4

Q 17: show that $u(x,y)$ is harmonic in some domain and find a harmonic conjugate $v(x,y)$ when $u(x,y) = 2x(1-y)$.

Q 18: Use established properties of moduli to show that when $|z_3| \neq |z_4|$.

$$\left| \frac{z_1 + z_2}{z_3 + z_4} \right| \leq \frac{|z_1| + |z_2|}{\left| |z_3| - |z_4| \right|}$$

Q 19: Represent graphically the set of values of z for which

    (a) $\left| \frac{z-3}{z+3} \right| = 2$
    (b) $\left| \frac{z-3}{z+3} \right| < 2$

Q 20: solve the equation

    (a) $z^2 + (2i-3)z + (5-i) = 0$

    (b) prove that $\sin^2 z + \cos^2 z = 1$

(4)

Q21: (a) If $f(z)$ is analytic at z_0, prove it must be continuous at z_0.

(b) show that $\frac{d}{dz}(\bar{z})$ does not exist anywhere i.e $f(z)$ is non-analytic anywhere.

Q22: (a) let $w = \sin z$. Show that Riemann equation is satisfied and prove $\frac{d}{dz}(\sin z) = \cos z$

(b) Evalute $\lim\limits_{z \to i} \dfrac{z^{10}+1}{z^6+1}$ Answer $\dfrac{5}{3}$

Q23: let $f(z) = \sqrt{r}\, e^{i\theta/2}$
$$(r>0 \;,\; \alpha < \theta < \alpha + 2\pi)$$

show that the Cauchy-Riemann equations are satisfied and find $f'(z)$.

Answer $f'(z) = \dfrac{1}{2f(z)}$

Q24. Find the principal value of

(a) $(1-i)^{4i}$ (b) i^{i}

Q25: At what points the function given by $\text{Log}(z^2+1)$ analytic?

Explain

Q26: Find all z such that $e^{z} = 1+i$, or explain why there are none.

Q27: Find all points at which the function f given by $f(z) = x^3 - i(1-y)^3$ is differentiable.

Q28: Find all points at which $f(z) = 2y - ix$ is differentiable.

Q29: Find the rectangular form of $(-1+i)^{100}$