Chapter-1

Electromagnetic Radiation

1. Introduction

Antennas are structures designed for radiating electromagnetic energy effectively in a prescribed manner. Without an efficient antenna, electromagnetic energy would be localized, and wireless transmission of information over long distances would be impossible.

An antenna may be a single straight wire or a conducting loop excited by a voltage source, an aperture at the end of a waveguide, or a complex array of these properly arranged radiating elements. Reflectors and lenses may be used to accentuate certain radiation characteristics. Among radiation characteristics of importance are field pattern, directivity, impedance, and bandwidth. These parameters will be examined when particular antenna types are studied.

To study electromagnetic radiation, we must call upon our knowledge of Max-well's equations and relate electric and magnetic fields to time-varying charge and current distributions. A primary difficulty of this task is that the charge and current distributions on antenna structures resulting from given excitations are generally unknown and very difficult to determine. In fact, the geometrically simple case of a straight conducting wire (linear antenna) excited by a voltage source in the middle has been a subject of extensive research for many years, and the exact charge and current distributions on a wire of a finite radius are extremely complicated even when the wire is assumed to be perfectly conducting. Fortunately, the radiation field of such an antenna is relatively insensitive to slight deviations in the current distribution, and a physically plausible approximate current on the wire yields useful results for nearly all practical purposes. We will examine the radiation properties of linear antennas with assumed currents.

By combining Maxwell's equations we can derive nonhomogeneous wave equations in **E** and in **H**. However, these equations tend to involve the charge and current densities in a complicated way. It is generally simpler to solve for the auxiliary potential functions **A** and *V* first. Using **A** and *V* in the following two equations:

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad \text{(T)}$$

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \qquad \text{(V/m).}$$

we can determine **H** and **E**. For harmonic time variation in a simple medium we have:

$$\mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A} \qquad \text{.... (1)}$$

$$\mathbf{E} = -\nabla V - j\omega \mathbf{A}. \qquad \text{.... (2)}$$

The potential functions **A** and *V* are themselves solutions of nonhomogeneous wave equations. For harmonic time dependence the ***phasor retarded potentials*** are,

$$\mathbf{A} = \frac{\mu}{4\pi} \int_{V'} \frac{\mathbf{J} e^{-jkR}}{R} \, dv', \qquad \text{.... (3)}$$

$$V = \frac{1}{4\pi\epsilon} \int_{V'} \frac{\rho e^{-jkR}}{R} \, dv', \qquad \text{.... (4)}$$

where $k = \omega\sqrt{\mu\epsilon} = 2\pi/\lambda$ is the wavenumber.

Of course, **A** and *V* are related by the Lorentz condition for potentials:

$$\nabla \cdot \mathbf{A} + j\omega\mu\epsilon V = 0.$$

and **J** and ρ are related by the equation of continuity:

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}.$$

or,

$$\nabla \cdot \mathbf{J} = -j\omega\rho. \qquad \text{.... (5)}$$

Hence there is no need for evaluating the integrals in both Eqs. (3) and (4). As a matter of fact, since **E** and **H** are related by the following equation:

$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E},$$

Thus,

$$\mathbf{E} = \frac{1}{j\omega\epsilon} \nabla \times \mathbf{H} \qquad \text{.... (6)}$$

We follow three steps in the determination of electromagnetic fields from a current distribution:

(1) determine **A** from **J** using Eq. (3);

(2) find **H** from **A** using Eq. (1);

(3) find **E** from **H** using Eq. (6).

Note that only Step 1 requires integration and that Steps 2 and 3 involve only straightforward differentiation. This is the procedure we will use in finding the radiation pattern of antennas.

We will first study the radiation fields and characteristic properties of an elemental electric dipole and of a small current loop (or magnetic dipole). We then consider finite-length thin linear antennas, of which the half-wavelength dipole is an important special case. The radiation characteristics of a linear antenna are largely determined by its length and the manner in which it is excited. To obtain more directivity and other desirable properties, a number of such antennas may be arranged together to form an ***antenna array.*** The geometrical configuration, the spacings between the array elements, as well as the relative amplitudes and phases of the excitations in the elements all affect the field pattern of the array. Some basic properties of simple arrays will be considered.

When an antenna is used as a receiving device, its function is to collect energy from an incoming electromagnetic wave and deliver it to a receiver. Any antenna that is useful for radiation is also useful for reception. We will use the ***reciprocity theorem*** to show that the pattern, directivity, input impedance, effective height, and effective aperture of an antenna are the same for transmitting as for receiving.

2- Radiation Fields of Elemental Dipoles (Hertzian Dipole)

In this section we study the radiation fields of the simplest types of all radiating systems-namely, ***elemental oscillating electric*** and ***magnetic dipoles***. We will find that the field solutions for electric and magnetic dipoles are **duals** of each other. As a consequence, the radiation properties of one can be deduced from those of the other without recalculation.

2.1 The Elemental Electric Dipole

Consider the elemental oscillating electric dipole (in free space), as shown in Fig.1, which consists of a short conducting wire of length *dl* terminated in two small conductive spheres or disks (capacitive loading). We assume the current in the wire to be uniform and to vary sinusoidally with time:

$$i(t) = I \cos \omega t = \mathcal{R}e\left[I e^{j\omega t}\right]. \qquad \text{.... (7)}$$

Since the current vanishes at the ends of the wire, charge must be deposited there. The relation between the charge and the current is

$$i(t) = \pm\frac{dq(t)}{dt}. \quad \text{.... (8)}$$

In phasor notation,

$$q(t) = \mathcal{R}e[Qe^{j\omega t}],$$

Thus, we have

$$I = \pm j\omega Q \quad \text{.... (9)}$$

$$Q = \pm\frac{I}{j\omega}, \quad \text{.... (10)}$$

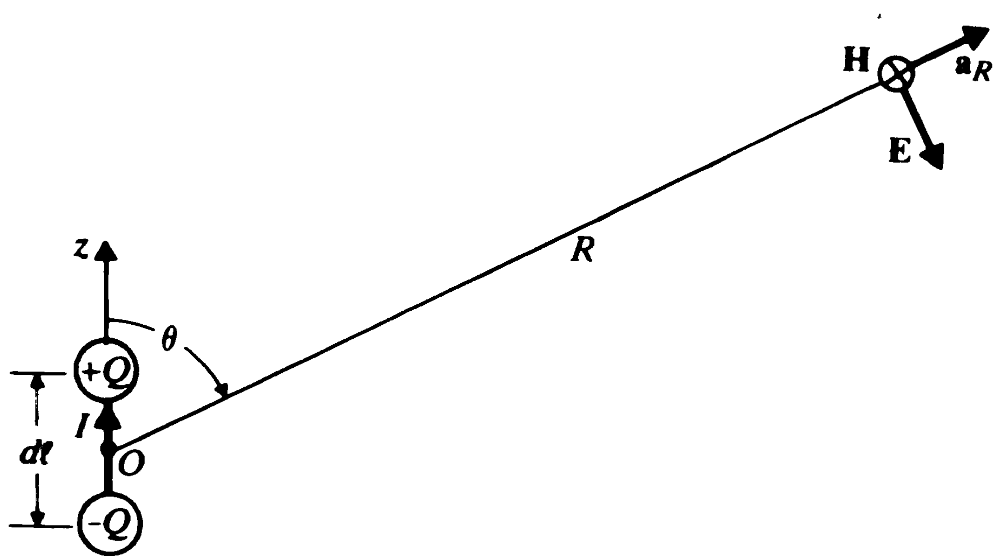

Fig.1: A Hertzian dipole

where, for the indicated current direction in Fig.1, the positive sign is for the charge on the upper end and the negative sign for the charge on the lower end. The pair of equal and opposite charges separated by a short distance effectively forms an electric dipole with a vector phasor electric moment

$$\mathbf{p} = \mathbf{a}_z Q\, d\ell \qquad (\mathrm{C \cdot m}). \qquad \text{.... (11)}$$

Such an oscillating dipole is called a ***Hertzian dipole.***

To determine the electromagnetic field of a Hertzian dipole, we follow the three steps outlined in Section-1. The phasor representation of the retarded vector potential is, from Eq. (3),

$$\mathbf{A} = \mathbf{a}_z \frac{\mu_0 I\, d\ell}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right), \qquad \text{.... (12)}$$

where $\beta = k_0 = \omega/c = 2\pi/\lambda.$ Since

$$\mathbf{a}_z = \mathbf{a}_R \cos\theta - \mathbf{a}_\theta \sin\theta, \qquad \text{.... (13)}$$

the spherical components of

$$\mathbf{A} = \mathbf{a}_R A_R + \mathbf{a}_\theta A_\theta + \mathbf{a}_\phi A_\phi$$

are given by:

$$A_R = A_z \cos\theta = \frac{\mu_0 I\, d\ell}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\cos\theta, \qquad \text{.... (14a)}$$

$$A_\theta = -A_z \sin\theta = -\frac{\mu_0 I\, d\ell}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\sin\theta, \qquad \text{.... (14b)}$$

$$A_\phi = 0. \qquad \text{.... (14c)}$$

From the geometry of Fig.1 we expect no variation with respect to the coordinate ϕ. By using cross product in spherical coordinate, we have:

$$\mathbf{H} = \frac{1}{\mu_0}\nabla \times \mathbf{A} = \mathbf{a}_\phi \frac{1}{\mu_0 R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$= -\mathbf{a}_\phi \frac{I\,d\ell}{4\pi}\beta^2 \sin\theta\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2}\right]e^{-j\beta R}. \qquad \text{.... (15)}$$

The electric field intensity can be obtained from Eq. (6):

$$\mathbf{E} = \frac{1}{j\omega\epsilon_0}\nabla \times \mathbf{H}$$

$$= \frac{1}{j\omega\epsilon_0}\left[\mathbf{a}_R \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}(H_\phi \sin\theta) - \mathbf{a}_\theta \frac{1}{R}\frac{\partial}{\partial R}(RH_\phi)\right], \qquad \text{.... (16)}$$

which gives

$$E_R = -\frac{I\,d\ell}{4\pi}\eta_0\beta^2 2\cos\theta\left[\frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3}\right]e^{-j\beta R}, \qquad \text{.... (16a)}$$

$$E_\theta = -\frac{I\,d\ell}{4\pi}\eta_0\beta^2 \sin\theta\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3}\right]e^{-j\beta R}, \qquad \text{.... (16b)}$$

$$E_\phi = 0, \qquad \text{.... (16c)}$$

where $\eta_0 = \sqrt{\mu_0/\epsilon_0} \cong 120\pi\ (\Omega)$.

Equations (15) and (16) constitute the electromagnetic field of a Hertzian dipole. Note that in deriving these expressions we used only the current in the dipole to find the vector potential **A**; the charges at the ends of the dipole did not enter into the calculations. We could, however, take an alternative approach by finding both **A** from Idl, as in Eq. (12), and the scalar potential V from the pair of equal and opposite charges using Eq. (4). The electric field intensity could then be determined from Eq. (2), instead of from Eq. (6). The result would be exactly the same as that obtained above.

- ***Near Field:*** In the region near to the Hertzian dipole (in the ***near zone),*** *βR=2πR/λ<<1*, the leading term in Eq. (15) is

$$H_\phi = \frac{I\,d\ell}{4\pi R^2}\sin\theta, \qquad \text{.... (17)}$$

where we have approximated the factor $e^{-j\beta R} = 1 - j\beta R - (\beta R)^2/2 + \cdots$ by unity. Equation (17) is exactly what would be obtained for the magnetic field intensity due to a current element *Idl* by applying the Biot-Savart law in magnetostatics.

The leading near-zone terms for the electric field intensity are, from Eqs. (16a) and (16b),

$$E_R = \frac{p}{4\pi\epsilon_0 R^3}\,2\cos\theta \qquad \text{.... (18a)}$$

and

$$E_\theta = \frac{p}{4\pi\epsilon_0 R^3}\sin\theta, \qquad \text{.... (18b)}$$

where the phasor relations (10) and (11) have been used. These expressions are identical to those of the electric field intensity due to an elemental electric dipole of a moment ***p*** in the z-direction, obtained by an application of the laws of electrostatics. The ***near-zone fields*** of an oscillating time-varying dipole are then ***quasi-static fields.***

- ***Far Field***: The region where $\beta R = 2\pi R/\lambda \gg 1$ is the ***far zone.*** The far-zone leading terms in Eqs. (15) and (16) are:

$$H_\phi = j\frac{I\,d\ell}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\beta \sin\theta \qquad \text{(A/m)}, \qquad \text{.... (19a)}$$

$$E_\theta = j\frac{I\,d\ell}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\eta_0\beta \sin\theta \qquad \text{(V/m)}. \qquad \text{.... (19b)}$$

Several important observations can be made on these ***far-zone fields.*** First, $\boldsymbol{E_\theta}$ and $\boldsymbol{H_\phi}$ are in space quadrature and in time phase. Second, their ratio $E_\theta/H_\phi=\eta_0$ is a constant equal to the intrinsic impedance of the medium (which is, in the present case, free space). The far-zone fields, then, have the same properties as those of a plane wave. This is not unexpected, since at very large distances from the dipole a spherical wavefront closely resembles a plane wavefront.

A third observation from Eqs. (19a,b) is that the magnitude of the far-zone fields varies inversely with the distance from the source. The phase of both E_θ and H_ϕ is a periodic function of R with a period that is the wavelength:

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f}. \qquad \text{.... (20)}$$

Note that the far-zone condition βR >>1 translates into $R>>\lambda/2\pi$; hence one has to be farther away from the dipole at lower frequencies in order to be in the far zone. (Other characteristics of far-zone fields will be discussed in Section-3).

2.2 The Elemental Magnetic Dipole

Let us now consider a small filamentary loop of radius *b* carrying a uniform time- harmonic current *i(t) = I cosωt* around its circumference, as shown in Fig.2. This is an elemental magnetic dipole with a vector phasor magnetic moment

$$\mathbf{m} = \mathbf{a}_z I\pi b^2 = \mathbf{a}_z m \qquad (\mathrm{A \cdot m^2}). \quad \text{…. (21)}$$

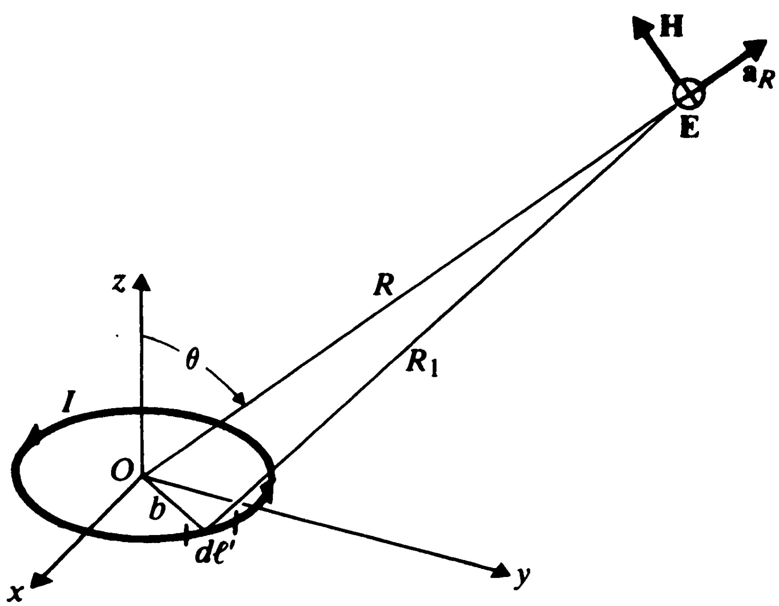

Fig.2: A magnetic dipole

To determine the electromagnetic field, we first find the vector potential. The procedure is as follows:

$$\mathbf{A} = \frac{\mu_0 I}{4\pi} \oint \frac{e^{-j\beta R_1}}{R_1}\, d\boldsymbol{\ell}'. \quad \text{…. (22)}$$

The integral in Eq.(22) is relatively difficult to carry out because R_1 changes with the location of $d\ell$ on the loop. For a small loop, the exponential factor in the numerator can be written as:

$$e^{-j\beta R_1} = e^{-j\beta R} e^{-j\beta(R_1 - R)}$$
$$\cong e^{-j\beta R}[1 - j\beta(R_1 - R)]. \qquad \text{.... (23)}$$

Substitution of Eq.(23) in Eq.(22) yields approximately:

$$\mathbf{A} = \frac{\mu_0 I}{4\pi} e^{-j\beta R}\left[(1 + j\beta R)\oint \frac{d\ell'}{R_1} - j\beta \oint d\ell'\right]. \qquad \text{.... (24)}$$

The second integral in Eq.(24) obviously vanishes. We have:

$$\mathbf{A} = \mathbf{a}_\phi \frac{\mu_0 m}{4\pi R^2}(1 + j\beta R)e^{-j\beta R} \sin\theta. \qquad \text{.... (25)}$$

The electric and magnetic field intensities can be determined by straightforward differentiation using Eq.(6) and (1) respectively:

$$E_\phi = \frac{j\omega\mu_0 m}{4\pi}\beta^2 \sin\theta\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2}\right]e^{-j\beta R}, \qquad \text{.... (26a)}$$

$$H_R = -\frac{j\omega\mu_0 m}{4\pi\eta_0}\beta^2 2\cos\theta\left[\frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3}\right]e^{-j\beta R}, \qquad \text{.... (26b)}$$

$$H_\theta = -\frac{j\omega\mu_0 m}{4\pi\eta_0}\beta^2 \sin\theta\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3}\right]e^{-j\beta R}. \qquad \text{.... (26c)}$$

Comparison of Eqs. (26a,b,c) with Eqs. (15) and (16a,b) reveals immediately the **dual** nature of the electromagnetic fields of electric and magnetic dipoles.

Let ($\mathbf{E_e}$, $\mathbf{H_e}$) denote the electric and magnetic fields of the electric dipole and ($\mathbf{E_m}$, $\mathbf{H_m}$) the electric and magnetic fields of the magnetic dipole. We have

$$\mathbf{E}_e = \eta_0 \mathbf{H}_m \qquad \text{.... (27)}$$

and

$$\mathbf{H}_e = -\frac{\mathbf{E}_m}{\eta_0} \qquad \text{.... (28)}$$

if the electric and magnetic dipole moments are related as follows:

$$I\,d\ell = j\beta m, \qquad \text{.... (29)}$$

where Equations (27) and (28) are results expected from the principle of **<u>duality</u>**. Thus $\beta = \omega\mu_0/\eta_0 = \omega\sqrt{\mu_0\epsilon_0}$. Hertzian electric dipole and elemental magnetic dipole are **<u>dual devices</u>**, and their electromagnetic fields are <u>dual solutions</u> of source-free Maxwell's equations. As a consequence of this duality, the discussions about the nature of the near and far fields of an electric dipole apply to the dual quantities of a magnetic dipole. In particular, the far-zone *(βR* >>1) fields of a magnetic dipole are

$$E_\phi = \frac{\omega\mu_0 m}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\beta\sin\theta \qquad \text{(V/m)}, \qquad \text{.... (30a)}$$

$$H_\theta = -\frac{\omega\mu_0 m}{4\pi\eta_0}\left(\frac{e^{-j\beta R}}{R}\right)\beta\sin\theta \qquad \text{(A/m)}. \qquad \text{.... (30b)}$$

We can see that the far-field intensities vary inversely as R and their ratio E_ϕ/H_θ equals the intrinsic impedance η_0 of free space.

Examination of the far-field $\boldsymbol{E_\theta}$ in Eq.(19b) of the electric dipole and in Eq. (30a) of the magnetic dipole reveals that they have the same pattern function $|\sin\theta|$ and are in both space and time quadrature. Thus it is possible to combine electric and magnetic dipoles to form an antenna that produces circular polarization.