93). جد قيمة $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$

علاقة $\longrightarrow$ $\tan^{-1}(y) + \tan^{-1}(x) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}*\frac{1}{3}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

---

94). جد قيمة $\sec^2[\tan^{-1}(x)]$

علاقة $\longrightarrow$ $\sec^2(x) = 1 + \tan^2(x)$

$$1 + \tan^2[\tan^{-1}(x)] = 1 + (\tan[\tan^{-1}(x)])^2$$

$$= 1 + x^2$$

---

95). جد قيمة $\sin\left(2\cos^{-1}\left(\frac{3}{5}\right)\right)$

$$\sin\left(2\cos^{-1}\frac{3}{5}\right) = 2\sin\left(\cos^{-1}\frac{3}{5}\right).\cos\left(\cos^{-1}\frac{3}{5}\right)$$

$$= 2.\sin\left(\cos^{-1}\frac{3}{5}\right).\frac{3}{5}$$

$\cos^{-1}\left(\frac{3}{5}\right) = y \rightarrow \cos(y) = \frac{3}{5} \rightarrow y = 53^\circ$ ط١)

$$= \frac{6}{5}.\sin(53^\circ)$$

$$= \frac{6}{5}.\frac{4}{5}$$

$$= \frac{24}{25}$$

ط٢)

$\cos^{-1}\left(\frac{3}{5}\right) = y \to \cos(y) = \frac{3}{5} \to \frac{\text{المجاور}}{\text{الوتر}}$

علاقة فيثاغورس

$(\text{الوتر})^2 = (\text{المجاور})^2 + (\text{المقابل})^2$

$(5)^2 = (3)^2 + (\text{المقابل})^2$

$25 = 9 + (\text{المقابل})^2$

$16 = (\text{المقابل})^2$

$4 = \text{المقابل}$

$\sin(y) = \frac{4}{5} \to \frac{\text{المقابل}}{\text{الوتر}}$

$\frac{4}{5} \cdot \frac{6}{5} = \frac{24}{25}$

---

96). جد قيمة $\sin^{-1}\left(\frac{3}{\sqrt{10}}\right) + \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

علاقة

$\sin^{-1}(x) + \sin^{-1}(y) = \sin^{-1}(x\sqrt{1-y^2}) + y\sqrt{1-x^2})$

$$\sin^{-1}(\frac{3}{\sqrt{10}}) + \sin^{-1}(\frac{2}{\sqrt{5}}) = \sin^{-1}[\frac{3}{\sqrt{10}} . \sqrt{1-\left(\frac{2}{\sqrt{5}}\right)^2}) + \frac{2}{\sqrt{5}} . \sqrt{1-\left(\frac{3}{\sqrt{10}}\right)^2}]$$

$$= \sin^{-1}[\frac{3}{\sqrt{10}} . \sqrt{1-\frac{4}{5}}) + \frac{2}{\sqrt{5}} . \sqrt{1-\frac{9}{10}}]$$

$$= \sin^{-1}[\frac{3}{\sqrt{10}} . \sqrt{\frac{1}{5}} + \frac{2}{\sqrt{5}} . \sqrt{\frac{1}{10}}]$$

$$= \sin^{-1}\frac{\sqrt{5}}{\sqrt{10}}$$

$$= \sin^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$$

97). جد قيمة $2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)$

علاقة $\tan^{-1}(y)+\tan^{-1}(x)=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$$2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{7}}{1-\frac{1}{3}\cdot\frac{1}{7}}\right)$$

$$=\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{2}\right)$$

$$=\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{2}\cdot\frac{1}{3}}\right)$$

$$=\tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right)=\tan^{-1}(1)=\frac{\pi}{4}$$

---

98). جد قيمة $\cos(2.\cos^{-1}x)$

$$\cos(2.\cos^{-1}x)=2\cos^2(\cos^{-1}x)-1$$

$$=2\left[\cos\left(\cos^{-1}x\right)\right]^2-1$$

$$=2x^2-1$$

---

99). جد y' للدالة $y=\cot^{-1}\left(\frac{2}{x}\right)-\tan^{-1}\left(\frac{x}{2}\right)$

$$y'=\frac{-1}{1+\left(\frac{2}{x}\right)^2}*\frac{-2}{x^2}-\frac{1}{1+\left(\frac{x}{2}\right)^2}*\frac{1}{2}$$

$$y'=\frac{2}{x^2\left(1+\frac{4}{x^2}\right)}-\frac{1}{2\left(1+\frac{x^2}{4}\right)}\rightarrow y'=\frac{2}{x^2+4}-\frac{1}{2+\frac{x^2}{2}}\rightarrow y'=\frac{2}{x^2+4}-\frac{1}{\frac{4+x^2}{2}}$$

$$y'=\frac{2}{x^2+4}-\frac{2}{4+x^2}$$

$$y'=\frac{0}{x^2+4}=0$$

100). جد y' للداله $y = x.\cos^{-1}(2x) - \frac{1}{2}\sqrt{1-4x^2}$

$$y' = x.\frac{-1}{\sqrt{1-(2x)^2}} * 2 + \cos^{-1}(2x) - \frac{1}{2} * \frac{1}{2\sqrt{1-4x^2}} * -8x$$

$$y' = \frac{-2x}{\sqrt{1-4x^2}} + \cos^{-1}(2x) + \frac{2x}{\sqrt{1-4x^2}}$$

$$y' = \cos^{-1}(2x)$$

---

101). جد y' للداله $y = \sin^{-1}(\cos\sqrt{x}) + \cos^{-1}(\sin\sqrt{x})$

$$y' = \frac{1}{\sqrt{1-\cos^2\sqrt{x}}} * -\sin(\sqrt{x}) * \frac{1}{2\sqrt{x}} + \frac{-1}{\sqrt{1-\sin^2\sqrt{x}}} * \cos\sqrt{x} * \frac{1}{2\sqrt{x}}$$

$$y' = \frac{-\sin(\sqrt{x})}{2\sqrt{x}.\sqrt{1-\cos^2\sqrt{x}}} - \frac{\cos\sqrt{x}}{2\sqrt{x}.\sqrt{1-\sin^2\sqrt{x}}}$$

$$y' = \frac{-\sin(\sqrt{x})}{2\sqrt{x}.\sqrt{\sin^2\sqrt{x}}} - \frac{\cos\sqrt{x}}{2\sqrt{x}.\sqrt{\cos^2\sqrt{x}}}$$

$$y' = \frac{-\sin(\sqrt{x})}{2\sqrt{x}.\sin\sqrt{x}} - \frac{\cos\sqrt{x}}{2\sqrt{x}.\cos\sqrt{x}}$$

$$y' = \frac{-1}{2\sqrt{x}} - \frac{1}{2\sqrt{x}}$$

$$y' = \frac{-2}{2\sqrt{x}} \to y' = -\frac{1}{\sqrt{x}}$$

102). $\tan^{-1}(xy) = \cot^{-1}(x+y)$ جد للداله y'

$$\frac{1}{1+(xy)^2}(x.y'+y) = \frac{-1}{1+(x+y)^2}(1+y')$$

$$\frac{x}{1+(xy)^2}.y' + \frac{y}{1+(xy)^2} = \frac{-1}{1+(x+y)^2} - \frac{1}{1+(x+y)^2}.y'$$

$$\frac{x}{1+(xy)^2}.y' + \frac{1}{1+(x+y)^2}.y' = \frac{-1}{1+(x+y)^2} - \frac{y}{1+(xy)^2}$$

$$y'\left(\frac{x}{1+(xy)^2} + \frac{1}{1+(x+y)^2}\right) = \frac{-1}{1+(x+y)^2} - \frac{y}{1+(xy)^2}$$

$$y' = \frac{\frac{-1}{1+(x+y)^2} - \frac{y}{1+(xy)^2}}{\frac{x}{1+(xy)^2} + \frac{1}{1+(x+y)^2}}$$

---

103). $(1-x^2)y'' - x.y' - 2 = 0$ اثبت ان $y = (\sin^{-1} x)^2$ اذا كانت

$$y' = 2.\sin^{-1}(x).\frac{1}{\sqrt{1-x^2}} \rightarrow y' = \frac{2.\sin^{-1}(x)}{\sqrt{1-x^2}}$$

$$y'' = 2\left[\frac{\sqrt{1-x^2}.\frac{1}{\sqrt{1-x^2}} - \sin^{-1}(x).\frac{1}{2\sqrt{1-x^2}}.-2x}{\left(\sqrt{1-x^2}\right)^2}\right] \rightarrow y'' = 2\left[\frac{1+\frac{x.\sin^{-1}(x)}{\sqrt{1-x^2}}}{1-x^2}\right] =$$

$$y'' = \frac{2+\frac{2.x.\sin^{-1}(x)}{\sqrt{1-x^2}}}{1-x^2}$$

$$(1-x^2)y'' - x.y' - 2 = (1-x^2)\frac{2+\frac{2.x.\sin^{-1}(x)}{\sqrt{1-x^2}}}{1-x^2} - x.\frac{2.\sin^{-1}(x)}{\sqrt{1-x^2}} - 2$$

$$= 2 + \frac{2.x.\sin^{-1}(x)}{\sqrt{1-x^2}} - \frac{2.x.\sin^{-1}(x)}{\sqrt{1-x^2}} - 2$$

$$= 0$$

104). اذا كانت $y = \sin(\tan^{-1} x)$ اثبت ان $(1+x^2)^2 y'' + 2x(1+x^2)y' + y = 0$

$$y' = \cos(\tan^{-1} x) \cdot \frac{1}{1+x^2} \rightarrow y' = \frac{\cos(\tan^{-1} x)}{1+x^2}$$

$$y'' = \frac{(1+x^2) \cdot \frac{-\sin(\tan^{-1} x)}{1+x^2} - \cos(\tan^{-1} x) \cdot 2x}{(1+x^2)^2} \rightarrow y'' = \frac{-\sin(\tan^{-1} x) - 2x \cdot \cos(\tan^{-1} x)}{(1+x^2)^2}$$

$$(1+x^2)^2 y'' + 2x(1+x^2)y' + y =$$

$$(1+x^2)^2 \frac{-\sin(\tan^{-1} x) - 2x \cdot \cos(\tan^{-1} x)}{(1+x^2)^2} + 2x(1+x^2)\frac{\cos(\tan^{-1} x)}{1+x^2} + \sin(\tan^{-1} x)$$

$$= -\sin(\tan^{-1} x) - 2x \cdot \cos(\tan^{-1} x) + 2x\cos(\tan^{-1} x) + \sin(\tan^{-1} x)$$

$$= 0$$

---

105). اذا كانت $y = e^{1+\cot^{-1}(x)}$ اثبت ان $(1+x^2)^2 y'' + (1+x^2)y' - 2xy = 0$

$$y' = e^{1+\cot^{-1}(x)} * \frac{-1}{1+x^2} \rightarrow y' = \frac{-e^{1+\cot^{-1}(x)}}{1+x^2}$$

$$y'' = \frac{(1+x^2) * -e^{1+\cot^{-1}(x)} * \frac{-1}{1+x^2} + e^{1+\cot^{-1}(x)} * 2x}{(1+x^2)^2} = \frac{e^{1+\cot^{-1}(x)} + 2x \cdot e^{1+\cot^{-1}(x)}}{(1+x^2)^2}$$

$$(1+x^2)^2 y'' + (1+x^2)y' - 2xy =$$

$$(1+x^2)^2 * \frac{e^{1+\cot^{-1}(x)} + 2x \cdot e^{1+\cot^{-1}(x)}}{(1+x^2)^2} + (1+x^2)\frac{-e^{1+\cot^{-1}(x)}}{1+x^2} - 2x \cdot e^{1+\cot^{-1}(x)}$$

$$= e^{1+\cot^{-1}(x)} + 2x \cdot e^{1+\cot^{-1}(x)} + -e^{1+\cot^{-1}(x)} - 2x \cdot e^{1+\cot^{-1}(x)}$$

$$= 0$$

احسب $\int \frac{1}{x\sqrt{5x^2-3}}dx$

106).

$$\int \frac{1}{x\sqrt{3\left(\frac{5}{3}x^2-1\right)}}dx = \int \frac{1}{x.\sqrt{3}.\sqrt{\frac{5}{3}x^2-1}}dx$$

$$= \frac{1}{\sqrt{3}}\int \frac{1}{x.\sqrt{\frac{5}{3}x^2-1}}dx$$

$$= \frac{1}{\sqrt{3}}\int \frac{\frac{5}{3}}{\frac{5}{3}.x.\sqrt{\frac{5}{3}x^2-1}}dx$$

$$= \frac{1}{\sqrt{3}}.\sec^{-1}\left(\frac{\sqrt{5}}{\sqrt{3}}.x\right) + c$$

احسب $\int \frac{1}{5+6x^2}dx$

107).

$$\int \frac{1}{5\left(1+\frac{6}{5}x^2\right)}dx = \frac{1}{5}\int \frac{1}{1+\left(\frac{\sqrt{6}}{\sqrt{5}}x\right)^2}dx$$

$$= \frac{\sqrt{5}}{\sqrt{6}}.\frac{1}{5}\int \frac{\frac{\sqrt{6}}{\sqrt{5}}}{1+\left(\frac{\sqrt{6}}{\sqrt{5}}x\right)^2}dx$$

$$= \frac{1}{\sqrt{30}}.\tan^{-1}\left(\frac{\sqrt{6}}{\sqrt{5}}x\right) + c$$

107). احسب $\int \frac{1}{\sqrt{36-12x^2}} dx$

$$\int \frac{1}{\sqrt{36\left(1-\frac{12}{36}x^2\right)}} dx = \int \frac{1}{\sqrt{36}\sqrt{1-\left(\frac{1}{\sqrt{3}}x\right)^2}} dx$$

$$= \int \frac{1}{6\sqrt{1-\left(\frac{1}{\sqrt{3}}x\right)^2}} dx$$

$$= \sqrt{3}.\frac{1}{6}\int \frac{1}{\sqrt{1-\left(\frac{1}{\sqrt{3}}x\right)^2}} * \frac{1}{\sqrt{3}} dx$$

$$= \frac{1}{2\sqrt{3}} . \sin^{-1}\left(\frac{1}{\sqrt{3}}x\right) + c$$

---

108). احسب $\int_{\frac{5}{4\sqrt{2}}}^{\frac{5}{4}} \frac{16}{\sqrt{25\left(1-\frac{16}{25}x^2\right)}} dx$

$$\int_{\frac{5}{4\sqrt{2}}}^{\frac{5}{4}} \frac{16}{\sqrt{25\left(1-\frac{16}{25}x^2\right)}} dx = \int_{\frac{5}{4\sqrt{2}}}^{\frac{5}{4}} \frac{16}{\sqrt{25}\sqrt{1-\left(\frac{4}{5}x\right)^2}} dx$$

$$= \int_{\frac{5}{4\sqrt{2}}}^{\frac{5}{4}} \frac{16}{5\sqrt{1-\left(\frac{4}{5}x\right)^2}} dx$$

$$= 4\int_{\frac{5}{4\sqrt{2}}}^{\frac{5}{4}} \frac{\frac{4}{5}}{\sqrt{1-\left(\frac{4}{5}x\right)^2}} dx$$

$$= 4.\left[\sin^{-1}\left(\frac{4}{5}x\right)\right]_{\frac{5}{4\sqrt{2}}}^{\frac{5}{4}} = 4.\left[\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \sin^{-1}(1)\right]$$

$$= 4.\left[\frac{\pi}{4} - \frac{\pi}{2}\right] = 4\,\frac{\pi - 2\pi}{4} = -\pi$$

109). احسب $\int_0^{\sqrt{2}} \frac{x}{4+x^2}\,dx$

$$\int_0^{\sqrt{2}} \frac{x}{4\left(1+\frac{1}{4}x^4\right)}\,dx = \frac{1}{4}\int_0^{\sqrt{2}} \frac{x}{1+\left(\frac{1}{2}x^2\right)^2}\,dx = \frac{1}{4}\left[\tan^{-1}\left(\frac{1}{2}x^2\right)\right]_0^{\sqrt{2}}$$

$$= \frac{1}{4}[\tan^{-1}(1) - \tan^{-1}(0)] = \frac{1}{4}\left[\frac{\pi}{4} - 0\right] = \frac{\pi}{16}$$

110). احسب $\int_{\frac{1}{\sqrt{3}}}^{1} \frac{6}{x\sqrt{4x^2-1}}\,dx$

$$6\int_{\frac{1}{\sqrt{3}}}^{1} \frac{1}{x\sqrt{(2x)^2-1}}\,dx = 6[\sec^{-1}(2x)]_{\frac{1}{\sqrt{3}}}^{1} = 6\left[\sec^{-1}(2) - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)\right]$$

$$= 6\left[\frac{\pi}{3} - \frac{\pi}{6}\right] = 2\pi - \pi = \pi$$

111). احسب $\int \frac{1}{\sqrt{20+8x-x^2}}\,dx$

$$\int \frac{1}{\sqrt{20+8x-x^2}}\,dx = \int \frac{1}{\sqrt{-(x^2-8x-20)}}\,dx = \int \frac{1}{\sqrt{-[(x^2-8x-20)+16-16]}}\,dx$$

$$= \int \frac{1}{\sqrt{-[(x^2-8x+16)-20-16]}}\,dx = \int \frac{1}{\sqrt{-[(x-4)^2-36]}}\,dx$$

$$= \int \frac{1}{\sqrt{36-(x-4)^2}}\,dx = \int \frac{1}{\sqrt{36\left(1-\frac{(x-4)^2}{36}\right)}}\,dx = \int \frac{1}{\sqrt{36}\sqrt{1-\left(\frac{x-4}{6}\right)^2}}\,dx$$

$$= \int \frac{1}{6\sqrt{1-\left(\frac{x-4}{6}\right)^2}}\,dx = \int \frac{\frac{1}{6}}{\sqrt{1-\left(\frac{x-4}{6}\right)^2}}\,dx$$

$$= \sin^{-1}\left(\frac{x-4}{6}\right) + c$$

112). احسب $\int \frac{1}{x^2+x+\frac{5}{4}}dx$

$$\int \frac{1}{x^2+x+\frac{5}{4}}dx = \int \frac{1}{\left(x^2+x+\frac{5}{4}\right)+\frac{1}{4}-\frac{1}{4}}dx = \int \frac{1}{\left(x^2+x+\frac{1}{4}\right)+\frac{5}{4}-\frac{1}{4}}dx$$

$$= \int \frac{1}{\left(x+\frac{1}{2}\right)^2+1}dx = \int \frac{1}{1+\left(x+\frac{1}{2}\right)^2}dx = \tan^{-1}\left(x+\frac{1}{2}\right)+c$$

113). احسب $\int \frac{1}{(2x+1)\sqrt{x^2+x}}dx$

$$\int \frac{1}{(2x+1)\sqrt{x^2+x+\frac{1}{4}-\frac{1}{4}}}dx = \int \frac{1}{(2x+1)\sqrt{\left(x+\frac{1}{2}\right)^2-\frac{1}{4}}}dx = \int \frac{1}{(2x+1)\sqrt{\left(\frac{2x+1}{2}\right)^2-\frac{1}{4}}}dx$$

$$\int \frac{1}{(2x+1)\sqrt{\frac{1}{4}(2x+1)^2-\frac{1}{4}}}dx = \int \frac{1}{(2x+1)\sqrt{\frac{1}{4}[(2x+1)^2-1]}}dx = \int \frac{2}{(2x+1)\sqrt{[(2x+1)^2-1]}}dx$$

$$= \sec^{-1}(2x+1)+c$$

114). جد y' للدالة $y = \int_{\sin x}^{1} \sqrt{1-t^2}\,dt$

$t = \sin x \rightarrow t' = \cos x$ ⟶ (نفرض)

$$y' = 0 - \sqrt{1-\sin^2 x} * \cos x$$

$$y' = -\sqrt{\cos^2 x} * \cos x$$

$$y' = -\cos x * \cos x = -\cos^2 x$$

115). جد y' للدالة $y = \int_{1}^{2x} \cot(t^2)\,dt$

$t = 2x \rightarrow t' = 2$ ⟶ (نفرض)

$$y' = \cot(2x)^2 * 2 - \cot(1) \rightarrow y' = 2.\cot 4x^2 - \cot(1) * 0$$

$$y' = 2.\cot 4x^2$$

116). جد y′ للدالة $x=\int_0^{\sin y}\sqrt{1-t^2}\,dt$

(نفرض) $t=\sin y \to t'=\cos y * y'$

$$1=\sqrt{1-\sin^2(y)}*\cos(y)*y'-0$$

$$1=y'.\sqrt{\cos^2(y)}*\cos y$$

$$1=y'.\cos(y)*\cos(y)$$

$$1=y'.\cos^2(y)$$

$$y'=\frac{1}{\cos^2(y)}=\sec^2(y)$$

---

117). اذا كانت $\int_1^x \tan^{-1}(t)\,dt$ اثبت ان $(1+x^2)\,y''+y'+\cot^{-1}(x)-1=\frac{\pi}{2}$

$$y'=\tan^{-1}(x)\to y''=\frac{1}{1+x^2}$$

$$L.S=(1+x^2)\,\frac{1}{1+x^2}+\tan^{-1}(x)+\cot^{-1}(x)-1$$

$$=1+\tan^{-1}(x)+\cot^{-1}(x)-1$$

$$=\tan^{-1}(x)+\cot^{-1}(x)=\frac{\pi}{2}=R.S$$

---

118). اذا كانت $\int_1^{\cos^{-1}x} t\,dt$ اثبت ان $(1+x^2)\,y''+xy'-1=0$

$$y'=\frac{-\cos^{-1}(x)}{\sqrt{1-x^2}}\to y''=\frac{\sqrt{1-x^2}*\frac{1}{\sqrt{1-x^2}}-\cos^{-1}(x)*\frac{1}{2\sqrt{1-x^2}}*-2x}{1-x^2}=\frac{1+\frac{x.\cos^{-1}(x)}{\sqrt{1-x^2}}}{1-x^2}$$

$$L.S=(1+x^2)\,\frac{1+\frac{x\,.\cos^{-1}(x)}{\sqrt{1-x^2}}}{1-x^2}+x.\frac{-\cos^{-1}(x)}{\sqrt{1-x^2}}-1$$

$$= 1 + \frac{x\,.\cos^{-1}(x)}{\sqrt{1-x^2}} - \frac{x.\cos^{-1}(x)}{\sqrt{1-x^2}} - 1$$

$$= 0 = R.S$$

116). $y = 2x.\tan^{-1}(x) - \ln(1+x^2)$ جد y' للداله

$$y' = 2x.\frac{1}{1+x^2} + \tan^{-1}(x) * 2 - \frac{1}{1+x^2} * 2x$$

$$y' = \frac{2x}{1+x^2} + 2.\tan^{-1}(x) - \frac{2x}{1+x^2}$$

$$y' = 2.\tan^{-1}(x)$$

117). $y = x[\sin(\ln x) + \cos(\ln x)]$ جد y' للداله

$$y = x.\sin(\ln x) + x.\cos(\ln x)$$

$$y' = x * \cos(\ln x) * \frac{1}{x} + \sin(\ln x) + x * -\sin(\ln x) * \frac{1}{x} + \cos(\ln x)$$

$$y' = \cos(\ln x) + \sin(\ln x) - \sin(\ln x) + \cos(\ln x)$$

$$y' = 2.\cos(\ln x)$$

118). $x = \ln(\sec y + \tan y)$ جد y' للداله

$$1 = \frac{1}{\sec(y)+\tan(y)} * [\sec(y)\,.\tan(y)\,.y' + \sec^2(y)\,.y']$$

$$1 = \frac{1}{\sec(y)+\tan(y)} * y'.\sec(y)\,[\tan(y) + \sec(y)]$$

$$1 = y'.\sec(y)$$

$$y' = \frac{1}{\sec(y)} = \cos(y)$$

119). جد y' للداله $y^x = x^y$

"قانون اشتقاق داله مرفوعه لقوة داله اخرى"

$$\frac{d}{dx}\left[f(x)^{g(x)}\right] = f(x)^{g(x)}\left[\frac{g(x)}{f(x)} * f'(x) + \ln f(x) * g'(x)\right]$$

$$y^x\left[\frac{x}{y}.y' + \ln(y).1\right] = x^y\left[\frac{y}{x}.1 + \ln(x).y'\right]$$

$$y^x\frac{x}{y}.y' + y^x.\ln(y) = x^y\frac{y}{x} + x^y.\ln(x).y'$$

$$y^x\frac{x}{y}.y' - x^y.\ln(x).y' = x^y\frac{y}{x} - y^x.\ln(y)$$

$$y'\left[y^x\frac{x}{y} - x^y.\ln(x)\right] = x^y\frac{y}{x} - y^x.\ln(y)$$

$$y' = \frac{x^y\frac{y}{x} - y^x.\ln(y)}{y^x\frac{x}{y} - x^y.\ln(x)}$$

---

120). جد y' للداله $\pi^y = x^\pi$

$$\pi^y\left[\frac{y}{\pi}.0 + \ln(\pi).y'\right] = \pi[x]^{\pi-1}$$

$$\pi^y.\ln(\pi).y' = \pi[x]^{\pi-1}$$

$$y' = \frac{\pi[x]^{\pi-1}}{\pi^y.\ln(\pi)}$$

---

121). احسب $\int \frac{1}{\sqrt{x}(1+\sqrt{x})}dx$

$$\int \frac{\frac{1}{\sqrt{x}}}{(1+\sqrt{x})}dx = 2\int \frac{\frac{1}{2\sqrt{x}}}{(1+\sqrt{x})}dx = 2.\ln\left(1+\sqrt{x}\right) + c$$

122). احسب $\int \frac{x.\ln(x^2+1)}{x^2+1}dx$

$$\int \frac{x}{x^2+1}.\ln(x^2+1)\,dx = \frac{1}{2}\int \frac{2x}{x^2+1}.\ln(x^2+1)\,dx$$

$$= \frac{1}{2}.\frac{[\ln(x^2+1)]^2}{2} + c$$

$$= \frac{[\ln(x^2+1)]^2}{4} + c$$

---

123). جد y′ للداله $y = \ln\left(\frac{\sqrt{x}.\sqrt[3]{x+3}}{\sin(x).\sec(x)}\right)$

$$y = \ln\left(\sqrt{x}.\sqrt[3]{x+3}\right) - \ln(\sin(x).\sec(x))$$

$$y = \ln\left(\sqrt{x}\right) + \ln\left(\sqrt[3]{x+3}\right) - \ln(\sin(x)) - \ln(\sec(x))$$

$$y = \ln(x)^{\frac{1}{2}} + \ln(x+3)^{\frac{1}{3}} - \ln(\sin(x)) - \ln(\sec(x))$$

$$y = \ln(x)^{\frac{1}{2}} + \ln(x+3)^{\frac{1}{3}} - \ln(\sin(x)) - \ln(\sec(x))$$

$$y' = \frac{1}{(x)^{\frac{1}{2}}} * \frac{1}{2}(x)^{-\frac{1}{2}} + \frac{1}{(x+3)^{\frac{1}{3}}} * \frac{1}{3}(x+3)^{-\frac{2}{3}} - \frac{1}{\sin(x)} * \cos(x) - \frac{1}{\sec(x)} * \sec(x).\tan(x)$$

$$y' = \frac{1}{2}(x)^{-\frac{1}{2}} * (x)^{-\frac{1}{2}} + \frac{1}{3}(x+3)^{-\frac{2}{3}} * (x+3)^{-\frac{1}{3}} - \cot(x) - \tan(x)$$

$$y' = \frac{1}{2}(x)^{-1} + \frac{1}{3}(x+3)^{-1} - \cot(x) - \tan(x)$$

$$y' = \frac{1}{2x} + \frac{1}{3(x+3)} - \cot(x) - \tan(x)$$

$$y' = \frac{1}{2x} + \frac{1}{3x+9} - \cot(x) - \tan(x)$$

124). جد y' للدالة $y = x^x$

$$y' = x^x\left[\frac{x}{x} * 1 + \ln(x) * 1\right]$$

$$y' = x^x[1 + \ln(x)]$$

$$y' = x^x + x^x \ln(x)$$

---

125). جد y' للدالة $y = [\sec(x)]^{\cos(x)}$

$$y' = \sec(x)^{\cos(x)}\left[\frac{\cos(x)}{\sec(x)} * \sec(x).\tan(x) + \ln(\sec(x)) * -\sin(x)\right]$$

$$y' = \sec(x)^{\cos(x)}\left[\cos(x).\frac{\sin(x)}{\cos(x)} - \sin(x).\ln(\sec(x))\right]$$

$$y' = \sec(x)^{\cos(x)}[\sin(x) - \sin(x).\ln(\sec(x))]$$

---

126). جد y' للدالة $y = [\sin(x)]^x$

$$y' = \sin^x(x)\left[\frac{x}{\sin(x)} * \cos(x) + \ln(\sin(x)) * 1\right]$$

$$y' = \sin^x(x)[x.\cot(x) + \ln(\sin(x))]$$

---

127). جد y' للدالة $y = \cot(x)^{\tan(x)}$

$$y' = -\csc^2(x^{\tan(x)}) * (x)^{\tan(x)}\left[\frac{\tan(x)}{x} * 1 + \ln(x) * \sec^2(x)\right]$$

$$y' = -(x)^{\tan(x)}.\csc^2(x^{\tan(x)})\left[\frac{\tan(x)}{x} + \sec^2(x).\ln(x)\right]$$

---

128). جد y' للدالة $y = 4 + (x)^{\tan^{-1}(2x)}$

$$y' = 0 + (x)^{\tan^{-1}(2x)}\left[\frac{\tan^{-1}(2x)}{x} * 1 + \ln(x) * \frac{1}{1+(2x)^2} * 2\right]$$

$$y' = (x)^{\tan^{-1}(2x)}\left[\frac{\tan^{-1}(2x)}{x} + \frac{2}{1+4x^2}.\ln(x)\right]$$

129). احسب $\int \frac{1}{x.\ln(x)}dx$

$$\int \frac{1}{x.\ln(x)}dx = \int \frac{\frac{1}{x}}{\ln(x)}dx = \ln[\ln(x)] + c$$

---

130). احسب $\int \frac{1}{\sec(2x)-\tan(2x)}dx$

$$\int \frac{1}{\sec(2x)-\tan(2x)}dx = \int \frac{1}{\sec(2x)-\tan(2x)} * \frac{\sec(2x)+\tan(2x)}{\sec(2x)+\tan(2x)}dx$$

$$= \int \frac{\sec(2x)+\tan(2x)}{\sec^2(2x)-\tan^2(2x)}dx$$

$$= \int \frac{\sec(2x)+\tan(2x)}{1}dx$$

$$= \int \sec(2x) + \tan(2x)\, dx$$

$$= \int \sec(2x)\, dx + \int \tan(2x)\, dx$$

$$= \int \sec(2x) * \frac{\sec(2x)+\tan(2x)}{\sec(2x)+\tan(2x)}dx - \frac{1}{2}\int \frac{\sin(2x)}{\cos(2x)}dx$$

$$= \frac{1}{2}\int \frac{\sec^2(2x)+\sec(2x).\tan(2x)}{\sec(2x)+\tan(2x)}dx - \frac{1}{2}\ln[\cos(2x)]$$

$$= \frac{1}{2}.\ln[\sec(2x) + \tan(2x)] - \frac{1}{2}.\ln[\cos(2x)] + c$$

---

131). احسب $\int \frac{\ln(x)}{x[1+(\ln x)^2]}dx$

$$\int \frac{\ln(x)}{x[1+(\ln x)^2]}dx = \int \frac{\ln(x)}{x} * \frac{1}{1+(\ln x)^2}dx$$

$$= \frac{1}{2}\int \frac{\frac{2\ln(x)}{x}}{1+(\ln x)^2}dx$$

$$= \frac{1}{2}.\ln[1 + (\ln x)^2] + c$$

132). احسب $\int \frac{1}{x[1+(\ln x)^2]} dx$

$$\int \frac{1}{x[1+(\ln x)^2]} dx = \int \frac{\frac{1}{x}}{1+(\ln x)^2} dx = \tan^{-1}(\ln x) + c$$

133). احسب $\int \frac{x^3+2x^2-x+1}{x+2} dx$

$$\int (x^2 - 1) + \frac{3}{x+2} dx = \frac{x^3}{3} - x + 3\ln(x+2) + c$$

$$\begin{array}{r|l} & x^2 - 1 \\ \hline x+2 & x^3 + 2x^2 - x + 1 \\ & \mp x^3 \mp 2x^2 \\ \hline & -x + 1 \\ & \pm x \pm 2 \\ \hline & 3 \end{array}$$

134). جد y' للداله $y = e^{\cot^{-1}(\tan x)}$

$$y' = e^{\cot^{-1}(\tan x)} * \frac{-1}{1+\tan^2 x} * \sec^2 x$$

$$y' = \frac{-\sec^2 x}{\sec^2 x} . e^{\cot^{-1}(\tan x)}$$

$$y' = -e^{\cot^{-1}(\tan x)}$$

135). جد y' للداله $\tan(y) = e^x . \ln(x)$

$$\sec^2(y) * y' = e^x * \frac{1}{x} + \ln(x) * e^x * 1$$

$$\sec^2(y) * y' = e^x[\frac{1}{x} + \ln(x)]$$

$$y' = \frac{e^x[\frac{1}{x} + \ln(x)]}{\sec^2(y)}$$

136). جد y' للداله $y = e^{[\ln(x)]^2}$

$$y = e^{[\ln(x)]^2} \to y' = e^{[\ln(x)]^2} * 2\ln(x) * \frac{1}{x} \to y' = \frac{2}{x} . e^{[\ln(x)]^2} . \ln(x)$$

137). جد y′ للداله $y = e^{\sec^{-1}(x)}$

$$y' = e^{\sec^{-1}(x)} * \frac{1}{x\sqrt{x^2-1}}$$

$$y' = \frac{e^{\sec^{-1}(x)}}{x\sqrt{x^2-1}}$$

---

138). جد y′ للداله $e^y = x^e$

$$e^y * y' = e.\, x^{e-1}$$

$$y' = \frac{e.x^{e-1}}{e^y} = e^{1-y}.\, x^{e-1}$$

---

139). جد مشتقة الدالة

$$y = [5]^{\int_1^{\sin x} \sin^{-1}(t)dt}$$

$$y' = [5]^{\int_1^{\sin x} \sin^{-1}(t)dt}\, [\sin^{-1}(\sin x) * \cos(x) - 0] * \ln(5)$$

$$y' = [5]^{\int_1^{\sin x} \sin^{-1}(t)dt} * x.\cos(x)\,.\ln(5)$$

140). $y = \log_5\left[\sec\left(3^{\sqrt{x}}\right) + \tan\left(3^{\sqrt{x}}\right)\right]$ جد y' للداله

"قانون اشتقاق لوغارتيم داله لاساس عدد ثابت"

$$\frac{d}{dx}[\log_c f(x)] = \frac{f'(x)}{f(x) * \ln(c)}$$

$$y = \log_5\left[\sec\left(3^{\sqrt{x}}\right) + \tan\left(3^{\sqrt{x}}\right)\right]$$

$$y' = \frac{1}{\left[\sec\left(3^{\sqrt{x}}\right)+\tan\left(3^{\sqrt{x}}\right)\right]\ln(5)} . \frac{3^{\sqrt{x}}}{2\sqrt{x}}\sec\left(3^{\sqrt{x}}\right).\tan\left(3^{\sqrt{x}}\right) + \sec^2\left(3^{\sqrt{x}}\right)\frac{3^{\sqrt{x}}}{2\sqrt{x}}$$

$$y' = \frac{1}{\left[\sec\left(3^{\sqrt{x}}\right)+\tan\left(3^{\sqrt{x}}\right)\right]\ln(5)} . \frac{3^{\sqrt{x}}}{2\sqrt{x}}\sec\left(3^{\sqrt{x}}\right)\left[\tan\left(3^{\sqrt{x}}\right) + \sec\left(3^{\sqrt{x}}\right)\right]$$

$$y' = \frac{\frac{3^{\sqrt{x}}}{2\sqrt{x}}\sec\left(3^{\sqrt{x}}\right)}{\ln(5)} = \frac{1}{2\sqrt{x}.\ln(5)} * 3^{\sqrt{x}}.\sec\left(3^{\sqrt{x}}\right)$$

---

141). $\log_e(e^y) = x^{\log_e(x)}$ جد y' للداله

$$\frac{1}{e^y * \ln(e)} * e^y * y' = x^{\log_e(x)}\left[\frac{\log_e(x)}{x} * 1 + \ln(x) * \frac{1}{x * \ln(e)}\right]$$

$$\frac{y'}{\ln(e)} = x^{\log_e(x)}\left[\frac{\log_e(x)}{x} + \frac{\ln(x)}{x.\ln(e)}\right]$$

$$y' = x^{\log_e(x)}\left[\frac{\log_e(x)}{x} + \frac{\ln(x)}{x.\ln(e)}\right].\ln(e)$$

142). $e^{y} - e^{-y} = 2x$ اثبت ان $y = \ln[x + \sqrt{x^2 + 1}$ اذا كانت

$$e^{\ln f(x)} = \ln\left(e^{f(x)}\right) = f(x)$$

$$e^{-\ln f(x)} = \frac{1}{f(x)}$$

$$L.S = e^{\ln[x+\sqrt{x^2+1}]} - e^{-\ln[x+\sqrt{x^2+1}]}$$

$$= \left(x + \sqrt{x^2 + 1}\right) - \frac{1}{x+\sqrt{x^2+1}}$$

$$= \frac{\left(x+\sqrt{x^2+1}\right)^2 - 1}{x+\sqrt{x^2+1}} = \frac{x^2+2x.\sqrt{x^2+1}+x^2+1-1}{x+\sqrt{x^2+1}} = \frac{2x^2+2x.\sqrt{x^2+1}}{x+\sqrt{x^2+1}}$$

$$= \frac{2x\left[x+\sqrt{x^2+1}\right]}{x+\sqrt{x^2+1}} = \frac{2x\left[x+\sqrt{x^2+1}\right]}{x+\sqrt{x^2+1}} = 2x = R.S$$

---

143). $y = \ln[x + \sqrt{x^2 + 1}$ اثبت ان $e^{y} - e^{-y} = 2x$ اذا كانت

$$e^{y} - e^{-y} = 2x \to [e^{y} - 2x - e^{-y} = 0]e^{y} \to (e^{y})^2 - 2x.e^{y} - 1 = 0$$

$$\left[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

$$e^{y} = \frac{2x\pm\sqrt{4x^2+4}}{2} \to e^{y} = \frac{2x\pm\sqrt{4(x^2+1)}}{2} \to e^{y} = \frac{2x\pm2\sqrt{x^2+1}}{2}$$

: $x < \sqrt{x^2 + 1}$ فان $e^{y} > 0$ اذن الاشاره السالبه تهمل

$$e^{y} = \frac{2x+2\sqrt{x^2+1}}{2} \to e^{y} = \frac{2\left[x+\sqrt{x^2+1}\right]}{2} \to e^{y} = x + \sqrt{x^2 + 1}$$

$$\left[e^{y} = x + \sqrt{x^2 + 1}\right]\ln \ \to \ln(e^{y}) = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$y.\ln(e)=\ln\left(x+\sqrt{x^2+1}\right)\rightarrow y=\ln\left(x+\sqrt{x^2+1}\right)$$

144). احسب $\int \frac{e^{\frac{1}{x}}}{x^2}dx$

$$\int \frac{e^{\frac{1}{x}}}{x^2}dx=\int e^{\frac{1}{x}}.\frac{1}{x^2}dx=-\int e^{\frac{1}{x}}.\frac{-1}{x^2}dx=-e^{\frac{1}{x}}+c$$

145). احسب $\int \cos(2x).e^{(\sin x+\cos x)^2}\,dx$

$$\int \cos(2x).e^{(\sin x+\cos x)^2}\,dx=\int \cos(2x).e^{(\sin^2 x+2\sin x.\cos x+\cos^2 x)}\,dx$$

$$=\int \cos(2x).e^{(1+2\sin x.\cos x)}\,dx$$

$$=\int \cos(2x).e^{(1+\sin 2x)}\,dx$$

$$=\frac{1}{2}\int 2.\cos(2x).e^{(1+\sin 2x)}\,dx$$

$$=\frac{1}{2}.e^{(1+\sin 2x)}+c$$

146). احسب $\int e^x.\tan^2(e^x)\,dx$

$$\int e^x.\tan^2(e^x)\,dx=\int e^x.(\sec^2(e^x)-1)dx$$

$$=\int e^x.[\sec(e^x)]^2-e^x dx$$

$$=\int e^x.[\sec(e^x)]^2dx-\int e^x dx$$

$$=\tan(e^x)-e^x+c$$

146). احسب $\int e^{\ln(\sin x)}.\cot(x)\,dx$

$$\int e^{\ln(\sin x)}.\cot(x)\,dx=\int \sin(x).\cot(x)\,dx$$

$$=\int \sin(x).\frac{\cos(x)}{\sin(x)}dx$$

$$=\int \cos(x)\,dx=\sin(x)+c$$

147). احسب $\int \frac{e^{\tan^{-1}(3x)}}{1+9x^2}dx$

$$\int \frac{e^{\tan^{-1}(3x)}}{1+9x^2}dx = \int \frac{1}{1+9x^2}.e^{\tan^{-1}(3x)}dx$$

$$= \frac{1}{3}\int \frac{3}{1+(3x)^2}.e^{\tan^{-1}(3x)}dx$$

$$= \frac{1}{3}.e^{\tan^{-1}(3x)} + c$$

---

148). احسب $\int \frac{e^{\sqrt{x^2+1}}}{\sqrt{x^2+1}}dx$

$$\int \frac{x.e^{\sqrt{x^2+1}}}{\sqrt{x^2+1}}dx = \int \frac{x}{\sqrt{x^2+1}}.e^{\sqrt{x^2+1}}dx$$

$$= \int \frac{2x}{2\sqrt{x^2+1}}.e^{\sqrt{x^2+1}}dx$$

$$= e^{\sqrt{x^2+1}} + c$$

---

149). احسب $\int \frac{\sin(x)}{\cos^2(x)}.e^{\sec(x)}dx$

$$\int \frac{\sin(x)}{\cos^2(x)}.e^{\sec(x)}dx = \int \frac{\sin(x)}{\cos(x)}.\frac{1}{\cos(x)}.e^{\sec(x)}dx$$

$$= \int \tan(x).\sec(x).e^{\sec(x)}dx$$

$$= e^{\sec(x)} + c$$

150). هل يمكن تطبيق مبرهنه رول $f(x)=|x^2-1|$ جد الثابت ان امكن.

اولا نستخرج الفتره المحدده للداله وذالك بجعل الداله مساويه للصفر.

$$f(x)=0 \rightarrow |x^2-1|=0 \rightarrow x^2-1=0 \rightarrow x^2=1 \rightarrow x=\pm 1$$

اذن الفتره هي:- $[-1,1]$

١)- الداله مستمره على الفتره المغلقه $[-1,1]$.

٢)- اختبار قابلية الاشتقاق على الفتره المفتوحه$(-1,1)$:-

$$f(x)=|x^2-1|=\begin{cases} x^2-1, 1\le x\le -1 \\ 1-x^2, -1<x<1 \end{cases}$$

$$f'^{(x)}=\begin{cases} 2x, 1\le x\le -1 \\ -2x, -1<x<1 \end{cases} \rightarrow f'^{(x)}=-2x$$

نلاحظ ان القابلية الاشتقاق على الفتره المفتوحه تنطبق على الجزء السفلي من تعرف المشتقه.

٣)-

$$f(-1)=|1-1|=0$$

$$f(1)=|1-1|=0$$

يوجد ثابت:-

$$f'(c)=0$$

$$-2c=0$$

$$c=0$$

وعليه يوجد مماس واحد للداله موازي للمحور السيني ضمن الفتره المغلقه.

151). هل يمكن تطبيق مبرهنه رول $f(x)=x^2(1-x)^2$ جد الثابت ان امكن.

اولا نستخرج الفتره المحدده للداله وذالك بجعل الداله مساويه للصفر.

$f(x) = 0 \rightarrow x^2(1-x)^2 = 0 \rightarrow \quad x^2 = 0 \;,\; (1-x)^2 = 0$

$x = 0 \;,\; (1-x)^2 = 0 \rightarrow 1 - x = 0 \rightarrow x = 1$

اذن الفتره هي:- $[0,1]$

١)- الداله مستمره على الفتره المغلقه $[0,1]$

٢)- اختبار قابلية الاشتقاق على الفتره المفتوحه $(0,1)$:-

$f(x) = x^2(1-2x+x^2) \rightarrow f(x) = x^2 - 2x^3 + x^4$

$f'(x) = 2x - 6x^2 + 4x^3$

٣)-

$f(0) = f(1) = 0$

يوجد ثابت:- $f'(c) = 0$

$2c - 6c^2 + 4c^3 = 0$

$2c(1 - 3c + 2c^2) = 0$

$2c = 0 \;,\; 1 - 3c + 2c^2 = 0$

$c = 0 \;,\; (2c+1)(c-1) = 0 \rightarrow c = \frac{1}{2} \;,\; c = 1$

$c = 0 , c = \frac{1}{2} , c = 1 \qquad 0 < c < 1$

اذن:- $c = \frac{1}{2}$

وعليه يوجد مماس واحد للداله موازي للمحور السيني ضمن الفتره المغلقه.

152). هل يمكن تطبيق مبرهنه رول $f(x) = \frac{x^2-4x}{x+2}$ على الفتره المغلقة $[0,4]$ ثم جد الثابت ان امكن.

١)- الداله تكون غير معرفة عند (٢-) وهو لا ينتمي للفتره المغلقة وعليه الداله مستمرة على الفتره المغلقة $[0,4]$.

٢)- اختبار قابلية الاشتقاق على الفتره المفتوحه $(0,4)$:-

$$f'(x) = \frac{2x^2-4x+4x-8-x^2+4x}{(x+2)^2} = \frac{x^2+4x-8}{(x+2)^2}$$

الداله تكون غير معرفة عند (٢-) وهو لا ينتمي للفتره المفتوحه $(0,4)$ وعليه الداله للاشتقاق على الفتره المفتوحه $(0,4)$.

٣)-

$$f(0) = \frac{(0)^2-4(0)}{0+2} = 0$$

$$f(4) = \frac{(4)^2-4(4)}{4+2} = 0$$

يوجد ثابت:-

$$f'(c) = 0$$

$$\frac{c^2+4c-8}{(c+2)^2} = 0 \rightarrow c^2 + 4c - 8 = 0 \qquad , \qquad x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$c = \frac{-4\pm\sqrt{16+32}}{2}$$

بما ان :- $0 < c < 1$
فان الاشارة السالبه تحذف.

$$c = \frac{-4+\sqrt{16+32}}{2} = \frac{-4+4\sqrt{1+2}}{2} = -2 + 2\sqrt{3} \in (0,4)$$

وعليه يوجد مماس واحد للداله موازي للمحور السيني ضمن الفتره المغلقة.

152). هل يمكن تطبيق مبرهنه رول $f(x) = \sin(3x)$ على الفتره المغلقة $[0, \frac{\pi}{3}]$ ثم جد الثابت ان امكن.

١)- الداله مستمره على الفتره المغلقه $[0, \frac{\pi}{3}]$.

٢)- اختبار قابلية الاشتقاق على الفتره المفتوحه $(0, \frac{\pi}{3})$:-

$$f'(x) = 3.\cos(3x)$$

الدالة قابله للاشتقاق على تلك الفتره.

٣)-

$$f(0) = f\left(\frac{\pi}{3}\right) = 0$$

يوجد ثابت:-

$$f'(c) = 0$$

$$3.\cos(3c) = 0 \to \cos(3c) = 0 \to c = \frac{\pi}{6} \in (0, \frac{\pi}{3})$$

---

153). احسب الغاية $\lim_{x \to 2} \frac{\cos(\frac{\pi}{x})}{x-2}$

"قانون لوبيتال لايجاد الغاية اذا كان التعويض المباشر يعطي صيغ غير محدده"

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

$$\lim_{x \to 2} \frac{\cos(\frac{\pi}{x})}{x-2} = \lim_{x \to 2} \frac{-\sin(\frac{\pi}{x}).\frac{\pi}{x^2}}{1}$$

$$= \lim_{x \to 2} -\sin\left(\frac{\pi}{x}\right).\frac{\pi}{x^2}$$

$$= -\sin\left(\frac{\pi}{2}\right).\frac{\pi}{4}$$

$$= -\frac{\pi}{4}$$

153). $\lim_{h\to 0} \frac{\cos(x+h)-\cos(x)}{h}$ احسب الغاية

$$\lim_{h\to 0} \frac{\cos(x+h)-\cos(x)}{h} = \lim_{h\to 0} \frac{-\sin(x+h)*(0+1)+\sin(x)*0}{1}$$

$$= \lim_{h\to 0} -\sin(x+h)$$

$$= -\sin(x)$$

---

154). $\sec^{-1}(-2) - \sec^{-1}(2)$ جد قيمة

$$\sec^{-1}(-x) = \pi - \sec^{-1}(x)$$

$$\sec^{-1}(-2) = \pi - \sec^{-1}(2)$$

$$\sec^{-1}(2) = y \to \sec(y) = 2 \to y = \frac{\pi}{3}$$

$$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}$$

---

155). $\cos[\sin^{-1}(0.6)]$ جد قيمة

$$\sin^{-1}(0.6) = y \to \sin(y) = 0.6 \to y = 37^\circ$$

$$\cos(y) = \cos(37^\circ) = \frac{4}{5}$$ ط١

$$\sin^{-1}(0.6) = y \to \sin(y) = 0.6 = \frac{6}{10} = \frac{3}{5} \to \frac{\text{المقابل}}{\text{الوتر}}$$ ط٢

$$(5)^2 = (3)^2 + (\text{المجاور})^2 \to 25 = 9 + (\text{المجاور})^2$$

$$(\text{المجاور})^2 = 16 \to \text{المجاور} = 4$$

$$\cos(y) = \frac{\text{المجاور}}{\text{الوتر}} = \frac{4}{5}$$

جد قيمة

156). $\cos\left[2.\sec^{-1}\left(\frac{1}{x}\right)\right]$

$$\cos[2x] = 2\cos^2[x] - 1$$

$$\cos\left[2.\sec^{-1}\left(\frac{1}{x}\right)\right] = 2\cos^2\left[\sec^{-1}\left(\frac{1}{x}\right)\right] - 1$$

$$\sec^{-1}\left(\frac{1}{x}\right) = \cos^{-1}(x)$$

$$2(\cos\,[\cos^{-1}(x)\,])^2 - 1$$

$$2x^2 - 1$$