carrying a current *I* is another vertical antenna. The image antenna has the same length, is equidistant from the ground, and carries the same current in the *same direction* as the original antenna. The electromagnetic field *in the upper half-space* due to the quarter-wave vertical antenna in Fig.7(a) is, then, the same as that of the half-wave antenna in Fig.7(b). The pattern function in Eq. (57) applies here for $0 \leq \theta \leq \pi/2$, and the radiation pattern drawn in dashed lines in Fig.7(b) is the upper half of that in Fig.6(a).
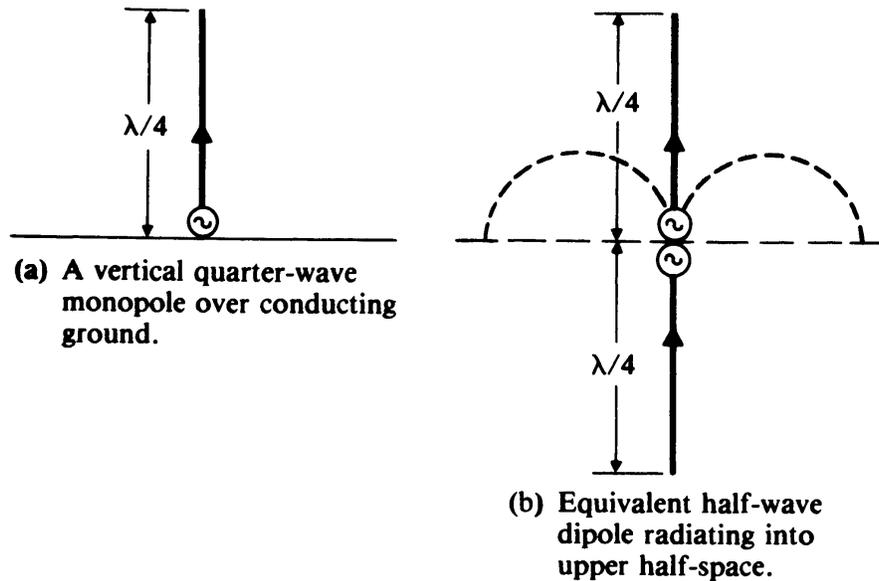

(a) A vertical quarter-wave monopole over conducting ground.

(b) Equivalent half-wave dipole radiating into upper half-space.

Fig.7: Quarter-wave monopole over a conducting ground and its equivalent half-wave dipole.

The magnitude of the time-average Poynting vector, $\mathcal{SP}_{av}$, in Eq. (60), holds for $0 \leq \theta \leq \pi/2$. Inasmuch as the quarter-wave antenna (a ***monopole)*** radiates only into the upper half-space, its total radiated power is only one-half that given in Eq. (62):

$$P_r = 18.27I_m^2 \quad (W).$$

Consequently, the radiation resistance is

$$R_r = \frac{2P_r}{I_m^2} = 36.54 \quad (\Omega), \qquad \dots (66)$$

which is one-half of the radiation resistance of a half-wave antenna in free-space.

To calculate directivity, we note that although the maximum radiation intensity U_{max} remains the same as that given in Eq. (64), the average radiation intensity is now $P_r/2\pi$. Thus,

$$D = \frac{U_{max}}{U_{av}} = \frac{U_{max}}{P_r/2\pi} = 1.64, \qquad \dots (67)$$

which is the same as the directivity of a half-wave antenna.

4. Effective Antenna Length

For thin linear antennas with a given current distribution it is sometimes convenient to define a quantity called the ***effective length,*** to which the far-zone field is proportional. Let us refer to the dipole antenna in Fig.5 and assume a general phasor current distribution *I(z)*. The far-zone field is then, from Eq. (54),

$$E_\theta = \eta_0 H_\phi = \frac{j30}{R} \beta e^{-j\beta R} \left\{ \sin\theta \int_{-h}^{h} I(z)e^{j\beta z \cos\theta} dz \right\}. \qquad \dots (68)$$

Let *I(0)* be the input current at the feed point of the antenna. We write Eq. (68) as

$$E_\theta = \eta_0 H_\phi = \frac{j30 I(0)}{R} \beta e^{-j\beta R} \ell_e(\theta), \qquad \text{.... (69)}$$

where

$$\ell_e(\theta) = \frac{\sin\theta}{I(0)} \int_{-h}^{h} I(z) e^{j\beta z \cos\theta} \, dz \qquad \text{.... (70)}$$

is the **effective length** of the transmitting antenna. (We will discuss the effective length of a receiving antenna presently.) As we see from Eq. (69), ℓ_e measures the effectiveness of the antenna as a radiator, and for a given current distribution the far-zone field is proportional to ℓ_e, which contains all the information about the directional properties of the antenna. In most practical situations the important value of the effective length is that at $\theta = \pi/2$, where

$$\ell_e = \frac{1}{I(0)} \int_{-h}^{h} I(z) \, dz \qquad \text{(m)}. \qquad \text{.... (71)}$$

Equation (71) indicates that ℓ_e is the length of an equivalent linear antenna with a uniform current $I(0)$ such that it radiates the same far-zone field in the $\theta = \pi/2$ plane.

EXAMPLE-5 Assume a sinusoidal current distribution on a center-fed, thin, straight half-wave dipole. Find its effective length. What is its maximum value?

Solution: For the assumed sinusoidal current distribution we use Eq. (51) for $I(z)$ and substitute it in Eq. (70), where $I(0) = I_m$ and $h = \lambda/4$. We have

$$\ell_e(\theta) = \sin\theta \int_{-\lambda/4}^{\lambda/4} \sin\beta\left(\frac{\lambda}{4} - |z|\right) e^{j\beta z \cos\theta} \, dz. \qquad \text{.... (72)}$$

The above integral has been evaluated in Eq. (56). Thus,

$$\ell_e(\theta) = \frac{2}{\beta}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]. \qquad \text{.... (73)}$$

The maximum value of $\ell_e(\theta)$ is at $\theta = \pi/2$, where the effective length is

$$\ell_e\left(\frac{\pi}{2}\right) = \frac{2}{\beta} = \frac{\lambda}{\pi}. \qquad \text{.... (74)}$$

We note from Eq. (74) that the maximum effective length of a half-wave dipole is less than its physical length $\lambda/2$.

A careful examination of Eq. (71) reveals a potential anomaly in the appearance of $I(0)$ in the denominator. When the half-length of a dipole is greater than $\lambda/4$ and approaches $\lambda/2$, $I(0)$ would be progressively less than I_m, which would not occur at $z = 0$. This could make ℓ_e much greater than $2h$. Thus the definition of effective length as given in Eqs.(70) and (71) is meaningful only for relatively short antennas that have a current maximum at the feed point.

The effective length of a receiving linear antenna is defined as the ratio of the open-circuit voltage V_{oc} induced at the antenna terminals and the electric field intensity $E_i = |\mathbf{E_i}|$ at the antenna that induces it:

$$\ell_e(\theta) = -\frac{V_{oc}}{E_i}, \qquad \text{.... (75)}$$

where the negative sign is to conform with the convention that the electric potential increases in a direction opposite to that of the electric field. The situation is illustrated in Fig.8. We will assume that $\mathbf{E_i}$, lies in the plane of incidence, since the component of $\mathbf{E_i}$, normal to the antenna does not induce a voltage across the antenna terminals. Obviously, the open-circuit voltage V_{oc} depends on E_i θ, and βh in a complicated way. It is possible

to use a reciprocity theorem to prove formally that *the effective length of an antenna for receiving is the same as that for transmitting*.

If the incoming electric field $\mathbf{E_i}$ is not parallel to the dipole, there is a polarization mismatch, and the magnitude of the open-circuit voltage will be

$$\left|V_{oc}\right| = \left|\boldsymbol{\ell}_e \cdot \mathbf{E}_i\right|, \qquad\qquad \text{.... (76)}$$

where $\boldsymbol{\ell}_e$ denotes the vector effective length. Obviously, $|V_{oc}|$ will be maximum when $\mathbf{E_i}$ is parallel to the dipole and will be zero if $\mathbf{E_i}$ is perpendicular to the dipole.
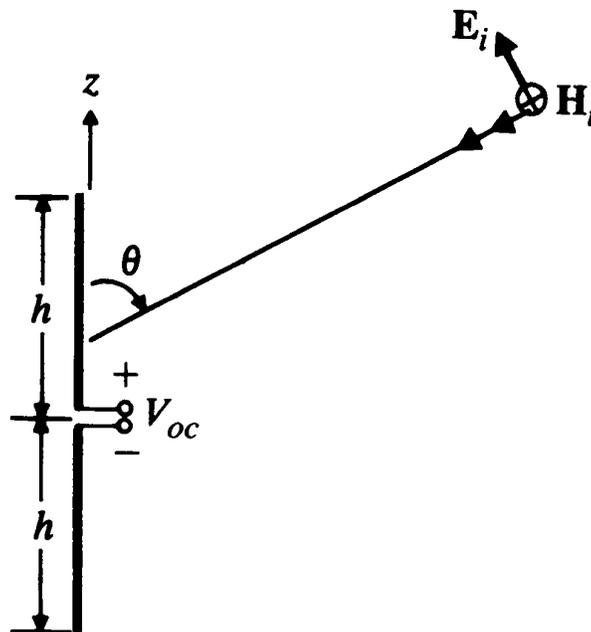

Fig.8: A linear antenna in the receiving mode.