Chapter Two

Pressure of fluids

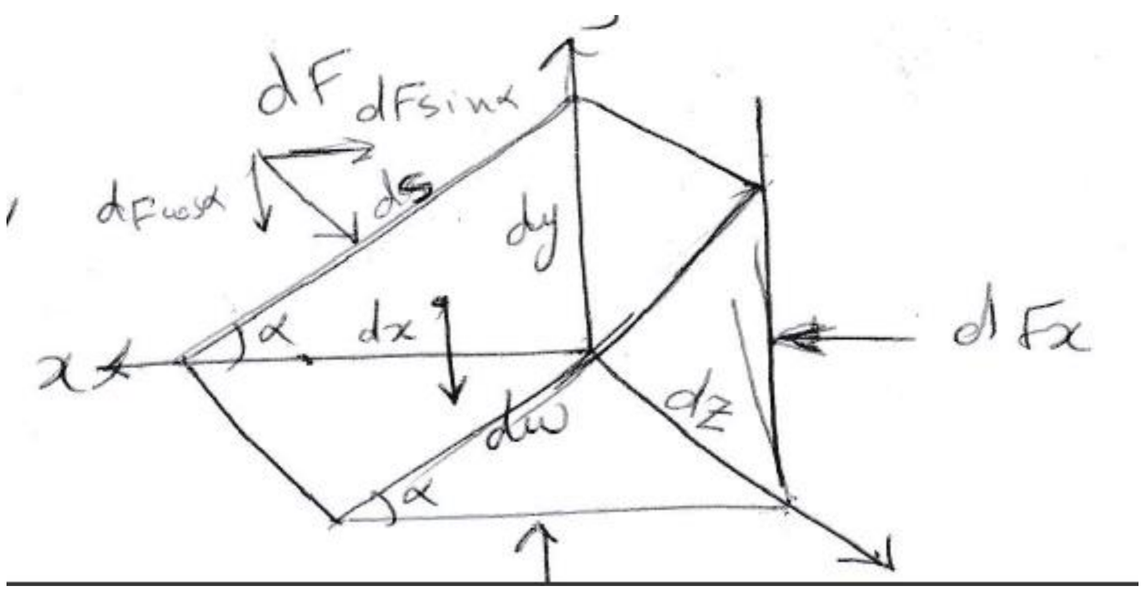
In this chapter we will deal with fluids at static, where there is no shear stress, hence only normal pressure forces are present.

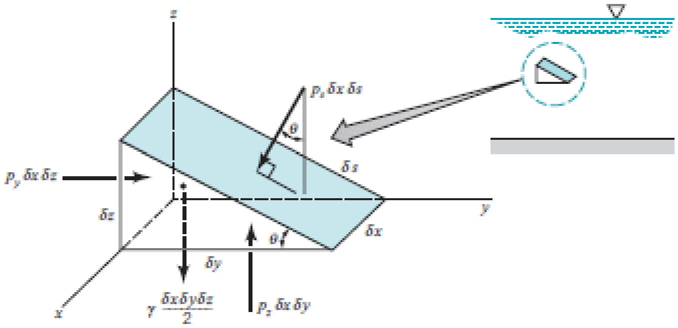
If F represent the total force exerted on finite area A, then

$$P = \frac{F}{A}$$

Pressure at point

Pressure is the compressive force per unit area. The fluid pressure intensity at appoint is of equal magnitude in all direction (Pascal’s law). This can be demonstrated by considering a small wedge shaped fluid element and this element at rest





We are considering the situation in which there are no shearing stresses, the only external forces acting on the wedge are due to the pressure and the weight. For simplicity the forces in the *x* direction are not shown, and the *z* axis is taken as the vertical axis so the weight acts in the negative *z* direction. Although we are primarily interested in fluids at rest, to make the analysis as general as possible, we will allow the fluid element to have accelerated motion. The assumption of zero shearing stresses will still be valid so long as the fluid element moves as a rigid body; that is, there is no relative motion between adjacent elements.
The equations of motion Newton's second law, F=m * a in the *y* and *z* directions are, respectively, where and are the average pressures on the faces, and are the fluid specific weight and density, respectively, and the accelerations. Note that a pressure must be multiplied by an appropriate area to obtain the force generated by the pressure. It follows from the geometry that

$$\sum F_x = 0$$

$$dFsin\,\alpha - dFx = 0$$

$$\sin\alpha = \frac{dy}{ds}$$

$Pd_y=P_x\, d_y$
$P=P_x$
$\sum F_y = 0$ Assume the wedge is very small the weight is very small so it is neglected

$dF_y-\ dF\cos\alpha =0$

$P_y*dz*dx= P\cos\alpha\, ds\, dz$

Cos α = (dx/ds)

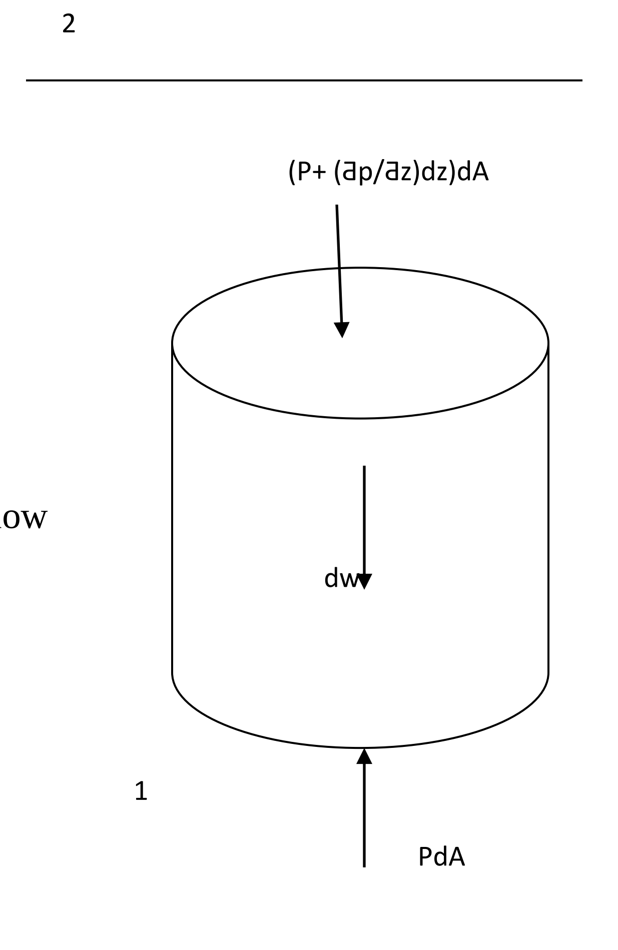
Py dx= Pdx

Py= P

The pressure is the same in all direction

P=Px=Py=Pz   Pascal's law

1 Pa = 1 N/m^2 kPa= 1000 Pa



Variation of pressure in a static fluid

Consider the differential element of a static shown below
W= m g

$$\sum Fz = 0$$

$$PdA - \left(p + \frac{\partial P}{\partial z} dz\right) dA - \rho g\ dz\ dA = 0$$

$$\frac{\partial p}{\partial z} = -\rho g$$

$$\partial P = -\rho g\ \partial z$$

For incompressible fluid density is constant, P change only with z direction

$$dp = -\rho g\ dz$$

Integrate

$$\int_{p1}^{p2} dP = -\rho g \int_{z1}^{z2} dz$$

$P_1 - P_2 = \rho g\,(Z_2 - Z_1)$

When point 2 on the free surface

$P_2 = P_{atm} = 0$

$Z_2 - Z_1 = h$

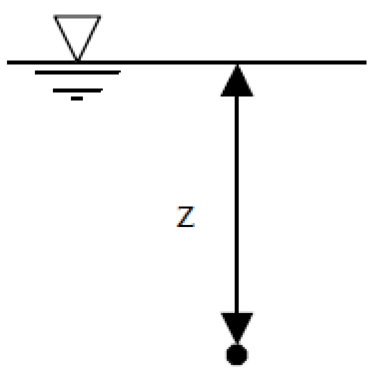
$P = \rho\, g\, h = \gamma\, h$

Where P fluid pressure (Pa)

h= depth of liquid (m)

ρ= density of liquid (kg/m^3)

<u>Note</u> In liquids the distance are measured vertically down from free surface

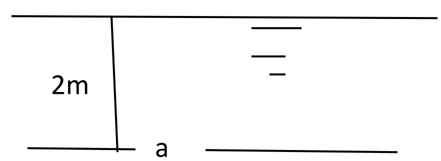


Example Calculate the pressure at piont a

Pa= ρ g h

Pa=1000*9.81 *2

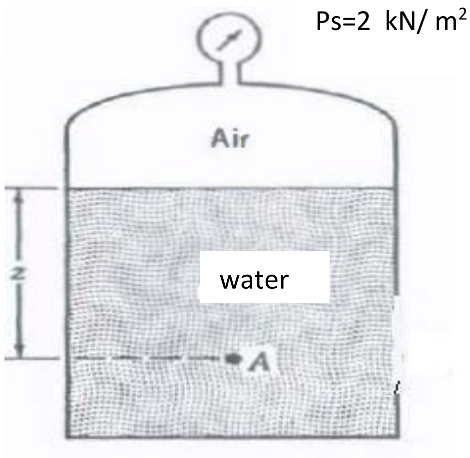
= 19620 N/m^2= 19.62 kN/m^2



If there is a pressure = Ps acting on the free surface

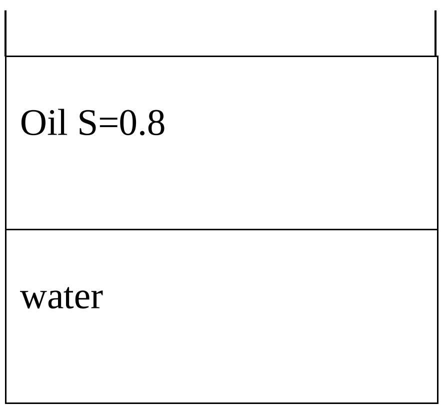
$P = = \rho\ g\ h + Ps$

Ex: Find the pressure at A if the total depth of water =70 cm and the z=50cm
$PA = \rho\ g\ h + Ps$
$= 9.81*(50/100)+2=6.905$ kPa

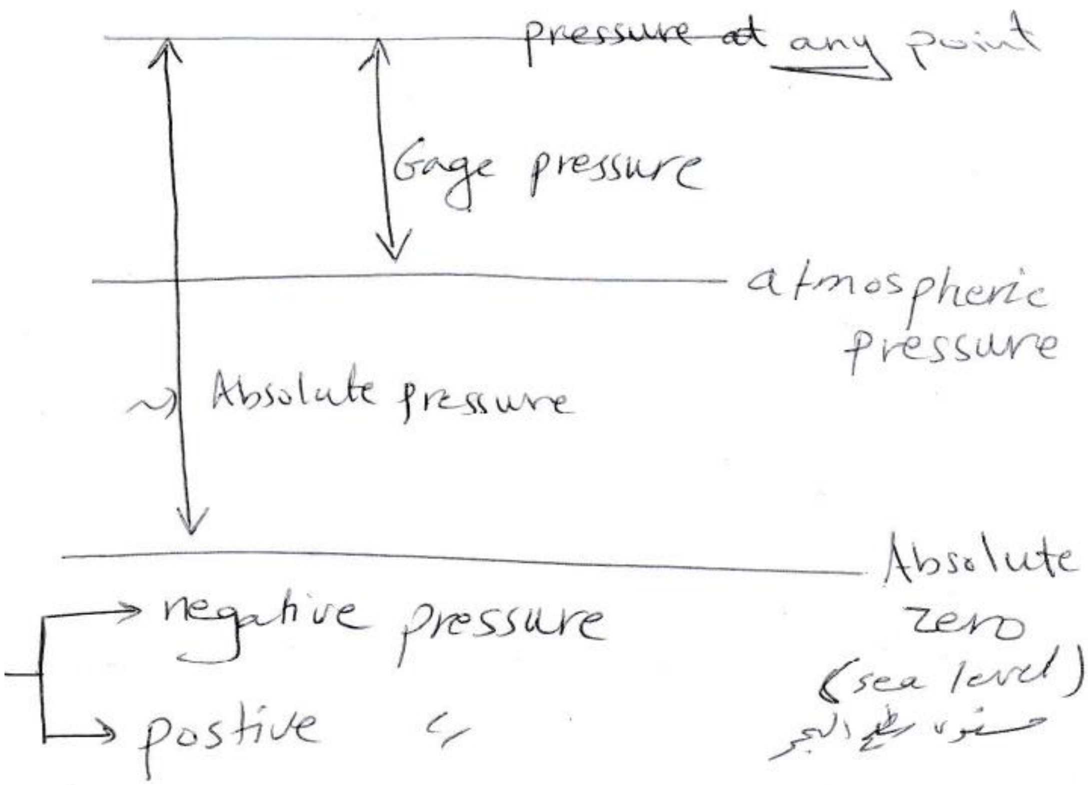


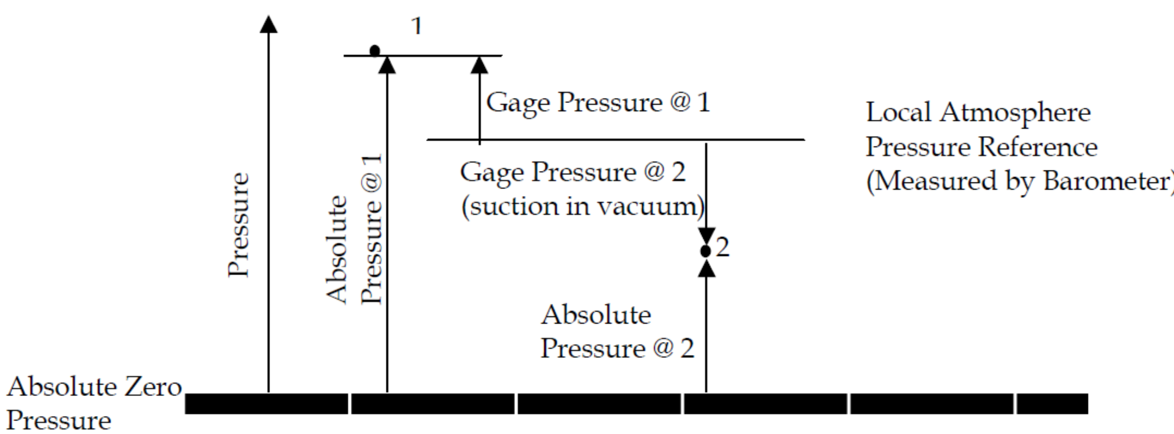
water

Ex: For the container shown below. Find the pressure at point a, b, c, d and e. If depth of water 1m and depth of oil 1m, point a at the mid of oil, point b at the end of oil, point c at the mid of water, point d at the end of water, and point e at the free surface.



Absolute and Gage Pressure





Absolute pressure is the pressure measured from the absolute zero. (sea level)
Gage pressure is the pressure measured relative to the atmospheric pressure.
The pressure at a point within a fluid mass will be designated as either an ***absolute*** **pressure** or a ***gage*** **pressure.**
Absolute pressures are always positive, but gage pressures can be either positive or negative depending on whether the pressure is above atmospheric pressure. A negative gage pressure is also referred to as a suction or ***vacuum*** **pressure**.

Measurements of the pressure

The various devices adopted for measuring fluid pressure may be classified into;

1- Manometers 2- Mechanical Gages

Manometers are pressure devices which are based on the principle of balancing the columns of liquid whose pressure is to be found by the same to another column of liquid. The manometers are classified into:

Simple manometers * differential manometers

Simple manometers

They measure pressure at a point in a fluid contained in a pipe or vessel.

Differential manometers

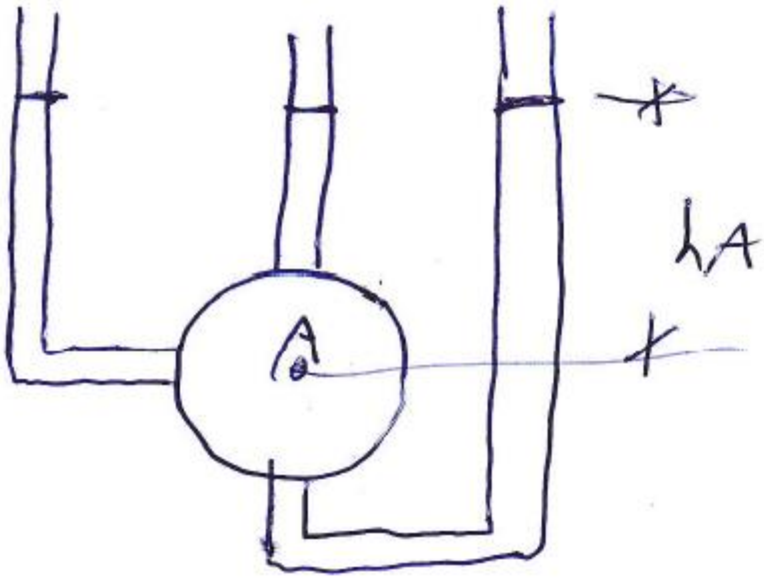
They measure the difference of pressure between any two points in a fluid contained in a pipe or a vessel.

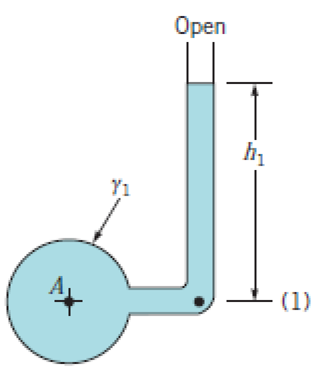
Simple Manometers

A simple manometer consists of a glass tube having one of its end connected to the point where the pressure is to be measured and the other remains open to the atmospheric Three common types of manometers include the piezometer tube, the U-tube manometer, and the inclined-tube manometer.

Piezomters

A piezomter is used for measuring small pressures of liquids. It consists of glass tube inserted in the wall of a pipe or vessel containg a liquid whose pressure is to be measured. Pioezometers measure gage pressure only since the surface of the liquid in the tube is subjected to atmospheric pressure.





If specific weight = γ

$P_A = \gamma h_A$

The pressure at any point in the liquid is indicated by the height of the liquid in the tube.

<u>Manometers</u>

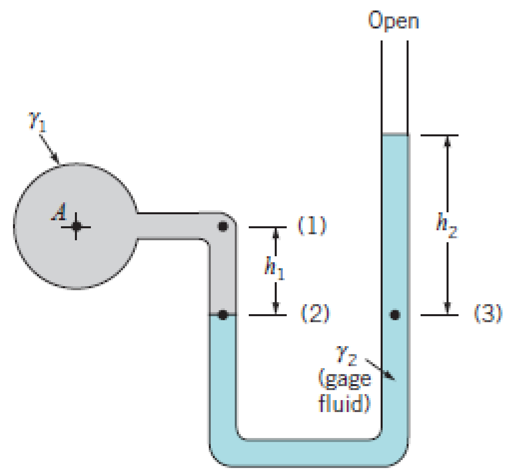
1- Simple Manometer 2- Micro Manometer 3- Differential Manometer 4- Inverted differntial manometer.

the first and seconed manometer are used to measure the pressure at point, the third anf forth manometers are measure the difference between two points.

1- <u>Simple Manometer</u> : for moderate and high pressure at point

Case A

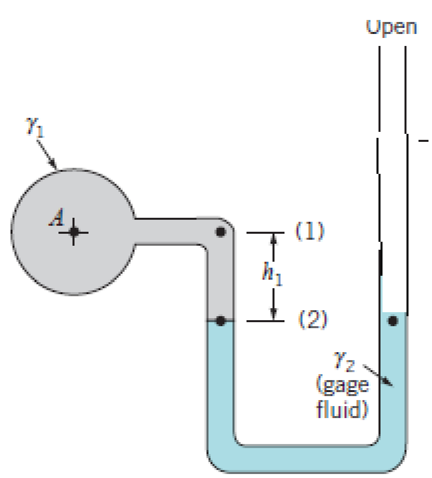
Pressure at 2= pressure at 3



$P_A + \rho_1 g h_1 = \rho_2 g h_2 + P_{atm}$
$P_A = \gamma_2\, h_2 - \gamma_1\, h_1$

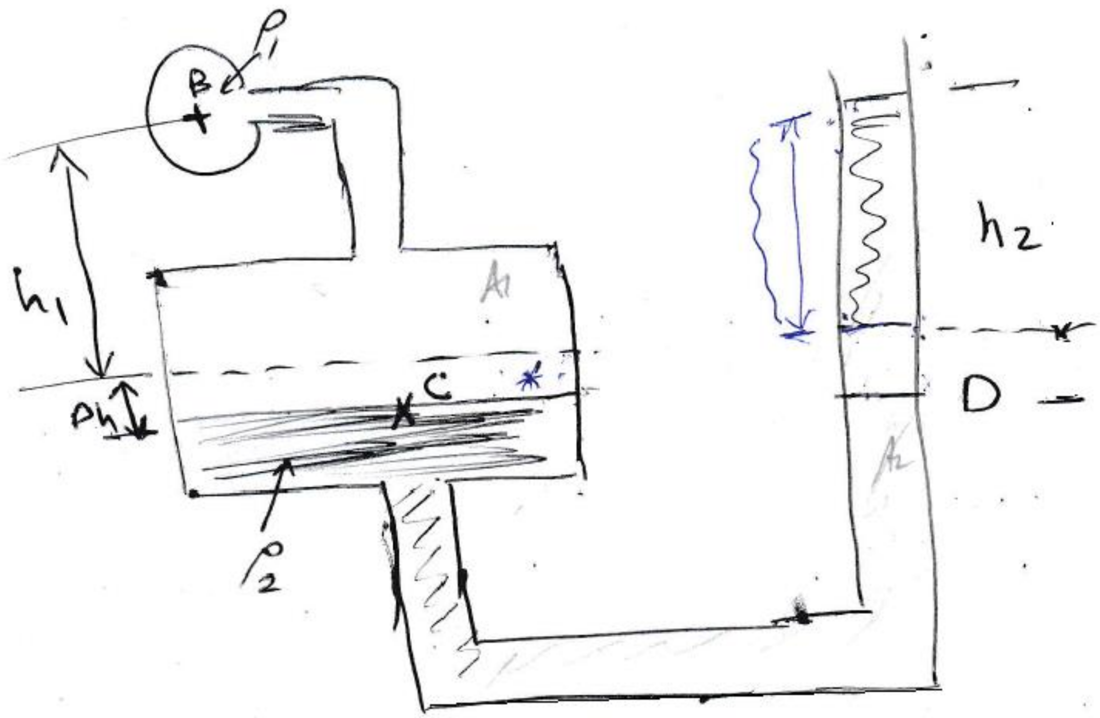
Case B

P2= P3



$P_A + \gamma_1 h_1 + \gamma_2\, h_2 = 0$
$PA = -(\gamma_1 h1 + \gamma_2\, h_2)$ or $PA = (\gamma_1 h_1 + \gamma_2\, h_2)$ Vacuum

2- <u>Micro- Manometer</u> used for the minimum pressure at high accuracy
A= cross sectional area of the basin
a= cross sectional area of the tube



Pc=PD

$P_B+\rho_1 g\ h_1+\rho_1 g\ \Delta h= \rho_2 g\ h_2+\rho_2 g\ \Delta h$

$P_{B=}\ \rho_2 g\ h_2 - \rho_1 g\ h_1 + \Delta h(\rho_2 g - \rho_1 g)$

Since $A\ \Delta h= a.\ h_2$

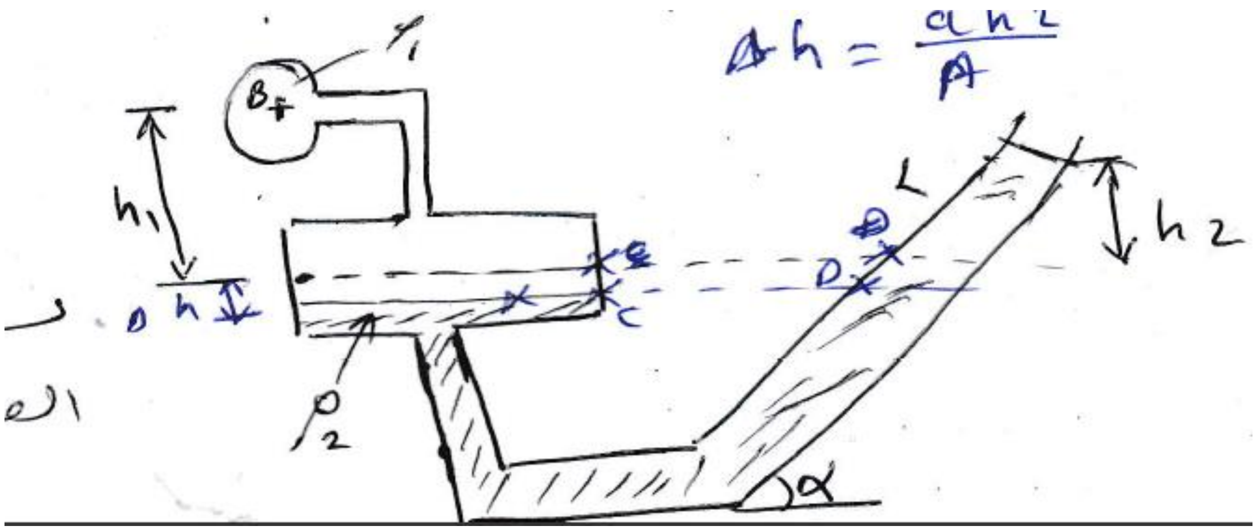
$\Delta h= (a/A)\ h_2$

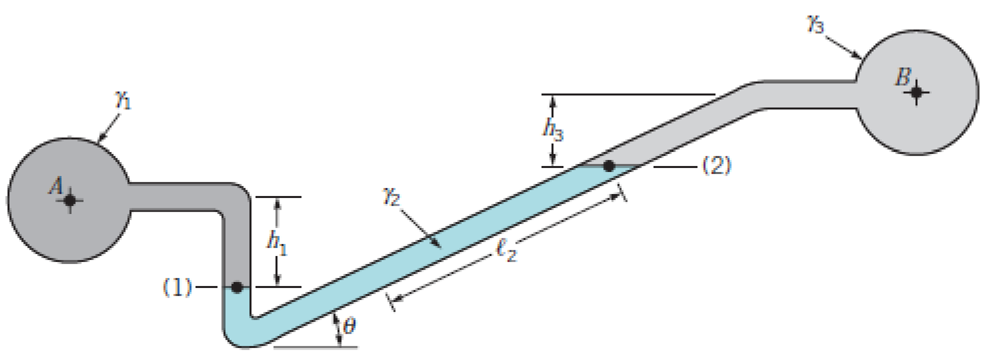
$P_{B=}\ \rho_2 g\ h_2 - \rho_1 g\ h_1 + (a/A)\ h_2\ (\rho_2 g - \rho_1 g)$

3- <u>Inclined Micromanometer</u>

Sin α = h2/L

h2= L sin Θ



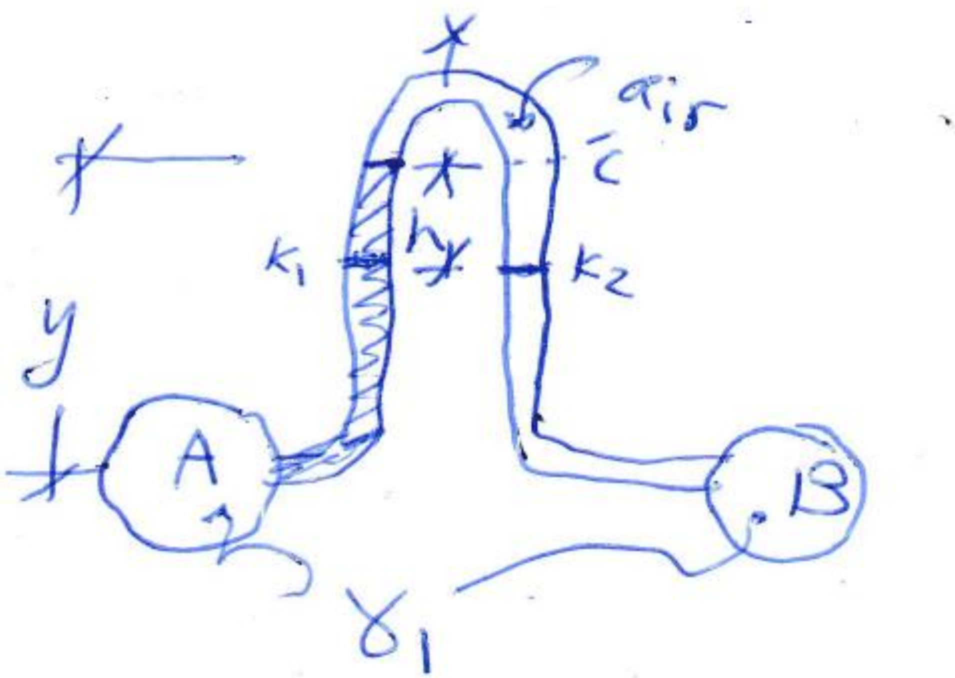


<u>Differntial Manometers</u>

They consist of a bent glass tube, the two ends of which are connected to each of the two gage points between which pressure difference is required. Some of common types of differential manometers are 1- Inverted U tube manometer 2- U tube differential manometer 3- Micromanometer

1- Inverted U tube manometer: it consists of a glass tube bent in U-shape and held inverted as shown below. This manometer is suitable to measure small difference pressure

Air lock



$P_{k1}=P_{k2}$

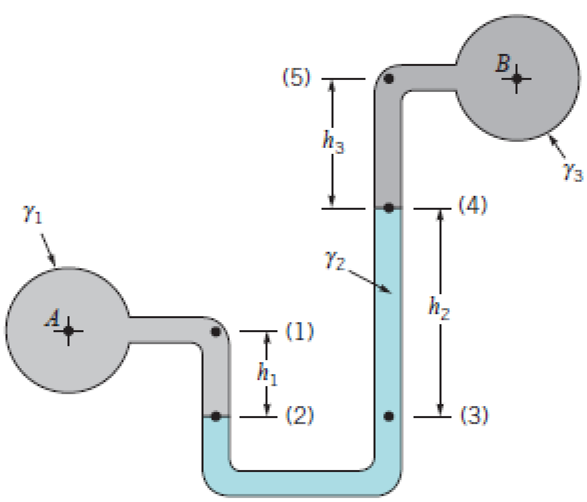
$P_A-Y\gamma= P_B-(y-h)\,\gamma$

$P_A-P_B= -(y-h)\,\gamma + Y\gamma$

$P_A-P_B= h\,\gamma$

Note : Air lock is provided at the top of the inverted U tube which facilities the rasing the liquid column in both the tube.

2- U- tube differential manometer



It consists of a glass tube bent in U-shape, the two ends of it are connected to the gage points between which the pressure difference is required to be considerd .

$P_2=P_3$

$P_A+ \gamma_1 h_1=P_B+ \gamma_3 h_3+ \gamma_2 h_2$

$P_A- P_B= \gamma_3 h_3+ \gamma_2 h_2 - \gamma_1 h_1$

The liquid usedin the barometer is usually mercury because it density is large to enable a resonable short tube to be used and its vapor pressure is negligible small at ordinary temperature.

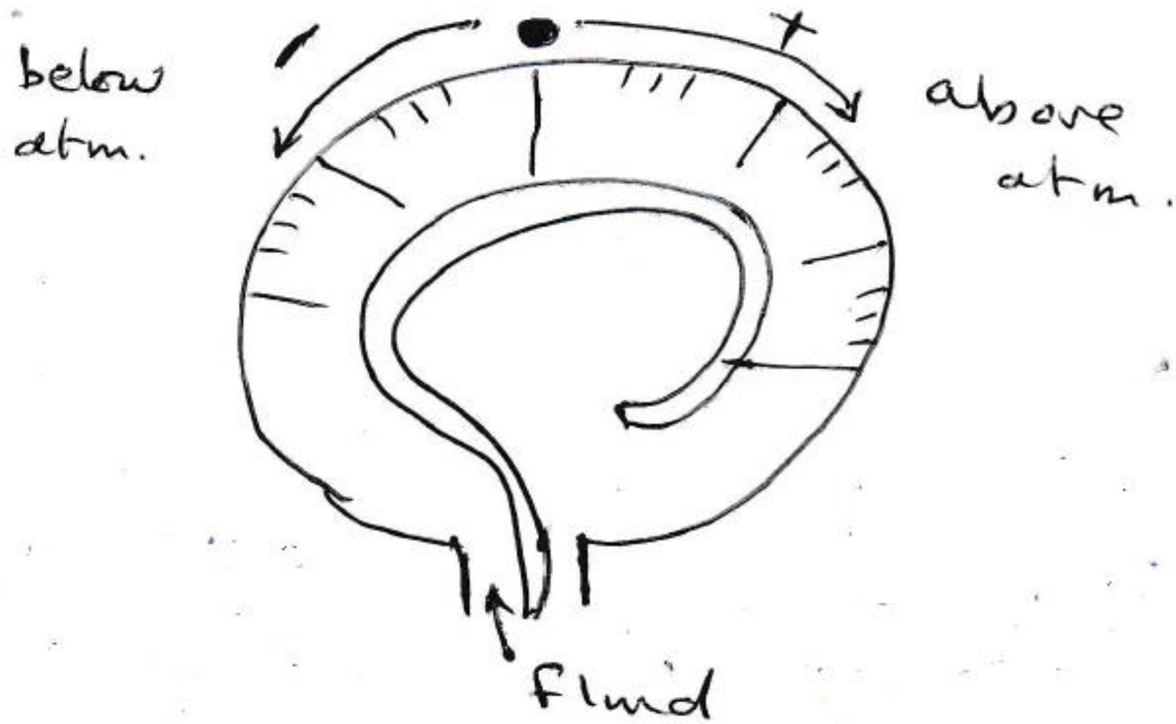
$Pa=101.3\ kN/m^2$

Or $Pa= 760$ mm of Hg

Or Pa= 10,3 m of H_2O

Mechanical gage (Bourden pressure gage)

Used for measure the relative air pressure and the other gases, the difference between the mechanical gage the pressure read immediately but the other gages the pressure can be find after some calculations.



Example:

For the inclined-tube manometer of Fig. shown below the pressure in pipe A is 0.6 psi. The fluid in both pipes A and B is water, and the gage fluid in the manometer has a specific gravity of 2.6. What is the pressure in pipe B corresponding to the differential reading shown? If h1= 8 cm, h2=20 cm , h3= 8 cm