SECTION1
((Understanding and computing simple interest))

Definition

Interest: is the rental fee charged by a lender to a business or an individual for the use of money .

The amount of interest charged is determined by three factors:

1-principal : Is the amount of money being borrowed or invested

2- **Rate** : the percent of interest charged on the money per year .

3- **Time** : Is the length of time of the loan, known as **time**.

Remark : The manner in which the interest is computed is an additional factor that influences the amount of interest . The two most commonly used methods in business today for computing interest are simple and compound.

Definition : (simple interest)

means that the interest is calculated *only once* for the entire time period of the loan. At the end of the time period, the borrower repays the principal plus the Interest .

Remark: Simple interest loans are usually made for short periods of time, such as a few days, weeks or month .

Definition : (Compound interest)

means that the interest is calculated more than once during the time period of the loan. When compound interest is applied to a loan, each succeeding time

period accumulates interest on the previous interest in addition to interest on the principal. Compound interest loans are generally for time periods of a year or longer.

EXERCISE 1: Find the amount of interest on each of the following loans.

principal	Rate %	Time
a) $ 4,000	7	$2\frac{1}{4}$
b) $ 45,000	$9\frac{3}{4}$	3 months
c) $130,000	10.4	42 months

3-2 ((CALCULATING SIMPLE INTEREST FOR LOANS WITH TERMS OF DAYS BY USING THE EXACT INTEREST AND ORDINARY INTEREST METHODS))

There are two methods for calculating the time factor, *T*, when applying the simple interest formula using days. Because time must be expressed in years, loans whose terms are given in days must be made into a fractional part of a year. This is done by dividing the days of a loan by the number of days in a year.

Simple Interest Formula __ Days

Exact Interest

The first method for calculating the time factor is known as **exact interest**. Exact interest uses *365 days* as the time factor denominator.

$$\mathbf{T} = \frac{number\ of\ days\ of\ a\ loans}{365}$$

Ordinary Interest

The second method for calculating the time factor is known as **ordinary interest**. Ordinary interest uses *360 days* as the denominator of the time factor.

$$\boldsymbol{T = \frac{number\ of\ days\ of\ a\ loans}{360}}$$

Example 2

Using the exact interest method, what is the amount of interest on a loan of $4,000 at 7% interest for 88 days?

SOLUTION

Because we are looking for exact interest, we will use 365 days as the denominator of the time factor

Interest = Principal × Rate × Time

Interest = 4,000 × .07 × $\frac{88}{365}$

Interest = $67.51

3-1 (Computing Simple Interest For Loans With Terms Of Years Or Months)

Simple interest is calculated by using a formula known as the simple interest formula. It is stated as

Interest = Principal × Rate × Time

I = PRT

Remark :When using the simple interest formula, the time factor, *T*, must be expressed in years or fractions of years .

(Simple interest –years or month)

Years:

When the time period of a loan is a year or longer, use the number of years as the time factor, converting fractional parts to decimals. For example, the time factor for a 2-year loan is 2 , 3-years is 3, $1\frac{1}{2}$ years is 1.5 ,$4\frac{3}{4}$ years is 4.75 and so on .

Months

When the time period of a loan is for a specified number of months, express the time factor

as a fraction of a year. The number of months is the numerator, and 12 months (1 year) is the denominator . A loan for 1 month would have a time factor of , $\frac{1}{12}$; a loan for 5 months would , $\frac{5}{12}$-and

18 months would use $\frac{18}{12}$ or $1\frac{1}{2}$ written as 1.5

EXAMPLE1

a. What is the amount of interest for a loan of $8,000 at 9% interest for 1 year?

Solution

To solve this problem, we apply the simple interest formula:

Interest = Principal × Rate × Time

Interest = 8,000 × 9% × 1

Interest = 8,000 × .09 × 1

Interest = $720

b. What is the amount of interest for a loan of $16,500 at $12\frac{1}{2}$ interest for 7 months?

SOLUTION

In this example, the rate is converted to ٠.125 and the time factor is expressed as a fraction of a year , $\frac{7}{12}$

Interest = Principal × Rate × Time

Interest = 16,500 × .125 × $\frac{7}{12}$

Interest = $1,203.13

EXERCISE 2

Joe Hale goes to a credit union and borrows $23,000 at 8% for 119 days. If the credit union calculates interest by the exact interest method, what is the amount of interest on the loan?

Example 3

Using the ordinary interest method, what is the amount of interest on a loan of $19,500 at 6% interest for 160 days?

SOLUTION

Because we are looking for ordinary interest, we will use 360 days as the denominator of the time factor in the simple interest formula:

Interest = Principal × Rate × Time

Interest = 19,500 × .06 × $\frac{160}{360}$

Interest = $520

EXERCISE 3

Karen Mitroff goes to the bank and borrows $15,000 at 91 $\frac{1}{2}$ % for 250 days. If the bank uses the ordinary interest method, how much interest will Karen have to pay?

Example 4

Allied Bank loaned Checkpoint Industries money at 8% interest for 90 days. If the amount of interest was $4,000, use the ordinary interest method to find the amount of principal borrowed

SOLUTION

To solve for the **principal**, we use the formula

$$Principal = \frac{Interest}{Rate \times Time} \qquad P = \frac{I}{R \times T}$$

$$P = \frac{4000}{0.08 \times \frac{90}{360}} \quad , P = \frac{4000}{0.02} \quad , \quad P = \$200{,}000$$

EXERCISE 4

Telex Electronics borrowed money at 9% interest for 125 days. If the interest charge was $560, use the ordinary interest method to calculate the amount of principal of the loan.

Example 5

Using the ordinary interest method, what is the rate of interest on a loan of $5,000 for 125 days if the amount of interest is $166? Round your answer to the nearest
hundredth of a percent.

SOLUTION

Remark: When we solve the simple formula for rate, the answer will be a decimal that must be converted to a percent. To solve for the **Rate,** we use the formula

$$Rate = \frac{Interest}{Principal \times Time} \qquad R = \frac{I}{P \times T}$$

$$R = \frac{166}{5000 \times \frac{125}{360}} \;, \; R = \frac{166}{1736.11111} \;, \quad R = 0.095616 \;, \quad R = 9.56\%$$

Exercise 5

Using the ordinary interest method, what is the rate of interest on a loan of $25,000 for 245 days if the amount of interest is $1,960? Round your answer to the nearest hundredth of a percent
.

Example 6

What would be the time period of a loan for $7,600 at 11% ordinary interest if the amount of interest is $290?

Solution

Remark :

When solving the simple interest formula for time, a whole number in the answer represents years and a decimal represents a portion of a year. The decimal should be converted to days by multiplying it by 360 for ordinary interest or by 365 for exact interest. Lending institutions consider any part of a day to be a full day. Therefore, any fraction of a day is rounded up to the next higher day even if it is less than 5 . To solve for the **Time**, we use the formula ,

$$T = \frac{Interest}{Principal \times RATE} \;, \qquad T = \frac{I}{P \times R} \;,$$

$$T = \frac{290}{7600 \times 0.11} \;, \qquad T = = \frac{290}{836} = 0.3468899$$

Because the answer is a decimal, the time is less than 1 year . Using ordinary interest, we multiply the entire decimal by 360 to
find the number of days of the loan.

$T = .3468899 \times 360$, Time = 124.8 days, or 125 days

EXERCISE 6

What is the time period of a loan for $15,000 at 9.5% ordinary interest if the amount of interest is $650?

3-3 ((Calculating The Maturity Value Of A Loan))

When the time period of a loan is over, the loan is said to mature. At that time, the borrower repays the original principal plus the interest . The total payback of principal and interest is known as the **maturity value** of a loan . Once the interest has been calculated, the maturity value can be found by using the formula:

Maturity value = Principal + Interest ,

$$MV = P + I$$

Remark : Maturity value can also be calculated directly without first calculating the interest by using the following formula :

:

Maturity value = Principal(1 + Rate × Time) , $MV = P(1 + RT)$

Example 7

What is the maturity value of a loan for $25,000 at 11% for $2\frac{1}{2}$% years?

Solution

Because this example asks for the maturity value, not the amount of interest, we will use the formula for finding maturity value directly

$MV = P(1 + RT)$.

MV = 25,000(1 +0 .11 × 2.5) ,

MV= 25,000(1.275)

MV = $31,875

EXERCISE 7

a. What is the amount of interest and the maturity value of a loan for $15,400 at $60\frac{1}{2}$% simple interest for 24 months?

b. Apollo Air Taxi Service borrowed $450,000 at 8% simple interest for 9 months to purchase a new airplane. find the maturity value of the loan.

3- 4 ((CALCULATING THE NUMBER OF DAYS OF A LOAN))

The first day of a loan is known as the **loan date**, and the last day is known as the **due date** or **maturity date**.

When these dates are known, the number of days of the loan can be calculated by using the "Days in Each Month"
chart and the steps that follow.

Days In Each Month

28 Days	30 Days	31 Days
February (29 leap year)	April June September November	January March May July August October December

STEPS FOR DETERMINING THE NUMBER OF DAYS OF A LOAN

1- Determine the number of days remaining in the first month by subtracting the loan date from the number of days in that month.

2-List the number of days for each succeeding whole month.

3- List the number of loan days in the last month.

4- Add the days from Steps 1, 2, and 3.

Example 8

Kevin Krease borrowed money from the Charter Bank on August 18 and repaid the loan on November 27.

What was the number of days of the loan?

Solution

Step 1. Days remaining in first month Aug. 31

Aug. - 81

____

13 August

13 days

Step 2 . Days in succeeding whole months September

30 days

October

31 days

STEP 3 . Days of a loan in last month November +

27 days

____

Step 4 . Add the days total

101 days

EXERCISE 8

a. A loan was made on April 4 and had a due date of July 18. What was the number of days of the loan?

b. Ryan McPherson borrowed $3,500 on June 15 at 11% interest. If the loan was due on October 9, what was the amount of interest on Ryan's loan using the exact interest method ?

3-5 ((DETERMINING THE MATURITY DATE OF A LOAN))

When the loan date and number of days of the loan are known, the maturity date can be found as follows:

STEPS FOR DETERMINING THE MATURITY DATE OF A LOAN

Step 1 . Find the number of days remaining in the first month by subtracting the loan date from the number of days in that month.

Step 2 . Subtract the days remaining in the first month (Step 1) from the number of days of the loan.

Step 3 . Continue subtracting days in each succeeding whole month until you reach a month with a difference less than the total days in that month. At that point , the maturity date will be the day that corresponds to the difference.

Example 9

What is the maturity date of a loan taken out on April 14 for 85 days?

Solution

Step 1. Days remaining in first month 30 days in April

_ 14 loan date April 14

______

Days remaining in April 16

Step 2 . Subtract remaining days in first month from days of the loan 85

_ 16 days remaining in April

______

Difference 69

Step 3 . Subtract succeeding whole months 69 Difference

_ 31 days in may

____

Difference 38

38 difference

_ 30 days in June

____

Difference 8

At this point, the difference, 8, is less than the number of days in the next month, July; therefore, the maturity date is July 8.

Exercise 9

a. What is the maturity date of a loan taken out on September 9 for 125 days?

b. On October 21, Jill Voorhis went to the Regal National Bank and took out a loan for $9,000 at 10% ordinary interest for 80 days. What is the maturity value and maturity date of this loan?

3-6((CALCULATING LOANS INVOLVING PARTIAL PAYMENTS BEFORE MATURITY))

Frequently, businesses and individuals who have borrowed money for a specified length of time find that they want to save

some interest by making one or more partial payments on the loan before the maturity date. The most commonly used method for this calculation is known as the **U.S. rule**. The rule states that when a partial payment is made on a loan, the payment is first

used to pay off the accumulated interest to date and the balance is used to reduce the principal. In this application, the ordinary interest method (360 days) will be used for all calculations .

STEPS FOR CALCULATING MATURITY VALUE OF A LOAN AFTER ONE OR MORE PARTIAL PAYMENTS

Step 1 . Using the simple interest formula with *ordinary* interest, compute the amount of interest due from the date of the loan to the date of the partial payment.

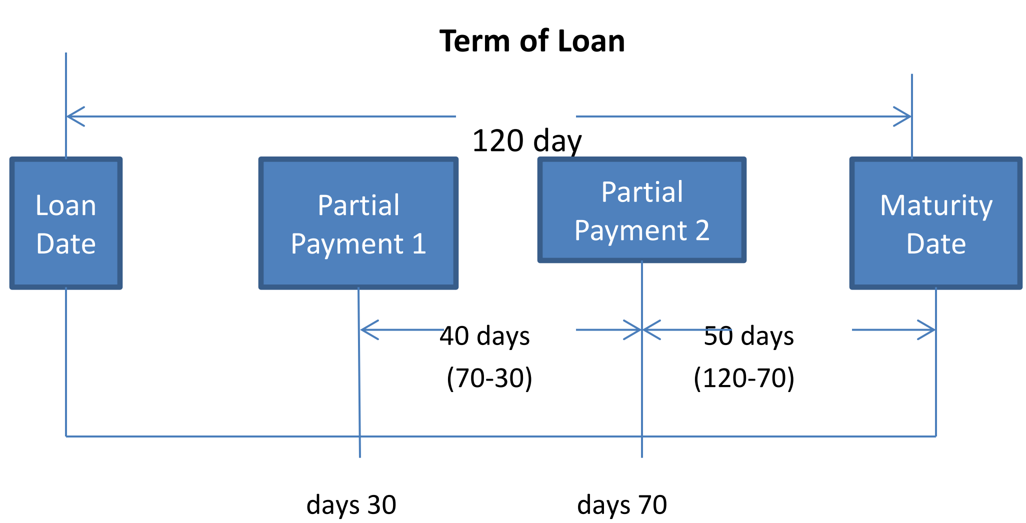
STEP 2 . Subtract the interest from Step 1 from the partial payment. This pays the interest to date.

Step 3 . Subtract the balance of the partial payment after Step 2 from the original principal of the loan. This gives the adjusted principal.

Step 4 . If another partial payment is made, repeat Steps 1, 2, and 3 using the adjusted principal and the number of days since the last partial payment.

Step 5 . The maturity value is computed by adding the interest since the last partial payment to the adjusted principal.

Remark: To help you visualize the details of a loan with partial payments, construct a timeline such as the one illustrated in Exhibit 10-1



Example 10

Ray Windsor borrowed $10,000 at 9% interest for 120 days. On day 30, Ray made a partial payment of $2,000. On day 70, he made a second partial payment of $3,000. What is the maturity value of the loan after the partial payments?

Solution

Step 1. Compute the interest from the date of the loan to the partial payment. In this problem, the first partial payment was made on day 30.

$I = PRT$, $I = 10{,}000 \times 0.09 \times \frac{30}{360} = 75$, $I = \$75$

Step 2. Subtract the interest from the partial payment.

$2,000 Partial payment
− 75 Accumulated interest
____
$1,925 Amount of partial payment left to reduce the principal

Step 3. Reduce the principal.

$10,000 Original principal
− 1,925 Amount of partial payment used to reduce principal
____
$8,075 Adjusted principal

Step 4. A second partial payment of $3,000 was made on day 70. We now repeat Steps 1, 2, and 3 to credit the second partial
payment properly. Remember, use the adjusted principal

and 40 days (70 − 30 = 40) for this calculation.

Step 1.

$I = PRT$,

$I = \$8{,}075 \times .09 \times \frac{40}{360}$

$I = \$80.75$ Accumulated interest since last partial payment

Step 2.

$3,000.00 Partial payment
− 80.75 Accumulated interest
____
$2,919. Amount of partial payment left to reduce principal

Step 3.

$8,075.00 Principal
− 2,919.25 Amount of partial payment used to reduce principal
____
$5,155.75 Adjusted principal

Step 5.

Once all partial payments have been credited, we find the maturity value of the loan by calculating the interest due from the last partial payment to the maturity date and adding it to the last adjusted principal.

Note: The last partial payment was made on day 70 of the loan; therefore, 50 days remain on the loan (120 − 70 = 50 days).

$I = PRT$

$I = \$5{,}155.75 \times 0.09 \times \frac{50}{360}$

I = $64.45 Interest from last partial payment to maturity date

Maturity Value = Principal + Interest

Maturity Value = $5,155.75 + $64.45

Maturity Value = $5,220.20

<u>Exercise 10</u>

Rita Peterson borrowed $15,000 at 12% ordinary interest for 100 days. On day 20 of the loan, she made a partial payment of $4,000. On day 60, she made another partial payment of $5,000. What is the maturity value of the loan after the partial payments?

SECTION 2
((COMPOUND INTEREST AND PERESENT VALUE))

COMPOUND INTEREST – The Time Value Of Money

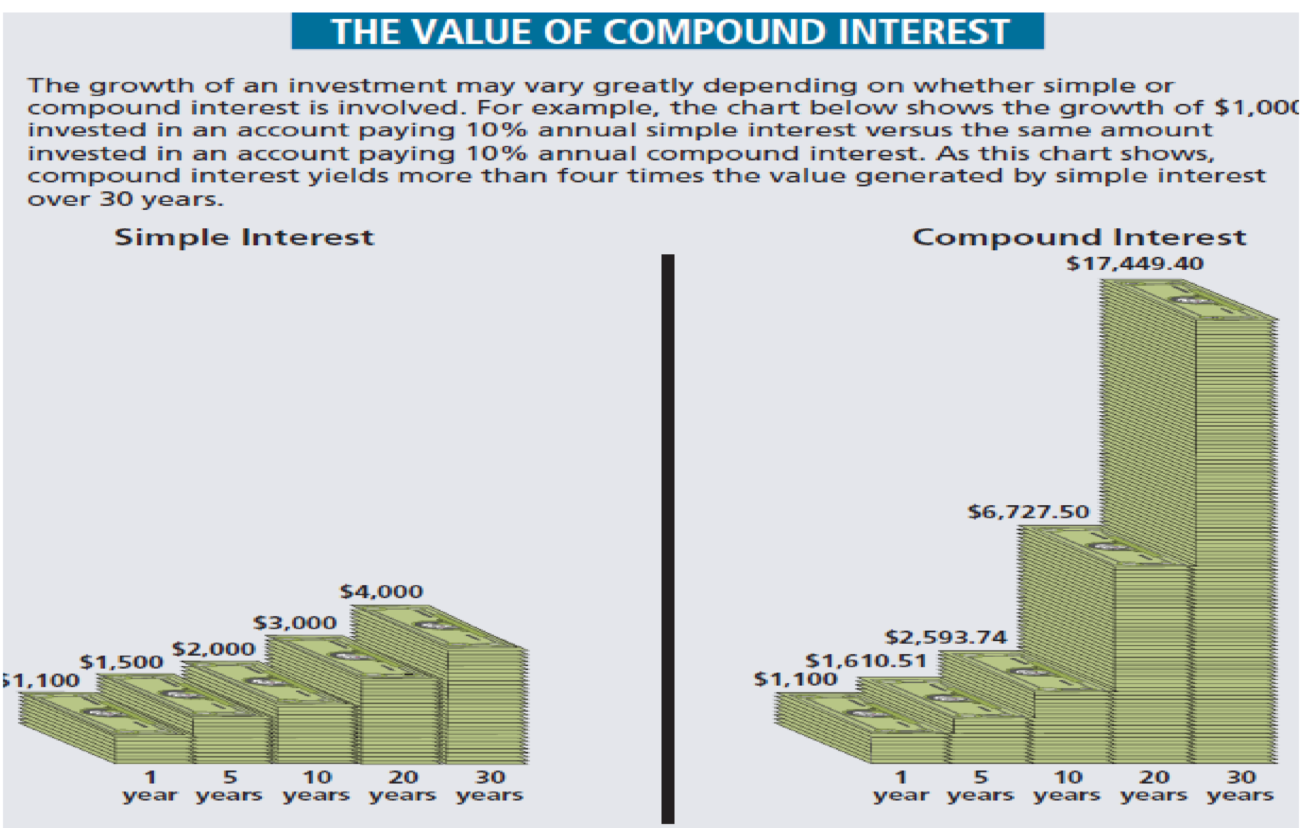
In section (2-1),we studied simple interest in which the formula *I* = *PRT* was applied once during

the term of a loan or an investment to find the amount of interest. In business, another common

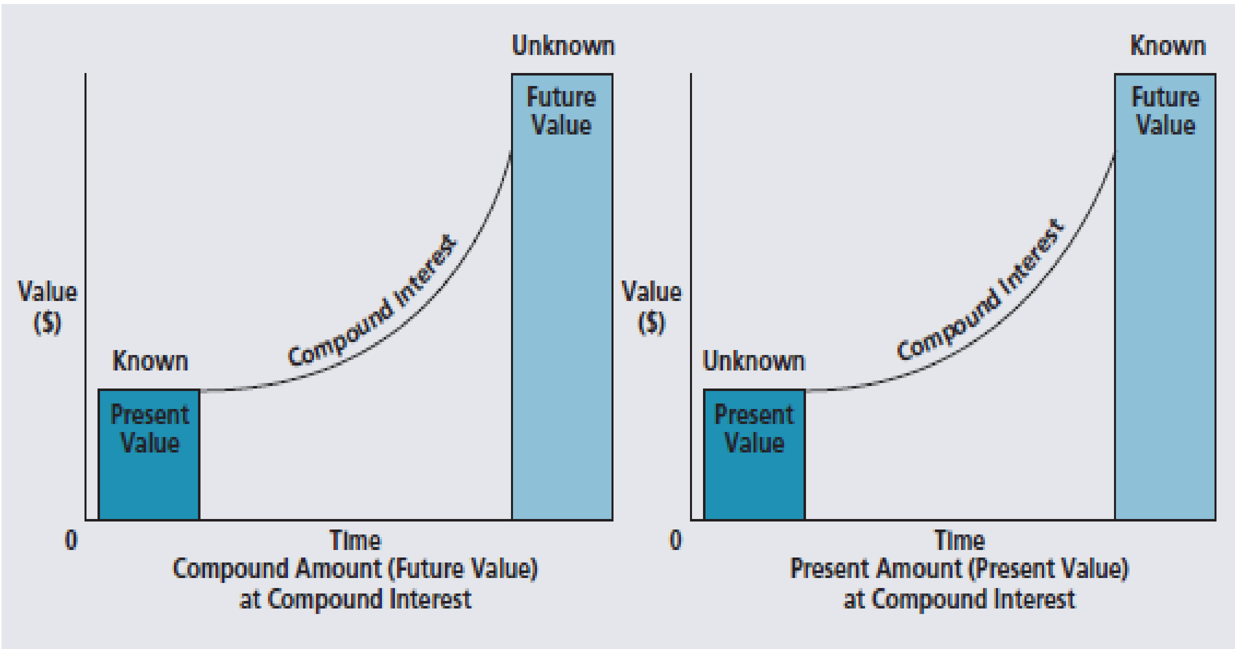
way of calculating interest is by using a method known as *compounding*, or **compound interest**, in which the interest calculation is applied a number of times during the term of the loan or investment. Compound interest yields considerably higher interest than simple interest does because the investor is earning interest on the interest. With compound interest, the interest earned for each period is reinvested or added to the previous principal before the next calculation or compounding. The previous principal plus interest then becomes the new principal for the next period. For example, $100 invested at 8% interest is worth $108 after the first year

($100 principal + $8 interest) . If the interest is not withdrawn, the interest for the next period will be calculated based on $108 principal.

As this compounding process repeats itself each period, the principal keeps growing by the amount of the previous interest. As the number of compounding periods increases, the amount of interest earned grows dramatically, especially when compared with simple interest . (see Exhibit 3-2-1) the time value of money .



In this section , you learn to calculate the compound amount (future value) of an investment
at compound interest when the present amount (present value) is known. You also learn to calculate the present value that must be deposited now at compound interest to yield a known



3-7 ((Manually Calculating Compound Amount (Future Value) And Compound Interest))

Compounding divides the time of a loan or an investment into compounding periods or simply periods. To manually calculate the compound amount or future value of an investment,
we must compound or calculate the interest as many times as there are compounding periods
at the interest rate per period.

For example, an investment made for 5 years at 6% compounded annually (once per year) would have five compounding periods (5 years × 1 period per year), each at 6%. If the same investment was compounded semiannually (two times per year), there would be 10 compounding
periods (5 years × 2 periods per year), each at 3% (6% annual rate ÷ 2 periods per year). The amount of compound interest is calculated by subtracting the principal from the compound amount.

Compound interest = Compound amount − Principal

Example 11

a. Katie Trotta invested $20,000 in a passbook savings account at 5% interest compounded annually for 2 years. Manually calculate the compound amount of the investment and the total amount of compound interest Katie earned.

Solution : To solve this compound interest problem manually, we must apply the simple interest formula twice because there are two compounding periods (2 years × 1 period per year) . Note how the interest from the first period is reinvested or added to the original principal to earn interest in the second period .

Original principal	$20,000.00	
Interest—period 1	+ 1,000.00	$(I = PRT = 20{,}000.00 \times .05 \times 1)$
Principal—period 2	21,000.00	
Interest—period 2	+ 1,050.00	$(I = PRT = 21{,}000.00 \times .05 \times 1)$
Compound Amount	$22,050.00	
Compound Amount	$22,050.00	
Principal	− 20,000.00	
Compound Interest Earned	$2,050.00	

b. Manually recalculate the compound amount and compound interest from the previous example by using semiannual compounding (two times per year). How much more interest would Katie earn if the bank offered semiannual compounding?

Solution

To solve this compound interest problem, we must apply the simple interest formula four times because there are four compounding periods (2 years × 2 periods per year). Note that the time factor is now $\frac{6}{12}$, or $\frac{1}{2}$, because semiannual compounding means every 6 months.

Original principal	$20,000.00	
Interest—period 1	+ 500.00	$(I = PRT = 20{,}000.00 \times .05 \times \frac{1}{2})$
Principal—period 2	20,500.00	
Interest—period 2	+ 512.50	$(I = PRT = 20{,}500.00 \times .05 \times \frac{1}{2})$
Principal—period 3	21,012.50	
Interest—period 3	+ 525.31	$(I = PRT = 21{,}012.50 \times .05 \times \frac{1}{2})$
Principal—period 4	21,537.81	
Interest—period 4	+ 538.45	$(I = PRT = 21{,}537.81 \times .05 \times \frac{1}{2})$
Compound Amount	$22,076.26	
Compound Amount	$22,076.26	
Principal	− 20,000.00	
Compound Interest	$2,076.26	

For the same investment values, semiannual compounding yields $26.26 more than annual compounding :

Interest with semiannual compounding	2,076.26
Interest with annual compounding	− 2,050.00
	$26.26

EXERCISE 11

Gail Parker invested $10,000 at 6% interest compounded semiannually for 3 years. Manually calculate the compound amount and the compound interest of Gail's investment.

3-8((CALCULATING COMPOUND AMOUNT (FUTURE VALUE) BY USING THE COMPOUND INTEREST FORMULA))

If your calculator has an exponential function key, y^x , you can calculate the compound amount of an investment by using the compound interest formula The compound interest. formula states:

$$A = P(1 + i)^n$$

where:

A = Compound amount

P = Principal

i = Interest rate per period (expressed as a decimal)

n = Total compounding periods (years × periods per year)

STEPS FOR SOLVING THE COMPOUND INTEREST FORMULA

STEP 1. Add the 1 and the interest rate per period, i.

STEP 2. Raise the sum from Step 1 to the nth (number of compounding periods) power by using the y^x key on your calculator.

STEP 3. Multiply the principal, P, by the answer from Step 2.

Example 12

Use the compound interest formula to calculate the compound amount of $5,000 invested at 10% interest compounded semiannually for 3 years.

Solution

In this example , Note that the rate per period**,** ***i***, is 5% (10% ÷ 2 periods per year) . The total number of periods, the exponent *n*, is 6 (3 years × 2 period per year).

$A = p(1+i)^n$

$A = 5,000\ (1+0.05)^6$

$A = 5,000(1.05)^6$

A = 5,000(1.3400956) = 6,700.4782 = $6,700.48

Exercise 12

Use the compound interest formula to calculate the compound amount of $3,000 invested at 8%
interest compounded quarterly for 5 years.

Exercise 13

Solve the following exercises and word problems by using the compound interest formula.

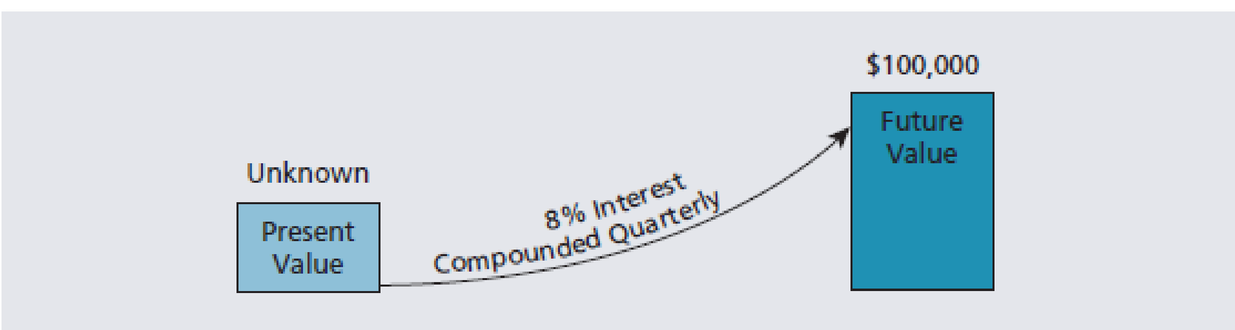
	Principal	Time Period (years)	Nominal Rate (%)	Interest Compounded	Compound Amount	Compound Interest
1-	$5,000	4	4.2	semiannually	$5,904.40	$904.40
2-	$700	8	1.5	monthly	______	______
3-	$2,800	$2\frac{1}{2}$	3.1	quarterly	______	______
4-	$12,450	10	2.6	annually	______	______
5-						

Section3

((PRESESNT VALUE))

In section 2, we learned how to find a future value when the present value was known. Let's take a look at the reverse situation. When a future value (an amount needed in the future)

is known, the present value is the amount that must be invested today to accumulate with compound interest to that future value. See exhibit 3



By Using The Present Value Formula))

If your calculator has an exponential function key , y^x, you can calculate the present value of an investment by using the present value formula. The present value formula states:

$$PV = \frac{A}{(1+i)^n}$$

Where : PV=Present value , A = Compound amount ,

I = Interest rate per period (expressed as a decimal) ,

n= Total compounding periods (years ×periods per year)

STEPS FOR SOLVING THE PRESENT VALUE FORMULA

Step1 . Add the 1 and the interest rate per period, *i*.

Step 2 . Raise the sum from Step 1 to the *n* th power by using the , y^x, key on your calculator.

Step 3 . Divide the compound amount, *A*, by the answer from Step 2.

Example 13

Use the present value formula to calculate the present value of $3,000 if the interest rate is 8% compounded quarterly for 6 years.

Solution

This problem is solved by substituting the investment information into the present value formula. It is important to solve the formula using the sequence of steps outlined. Note the rate per period, **I** , is 2% (8% ÷ 4 periods per year) . The total number of periods, the exponent **n**, is 24 (6 years × 4 periods per year) .

$$PV=\frac{A}{(1+i)^n} \quad , \quad PV=\frac{3000}{(1+0.02)^{24}}$$

$$PV=\frac{3000}{(1.02)^{24}} \quad , \quad PV=\frac{3000}{1.608437249} =\$\ 1867.16$$

Exercise 14

Sam and Rosa Alonso want to accumulate $30,000, 17 years from now as a college fund for their baby son, Michael. Use the present value formula to calculate how much they must invest now at an interest rate of 8% compounded semiannually to have $30,000 in 17 years.

Exercise 15

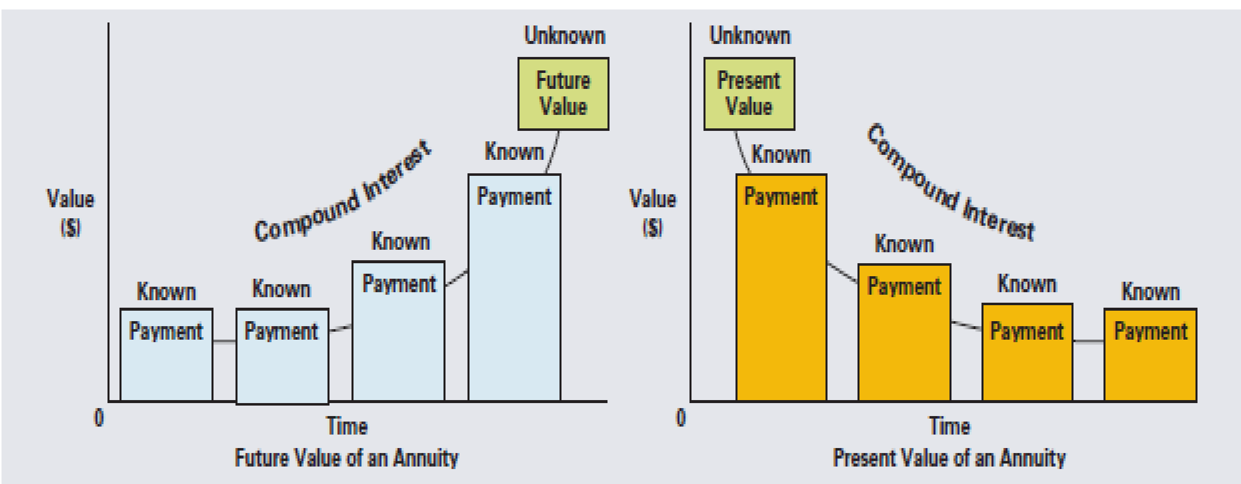
Solve the following exercises and word problems by using the present value formula

	Principal	Term of Investment	Nominal Rate (%)	Interest Compounded	Present Value	Compound Interest
1-	$4,500	7 years	3.8	annually	$3,466.02	$1,033.98
2-	$15,000	8 years	4.5	monthly	______	______
3-	$18,900	10 years	1.9	semiannually	______	______
4 -	$675	15 months	2.7	quarterly	______	______

SECTION 4
((FUTURE VALUE OF AN ANNUITY: ORDINARY AND ANNUITY DUE))

The concepts relating to compound interest in section (**3-2)**were mainly concerned with lump sum investments or payments. Frequently in business, situations involve a series of equal periodic payments or receipts rather than lump sums. These are known as annuities. An annuity is the payment or receipt of equal cash amounts per period for a specified amount of time.

In this section , you learn to calculate the future value of an annuity, the amount accumulated at compound interest from a series of equal periodic payments. You also learn to calculate the present value of an annuity, the amount that must be deposited now at compound interest to yield a series of equal periodic payments. Exhibit 3-3-1 graphically shows the difference between the future value of an annuity and the present value of an annuity .



As with compound interest, annuities can be calculated manually, and by formulas. Manual computation is useful for illustrative purposes; however, it is too tedious because it requires a calculation for each period. As in section (**3-2)** , there are formulas to calculate annuities; however, they require calculators with the exponential function key , y^x, and the change-of-sign key, + / −. These optional Performance Objectives are for students with business, financial, or scientific calculators .

3-9((Manually Calculating Compound Amount (Future Value) Of An Ordinary Annuity))

Annuities are categorized into **annuities certain** and **contingent annuities**. **Annuities certain** are annuities that have a specified number of periods, such as $200 per month for 5 years or $500 semiannually for 10 years. **Contingent annuities** are based on an uncertain time period, such as a retirement plan that is payable only for the lifetime of the retiree. This chapter is concerned only with annuities certain.

When the annuity payment is made at the end of each period, it is known as an **ordinary annuity**. When the payment is made at the beginning of each period, it is called an **annuity due**. A salary paid at the end of each month is an example of an ordinary annuity. A mortgage payment or rent paid at the beginning of each month is an example of an annuity due.

The **future value of an annuity** is also known as the **amount of an annuity**. It is the total of the annuity payments plus the accumulated compound interest on those payments.
For illustrative purposes, consider the following annuity calculated manually.

EXAMPLE 14

What is the future value of an ordinary annuity of $10,000 per year for 4 years at 6% interest compounded annually?

SOLUTION

Because this is an ordinary annuity, the payment is made at the end of each period (in this case, years). Each interest calculation uses I = PRT, with R = .06 and T = 1 year.

Time	Balance	
Beginning of period 1	0	
	+ 10,000.00	First annuity payment (end of period 1)
End of period 1	10,000.00	
Beginning of period 2	10,000.00	
	600.00	Interest earned, period 2 (10,000.00 × .06 × 1)
	+ 10,000.00	Second annuity payment (end of period 2)
End of period 2	20,600.00	
Beginning of period 3	20,600.00	
	1,236.00	Interest earned, period 3 (20,600.00 × .06 × 1)
	+ 10,000.00	Third annuity payment (end of period 3)
End of period 3	31,836.00	
Beginning of period 4	31,836.00	
	1,910.16	Interest earned, period 4 (31,836.00 × .06 × 1)
	+ 10,000.00	Fourth annuity payment (end of period 4)
End of period 4	$43,746.16	Future value of the ordinary annuity

3-10 ((CALCULATING THE FUTURE VALUE OF AN ORDINARY ANNUITY AND AN ANNUITY DUE BY FORMULA))

Students with financial, business, or scientific calculators may use the following formulas to solve for the future value of an ordinary annuity and the future value of an annuity due.

Future value of an ordinary annuity	Future value of an annuity due
$FV = Pmt \times \frac{(1+i)^n - 1}{i}$	$FV = Pmt \times \frac{(1+i)^n - 1}{i} \times (1+i)$

where:

FV = future value

Pmt = annuity payment

i = interest rate per period (nominal rate ÷ periods per year)

n = number of periods (years × periods per year)

Example 15

a. What is the future value of an ordinary annuity of $100 per month for 3 years at 6% interest compounded monthly?

b. What is the future value of this investment if it is an annuity due?

Solution

a. For this future value of an ordinary annuity problem, we use i = .5%(6% ÷ 12) and n = 36 periods (3 years × 12 periods per year) .

$$FV = Pmt \times \frac{(1+i)^n - 1}{i}$$

$$FV = 100 \times \frac{(1+.005)^{36} - 1}{.005}$$

$$FV = 100 \times \frac{(1.005)^{36} - 1}{.005}$$

$$FV = 100 \times \frac{1.196680525 - 1}{.005}$$

$$FV = 100 \times \frac{.196680525}{.005}$$

$$FV = 100 \times 39.336105 = \underline{\underline{\$3,933.61}}$$

b. To solve the problem as an annuity due rather than an ordinary annuity, multiply (1 + i) , for one extra compounding period, by the future value of the ordinary annuity.

$$FV_{\text{annuity due}} = (1 + i) \times FV_{\text{ordinary annuity}}$$
$$FV_{\text{annuity due}} = (1 + 0.005) \times 3{,}933.61$$
$$FV_{\text{annuity due}} = (1.005) \times 3{,}933.61 = \$3{,}953.28$$

Exercise 16

Katrina Byrd invested $250 at the end of every 3-month period for 5 years at 8% interest compounded quarterly.

a. How much is Katrina's investment worth after 5 years?

b. If Katrina had invested the money at the beginning of each 3-month period rather than at the
end, how much would be in the account?

Exercise 17

Solve the following exercises by using formulas.

Ordinary Annuities

	Annuity Payment	Payment Frequency	Time Period (years)	Nominal Rate (%)	Interest Compounded	Future Value of the Annuity
16.	$2,000	every 6 months	3	3.0	semiannually	$12,459.10
17.	$300	every month	8	6.0	monthly	______
18.	$1,800	every 3 months	$3\frac{1}{2}$	4.0	quarterly	______

Annuities Due

	Annuity Payment	Payment Frequency	Time Period (years)	Nominal Rate (%)	Interest Compounded	Future Value of the Annuity
19.	$675	every month	5	1.5	monthly	$42,082.72
20.	$4,800	every 3 months	3	6.0	quarterly	______
21.	$7,000	every year	10	3.2	annually	______

SECTION 5

((Present Value Of An Annuity : Ordinary and Annuity Due))

In **Section 4** of this chapter, we learned to calculate the future value of an annuity. This business situation requires that a series of equal payments be made into an account , such as a savings account. The annuity starts with nothing and accumulates at compound interest to a future amount. Now consider the opposite situation. What if we wanted an account from which we could withdraw a series of equal payments over a period of time? This business situation requires that a lump sum amount be deposited at compound interest now to yield the specified annuity payments. The lump sum that is required up front is known as the **present value of an annuity.**

Just as in **Section 4**, these annuities can be ordinary, whereby withdrawals from the account are made at the end of each period, or annuity due, in which the withdrawals are made at the beginning. As with the future value of an annuity , these annuities can be solved by using formulas requiring a calculator with a y^x key .

3-11 ((CALCULATING THE PRESENT VALUE OF AN ORDINARY ANNUITY AND AN ANNUITY DUE BY FORMULA))

Note that the annuity due formula is the same as the ordinary annuity formula except that it is multiplied by (1 + i). This is to account for the fact that with an annuity due, each payment earns interest for one additional period because payments are made at the beginning of each period, not the end.

Present value of an ordinary annuity

$$PV = Pmt \times \frac{1-(1+i)^{-n}}{i}$$

Present value of an annuity due

$$PV = Pmt \times \frac{1-(1+i)^{-n}}{i} \times (1+i)$$

where:

PV = present value (lump sum)

Pmt = annuity payment

i = interest rate per period (nominal rate ÷ periods per year)

n = number of periods (years × periods per year)

Example 16

a. What is the present value of an ordinary annuity of $100 per month for 4 years at 6% interest compounded monthly?

b. What is the present value of this investment if it is an annuity due?

Solution

a. For this present value of an ordinary annuity problem, we use i = .5% (6% ÷ 12) and n = 48 periods (4 years × 12 periods per year) .

$$PV = Pmt \times \frac{1-(1+i)^{-n}}{i}$$

$$PV = 100 \times \frac{1-(1+.005)^{-48}}{.005}$$

$$PV = 100 \times \frac{1-(1.005)^{-48}}{.005}$$

$$PV = 100 \times \frac{1-.7870984111}{.005}$$

$$PV = 100 \times \frac{.2129015889}{.005}$$

$$PV = 100 \times 42.58031778 = \underline{\underline{\$4{,}258.03}}$$

b. To solve as an annuity due rather than an ordinary annuity, multiply the present value of the ordinary annuity by (1 + i) for one extra compounding period.

$$PV_{\text{annuity due}} = (1+i) \times PV_{\text{ordinary annuity}}$$

$$PV_{\text{annuity due}} = (1+.005) \times 4{,}258.03$$

$$PV_{\text{annuity due}} = (1.005) \times 4{,}258.03 = \underline{\underline{\$4{,}279.32}}$$

Exercise 18

Use the present value of an annuity formula to solve the following.

a. Angus McDonald wants $500 at the end of each 3-month period for the next 6 years. If Angus's bank is paying 8% interest compounded quarterly , how much must he deposit now to receive the desired ordinary annuity?

b. If Angus wants the payments at the beginning of each 3-month period rather than at the end,
how much should he deposit?

Exercise 19

Solve the following exercises by using formulas.

Present value of an ordinary annuity

	Annuity Payment	Payment Frequency	Time Period (yrs)	Nominal Rate (%)	Interest Compounded	Present Value of the Annuity
1-	$500	every 3 months	$3\frac{1}{4}$	6.0	quarterly	$5,865.77
2-	$280	every month	5	3.0	monthly	______
3 -	$950	every year	8	2.9	annually	______

Present value of an annuity due

	Annuity Payment	Payment Frequency	Time Period (yrs)	Nominal Rate (%)	Interest Compounded	Present Value of the Annuity
1-	$1,100	every year	5	5.8	annually	$4,929.14
2-	$425	every month	$4\frac{3}{4}$	4.5	monthly	______
3-	$700	every 6 months	7	3.6	semiannually	______

SECTION 6

((Singing Fund And Amortization))

Sinking funds and amortization are two common applications of annuities. In the previous sections of this chapter, the amount of the annuity payment was known and you were asked to calculate the future or present value (lump sum) of the annuity. In this section, the future or present value of the annuity is known and the amount of the payments is calculated.

A sinking fund situation occurs when the future value of an annuity is known and the payment required each period to amount to that future value is the unknown. Sinking funds are accounts used to set aside equal amounts of money at the end of each period at compound interest for the purpose of saving for a future obligation.

Amortization is the opposite of a sinking fund. Amortization is a financial arrangement whereby a lump-sum obligation is incurred at compound interest now (present value) and is paid off or liquidated by a series of equal periodic payments for a specified amount of time. With amortization, the amount of the loan or obligation is given and the equal payments that will amortize, or pay off, the obligation must be calculated.

3-12 ((CALCULATING THE PRESENT VALUE OF AN ORDINARY ANNUITY AND AN ANNUITY DUE BY FORMULA))

sinking fund payments may be calculated by using the formula

$$\text{Sinking fund payment} = FV \times \frac{i}{(1+i)^n - 1}$$

where:

FV = amount needed in the future

i = interest rate per period (nominal rate ÷ periods per year)

n = number of periods (years × periods per year)

Example17

Ocean Air Corporation needs $100,000 in 5 years to pay off a bond issue. What sinking fund payment is required at the end of each month at 12% interest compounded monthly to meet this financial obligation?

Solution:

To solve this singing fund problem . We use 1% interest rate per period (12% ÷ 12)and 60 periods (5 years × 12 periods per year).

$$\text{Sinking fund payment} = \text{Future value} \times \frac{i}{(1+i)^n - 1}$$

$$\text{Sinking fund payment} = 100{,}000 \times \frac{.01}{(1+.01)^{60} - 1}$$

$$\text{Sinking fund payment} = 100{,}000 \times \frac{.01}{.8166967}$$

$$\text{Sinking fund payment} = 100{,}000 \times .0122444 = \underline{\underline{\$1{,}224.44}}$$

Exercise 20

Lake Louise Ski Rental Center will need $40,000 in 6 years to replace aging equipment. What sinking fund payment is required at the end of each month at 6% interest compounded monthly to amount to the $40,000 in 6 years?

3-13 ((CALCULATING AMORTIZATION PAYMENTS BY FORMULA))

amortization payments may be calculated by using the formula

$$\text{Amortization payment} = PV \times \frac{i}{1-(1+i)^{-n}}$$

where:

PV = amount of the loan or obligation

i = interest rate per period (nominal rate ÷ periods per year)

n = number of periods (years × periods per year)

Example 19

What amortization payment is required each month at 18% interest to pay off $5,000 in 3 years?

Solution : To solve this amortization problem, we use 1.5% interest rate per period (18% ÷ 12) and 36 periods (3 years × 12 periods per year) .

$$\text{Amortization payment} = \text{Present value} \times \frac{i}{1-(1+i)^{-n}}$$

$$\text{Amortization payment} = 5{,}000 \times \frac{.015}{1-(1+.015)^{-36}}$$

$$\text{Amortization payment} = 5{,}000 \times \frac{.015}{.4149103}$$

$$\text{Amortization payment} = 5{,}000 \times .0361524 = \underline{\underline{\$180.76}}$$

Exercise 21

Apex Manufacturing recently purchased a new computer system for $150,000. What amortization payment is required each month at 12% interest to pay off this obligation in 8 years?

Exercise 22 : Solve the following exercises by using the sinking fund or amortization formula.

Sinking fund payment

	Sinking Fund Payment	Payment Frequency	Time Period (yrs)	Nominal Rate (%)	Interest Compounded	Future Value (Objective)
1-	$345.97	every 3 months	5	6.0	quarterly	$8,000
2-	______	every month	8	1.5	monthly	$5,500
3-	______	every 6 months	$3\frac{1}{2}$	4.0	semiannually	$1,900

Amortization Payment

	Loan Payment	Payment Frequency	Time Period (yrs)	Nominal Rate (%)	Present Value (Amount of Loan)
1-	$3,756.68	every year	10	10.6	$22,500
2-	______	every 3 months	4	8.8	$9,000
3-		every month	6	9.0	$4,380