

Q1: Express the number $\frac{-1+3i}{2+5i}$ in the form $x+iy$.

Q2: Find the roots of equation $x^2+x+1=0$

Q3: Write the following number in polar form (a) $z=1+i$ (b) $w=\sqrt{3}-i$

and find the product of z and w .

Q4: Find $\frac{1 - \left(\frac{1}{2} + \frac{1}{2}i\right)^{10}}{2 \cdot (\sqrt{3} + 1)^7}$

Answer $\frac{1}{32}i$

Answer $(-64)(\sqrt{3}+i)$

Q5: Find the six roots of $z = -8$ and graph these roots in complex plane

Q6: Find the modulus and principle argument of

$$z = \left(\frac{\sqrt{3} + i}{1+i} \right)^{17}$$

Answer

$$|z| = 2^{\frac{17}{2}}$$

$$\text{Arg } z = \frac{-17}{12}\pi + 2k\pi$$

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Q7: Let $\tan z = \frac{\sin z}{\cos z}$ show that

$$\tan z = \frac{\sin 2x + i \sinh 2y}{\cos x + \cosh 2y}$$

Q8: prove that $u = e^{-x} (x \sin y - y \cos y)$ and find v such that $f(z) = u + iv$ is analytic.

Q9: find 1. $(\sqrt{3} + i)^{1/2}$
2. $\sqrt{-15 - 8i}$

Answer: $(1 - 4i)$
 $(-1 + 4i)$

Q10: Show that the hyperbola $x^2 - y^2 = 1$ can be written $z^2 + \bar{z}^2 = 2$

Q11: Evaluate (a) $\int_C \bar{z} dz$ from $z=0$ to $z=4+2i$

along the curve given by $z = t^2 + it$

(b) solve the equation $z^2 = 1 + i$

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Q₁₂: let $z = x + iy$ and $f(z) = x^2 - y^2 - 2y + i(2x - 2xy)$.
write $f(z)$ as a function of only z and \bar{z} .

Q₁₃: Using the exponential form to show that $(-1 + i)^7 = -8(1 + i)$.

Q₁₄ sketch the set of points determined by $|z - 1 + i| = 1$

Q₁₅: show that ~~$\sin z = \cos x \cosh y + i \cos x \sinh y$~~

$$\cos z = \cos x \cosh y - i \sin x \sinh y$$

$$\sin z = \sin x \cosh y + i \cos x \sinh y$$

Q₁₆: prove the identities

$$\frac{\sin 5\theta}{\sin \theta} = 16 \cos^4 \theta - 12 \cos^2 \theta + 1$$

if $\theta \neq 0, \pi, 2\pi, \dots$