Dynamics of Coupled Lasers.Theory and Applications

Outline

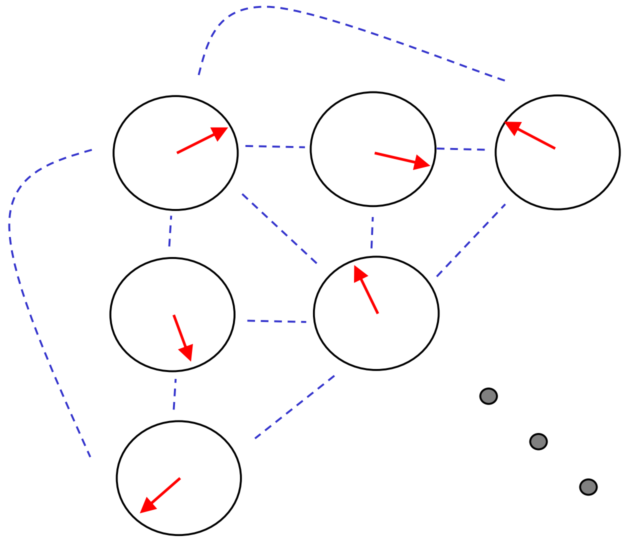
- Brief intro
- Types of synchronization
  - Phase synchronization (frequency locking)
  - Complete
  - Generalized
- Synchronization of coupled lasers
- Phase synchronization of limit cycle oscillators
- Summary and conclusions

Synchronization – What is it?

Many things in nature oscillate

Many things in nature are connected

Definition: Synchronization is the adjustment of rhythms of oscillating objects due to their weak interactions*



*A. Pikovsky, M. Rosenblum and J. Kurths, *Synchronization*, Cambridge Univ. Press 2001

Why Synchronization is Interesting

Physical systems: Clocks; Pattern formation;
Dynamics of coherent structures
in spatially extended systems (Epidemics, Neurons,
Lasers, continuum mechanics,…)

Engineering: Communication systems;
Manufacturing processes
Coupled fiber lasers for welding
Coupled chemical reactors for etching

Biological systems: Healthy dynamical rhythms;
Dynamical diseases;
Population dynamics

Defense Applications: New tunable radiation sources
THz sources for IED detection
Secure communications
Communicating autonomous vehicles

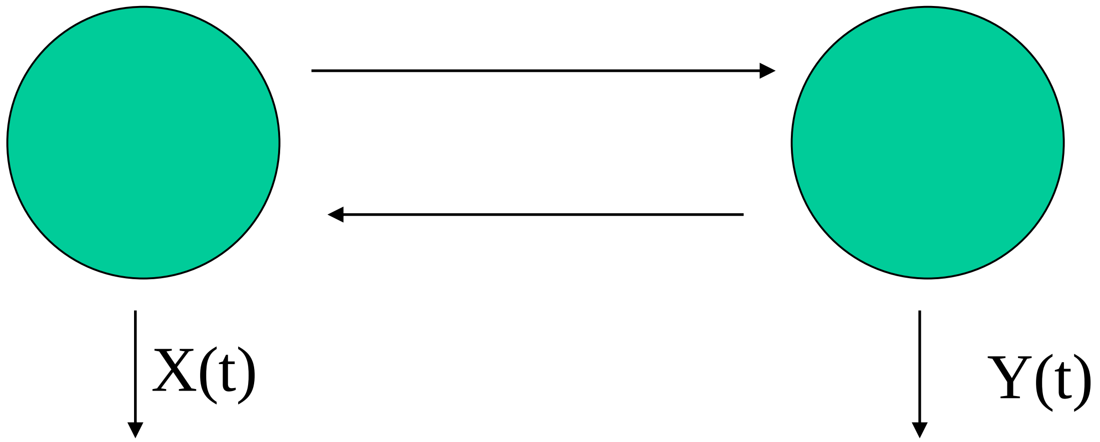
Complete Synchronization

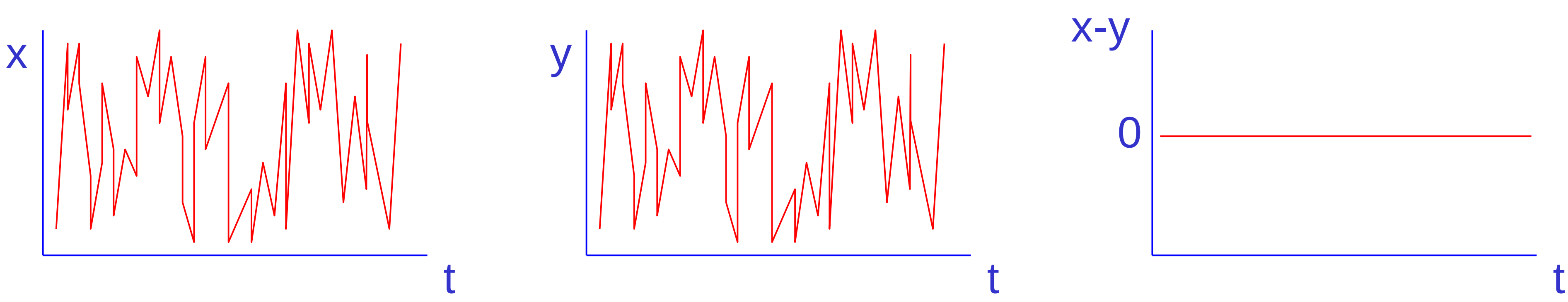
Complete or identical synchronization (easiest to understand)

The difference between states of systems goes asymptotically to zero as time goes to infinity.

$$\lim_{t\to\infty}\left|X(t)-Y(t)\right|=0$$

Amplitudes and phases are identical





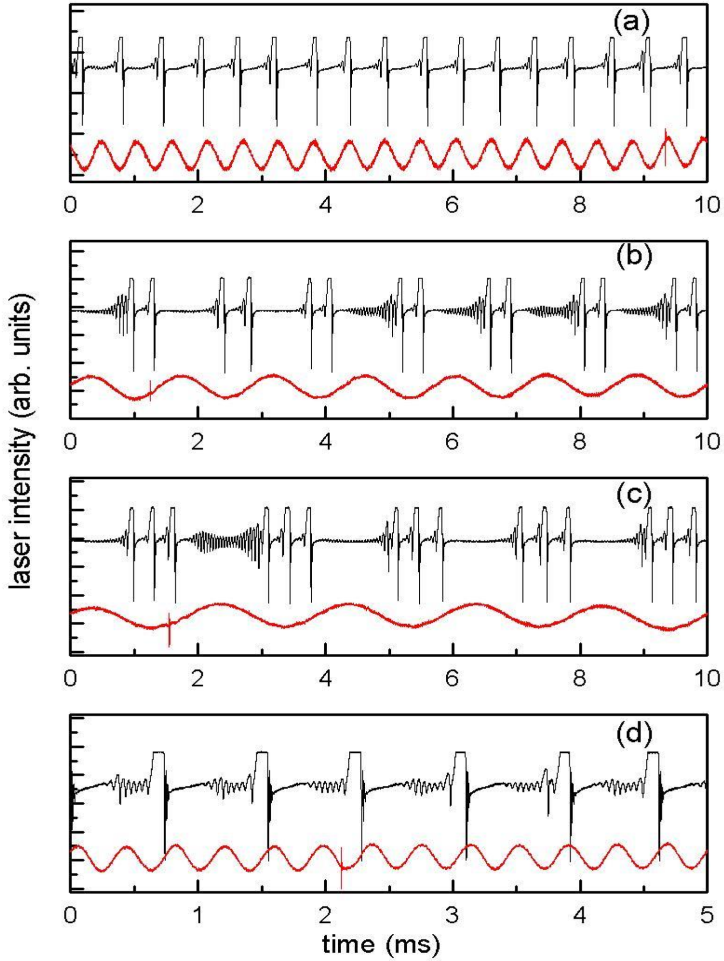
Phase synchronization

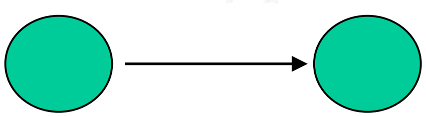
Phases have a functional relationship

If phases are locked, or entrained,
Then dynamics is in phase synchrony
Frequency locking

$$\left|m\dot{\phi}_1 - n\dot{\phi}_2\right| < const$$

Unidirectional Coupling in a Laser (Meucci)
Synchronization phases of one oscillator to an external oscillator

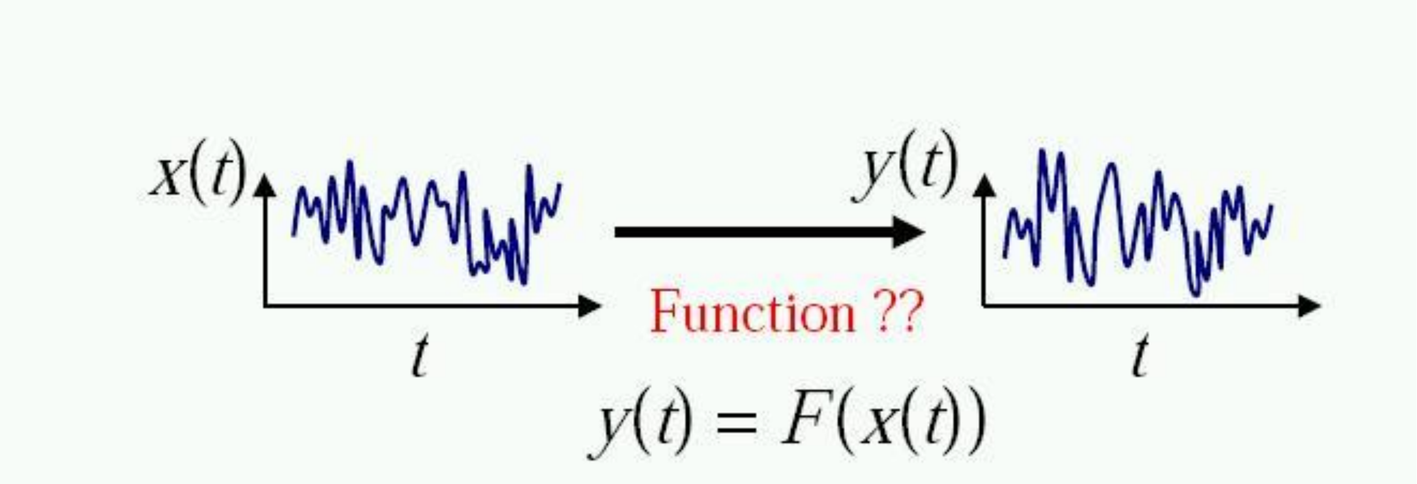




Generalized synchronization

Systems exhibit quite different temporal evolutions,
There exists a functional relation between them.

N. F. Rulkov et al. Phys. Rev. **E51** 980, (1995)



Detecting generalized synchronization is difficult to implement in experiments
Good for large changes in time scales

Generalized synchronization

The auxiliary system method:

Two or more replicas of the response system are available (i.e. obtained starting from different initial conditions)
Complete synchronization between response systems implies generalized synchronization between response and drive systems.

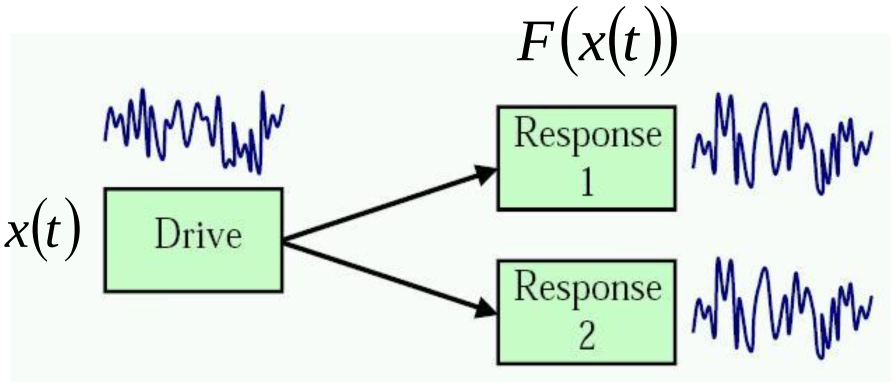
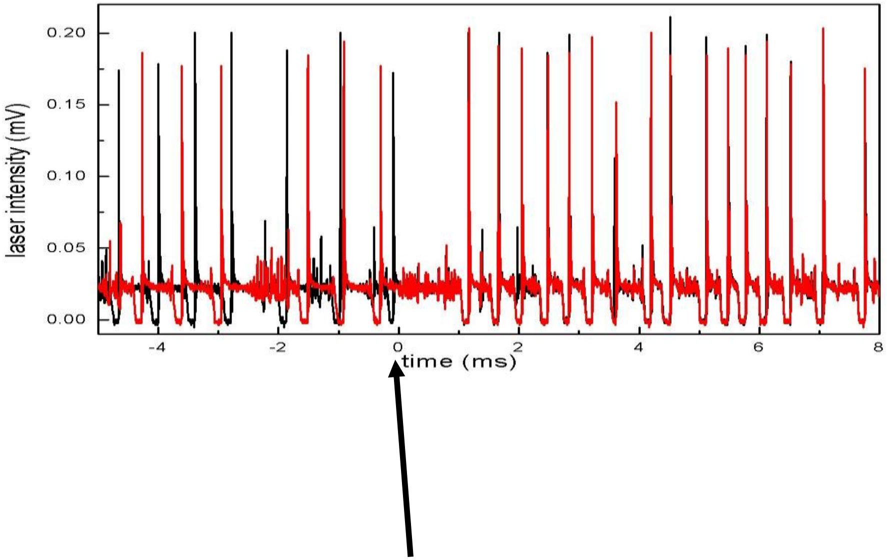


Figure 1.3: Schematic for generalized synchronization detected using the auxiliary method. Note that the drive signal is not synchronized to the response but that the response signals are synchronized to each other[15].

Experimental evidence of NIS
CO_2 laser (Meucci)

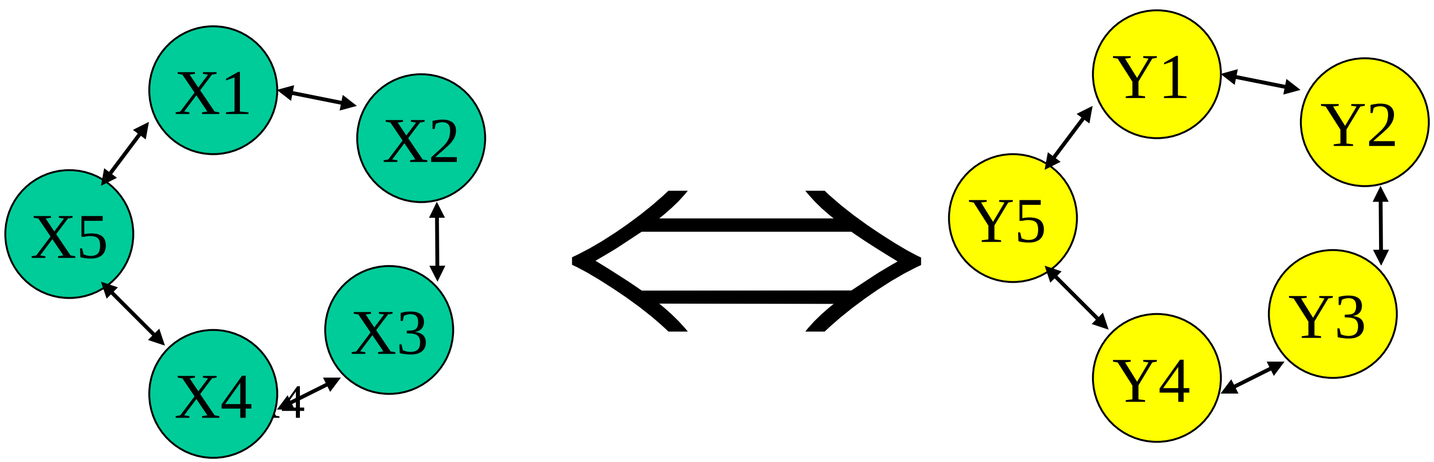


Start of the common noise signal

Application 1: coupled arrays of limit cycle oscillators

Coupled arrays of Limit cycle oscillators

- How diffusive coupling leads to different types of **phase-locked synchronization**
- The effect of global coupling and **generalized synchronization** via bifurcation analysis



Landsman and Schwartz, PRE 74, 036204 (2006)

Application 2: coupled lasers

- Mutually coupled, time-delayed semi-conductor lasers
  - **Generalized synchronization** can be used to understand **complete synchronization** of a group of lasers

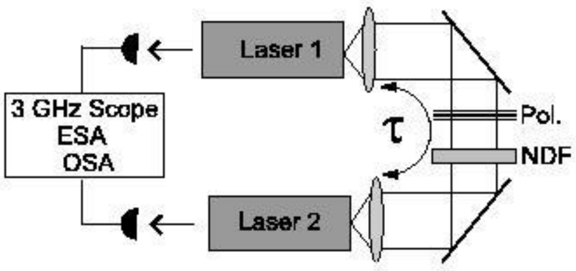
A.S. Landsman and I.B. Schwartz, *PRE* 75, 026201 (2007), http://arxiv.org/abs/nlin/0609047

Coherent power through delayed coupling architecture

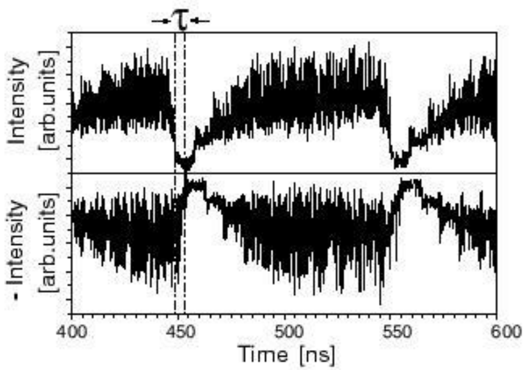
Experiments with Delayed Coupling – N=2

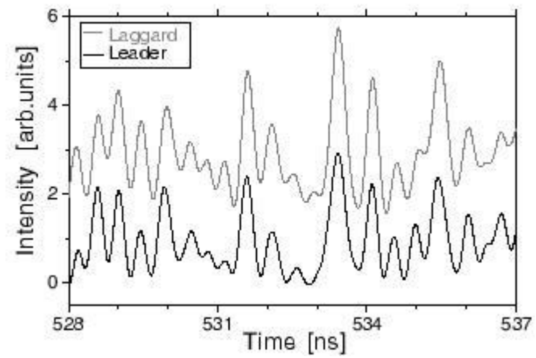
Coupled lasers do not have a stable coherent in-phase state

<u>Two</u> delay coupled semiconductor lasers: <u>experiment</u> showing stable out-of-phase state



Time series synchronized after being shifted by coupling delay

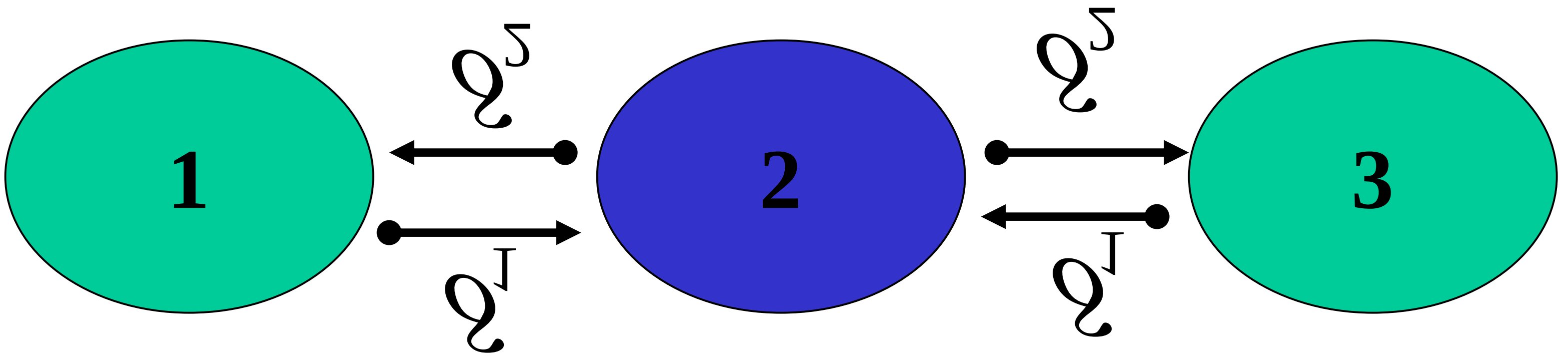




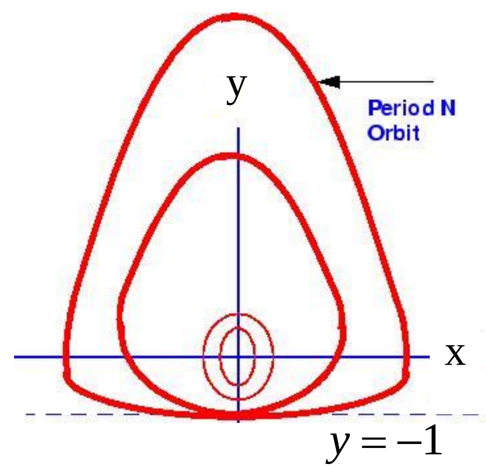
Heil et al PRL, 86 795 (2001)

Leaders and followers switch over time

Chaotic Synchronization of 3 semiconductor lasers with mutual, time delayed coupling



Scaled equations of a single, uncoupled laser:



$$dy/dt = x(1 + y)$$

$$dx/dt = -y - \varepsilon x(a + by)$$

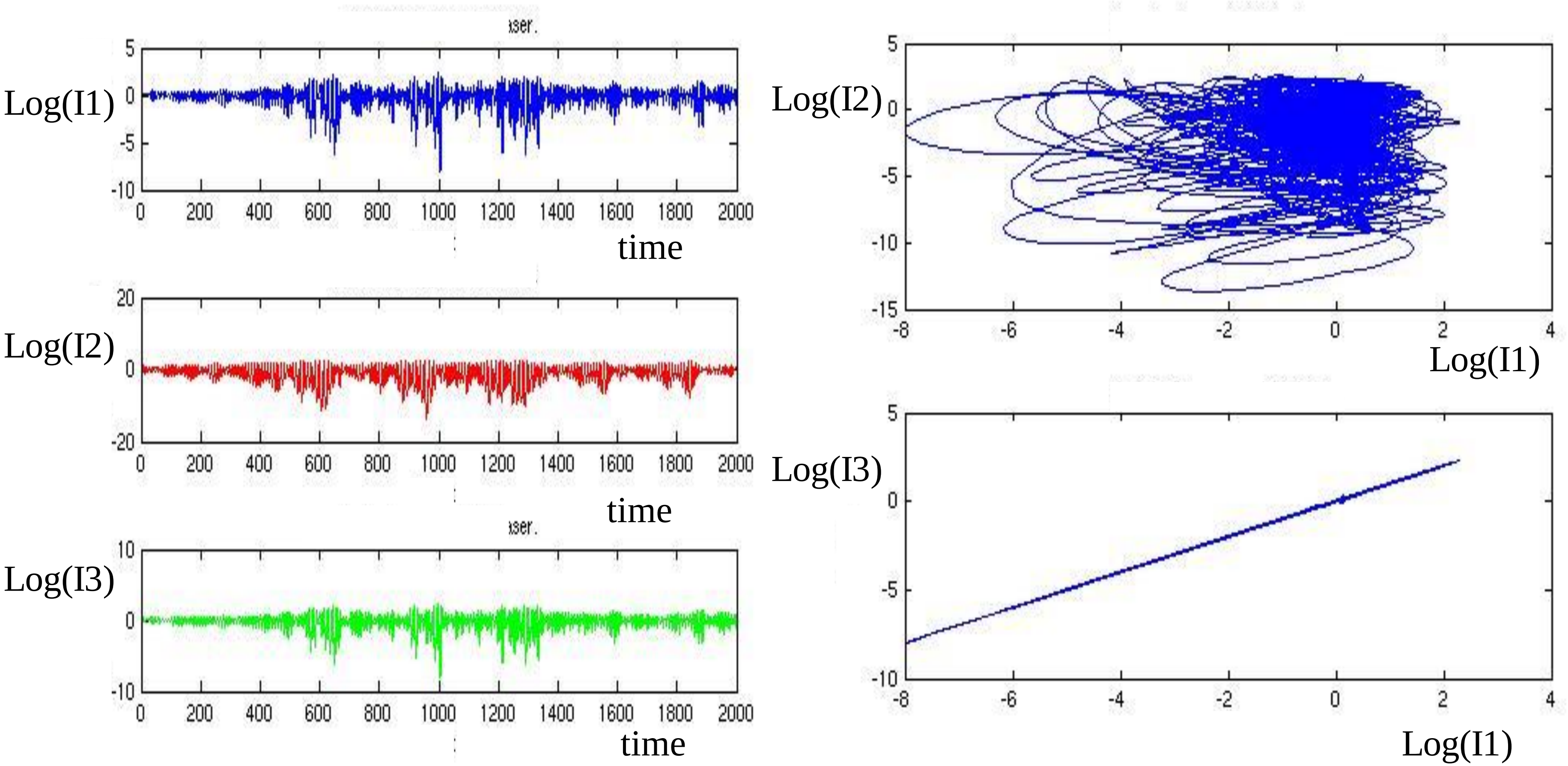
y - intensity

x - inversion

Weak dissipation: $\varepsilon << 1$

Problem:

Explain synchronization of outside lasers in a diffusively coupled, time-delayed, 3-laser system, with no direct communication between outside lasers



3 mutually coupled lasers, with delays

Laser 1

$$dy_1/dt = x_1(1+y_1)$$

$$dx_1/dt = -y_1 - \varepsilon x_1(a_1 + by_1) + \delta_2 y_2(t-\tau)$$

2

$$dy_2/dt = \beta x_2(1+y_2)$$

$$dx_2/dt = -y_2 - \varepsilon\beta\cdot x_2(a_2 + by_2) + \delta_1\cdot(y_1(t-\tau) + y_3(t-\tau))$$

Laser 3

$$dy_3/dt = x_3(1+y_3)$$

$$dx_3/dt = -y_3 - \varepsilon x_3(a_1 + by_1) + \delta_2 y_2(t-\tau)$$

coupling strengths: $\delta_1; \delta_2$

delay: τ

dissipation: $\varepsilon << 1$

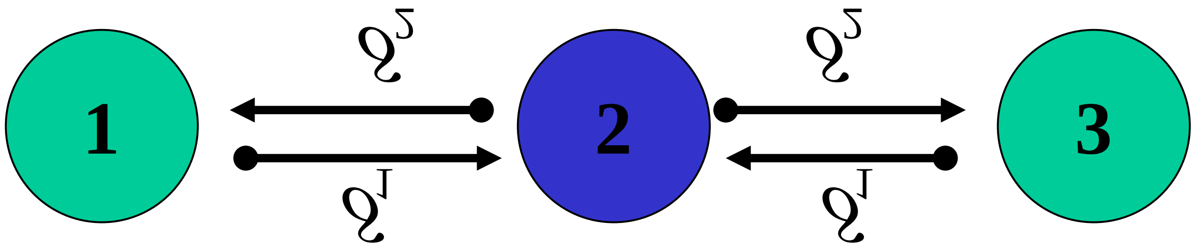
detuning: β

y - intensity

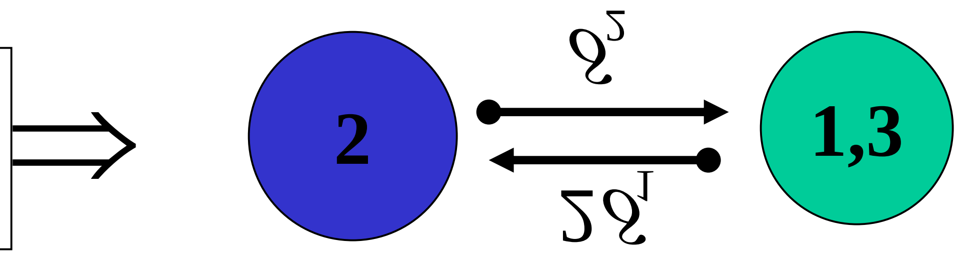
x - inversion

Synchronized state

Dynamics can be reduced to two coupled lasers



Above dynamics equivalent to



Detuning: $\beta > 1$ $\beta < 1$

Laser 2 leads **Laser 2 lags**

Synchronization over the delay time is similar to generalized synchronization

Outside lasers can be viewed as identical, dissipative driven system during the time interval 2τ

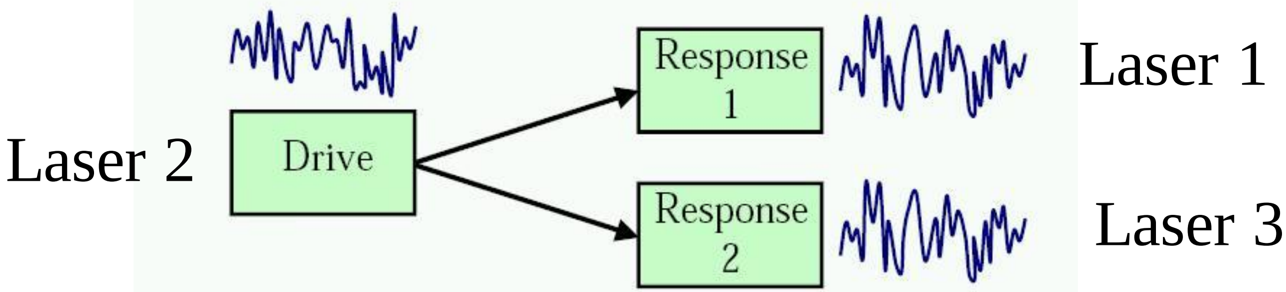
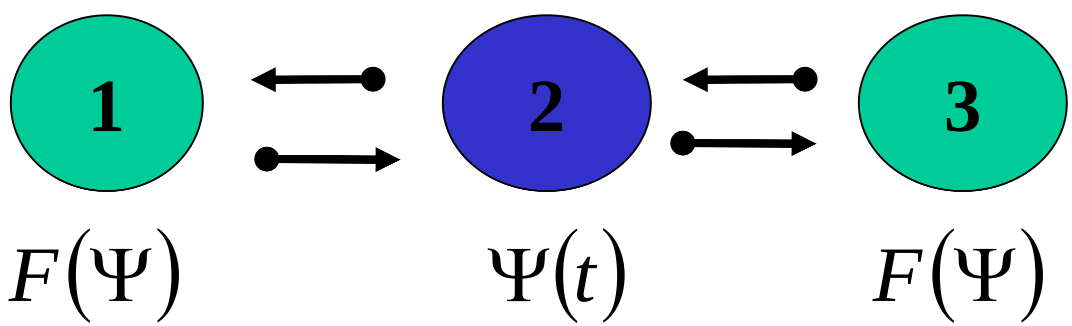


Figure 1.3: Schematic for generalized synchronization detected using the auxiliary method. Note that the drive signal is not synchronized to the response but that the response signals are synchronized to each other[15].

Stable synchronous state:



Analysis of dynamics close to the synchronization manifold

Outer lasers identical ⟹

Symmetry:
$x_1, y_1 \Leftrightarrow x_3, y_3$

Synchronized solution:

$$x_1(t) = x_3(t) = X(t)$$
$$y_1(t) = y_3(t) = Y(t)$$

The outer lasers synchronize if the Lyapunov exponents transverse to the synchronization manifold are negative

Linearized dynamics transverse to the synchronization manifold

The synchronous state, $\{X, Y\}$ is not affected by $\{\Delta x, \Delta y\}$ over the time interval of 2τ

⟹

$\{X, Y\}$ acts like a driving signal for $\{\Delta x, \Delta y\}$

Phase-space volume

Abel's Formula ⟶

$$W(t) = \exp\left(\int_{t_1}^{t} \{X(s) - \varepsilon(a_1 + b_1 Y(s))\} \cdot ds \right)$$

Transverse Lyapunov exponents

<u>for sufficiently long delays</u>:

Contracting phase-space volume ⟶ $W(t) \approx \exp\left(-\int_{t_1}^{t} \varepsilon \cdot \left(a_1 + b_1 Y(s)\right) \cdot ds\right)$

Lyapunov exponents ⟶ $\lambda_1 + \lambda_2 = \lim \frac{1}{t} \log |W(t)|$

$$\lambda_1 + \lambda_2 \approx -\varepsilon\left(a_1 + b_1 \overline{Y}\right)$$

linear dependence of Lyapunov exponents on ε

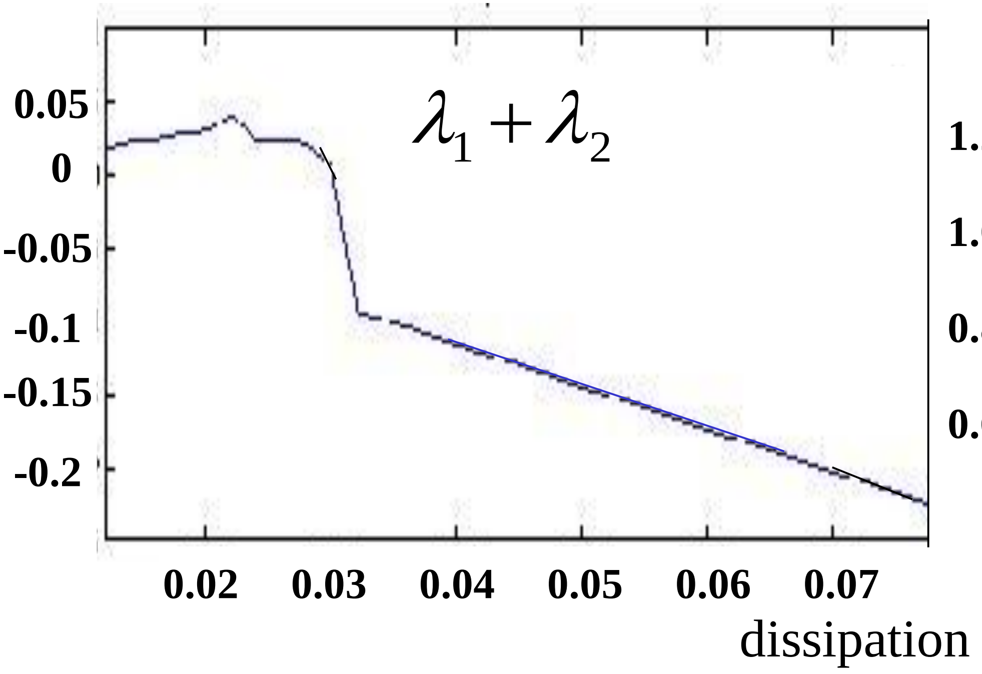
Synchronization due to dissipation in the outer lasers!

Effect of dissipation on synchronization: Numerical results

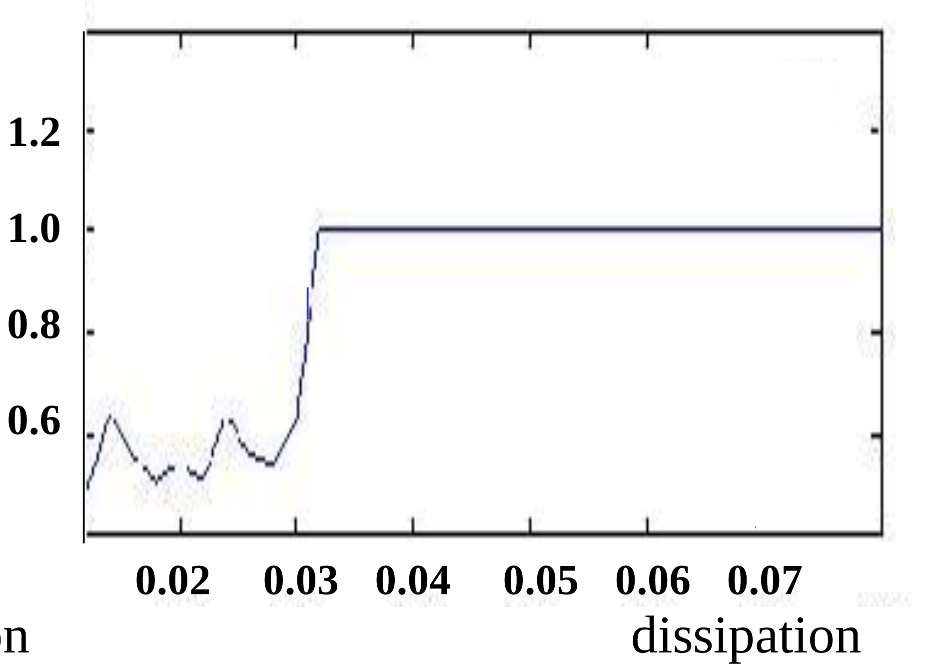
$$\lambda_1 + \lambda_2 \approx -\varepsilon\left(a_1 + b_1\bar{Y}\right)$$

$a_1 = 2, b_1 = 1, \bar{Y} \approx 1, \tau = 120$

Sum of Lyapunov exponents



Correlations



Dependence of synchronization on parameters: $\varepsilon, \tau, \delta_1 \delta_2$

Condition for negative Lyapunov exponents → $2\tau \cdot \varepsilon \left(a_1 + b_1 \bar{Y}\right) >> \left|X(t)\right|$

Maximum fluctuations in $\left|X(t)\right|$ depend on $\delta_1 \delta_2$

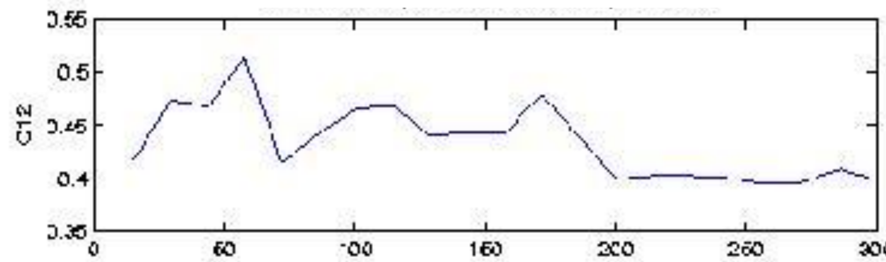
Less synchronization for increased coupling strengths, $\delta_1 \delta_2$

Better synchronization for longer delays, τ

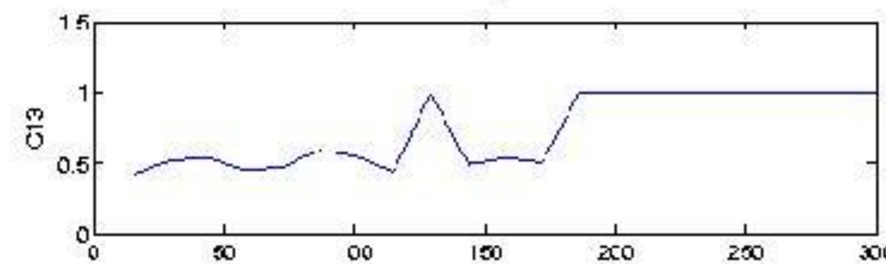
Better synchronization with increased dissipation, ε

Numerical results for synchronization as a function of delay

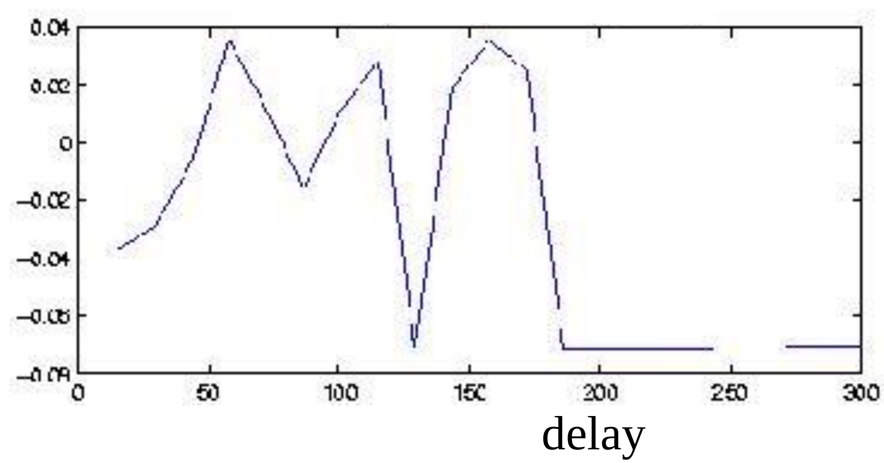
Correlations between the outer and the middle laser



Correlations between outer lasers



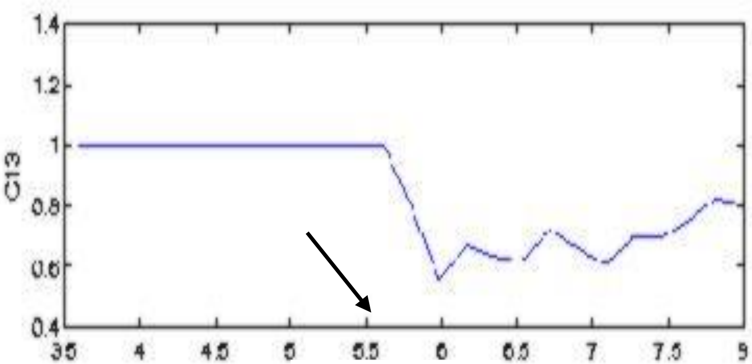
Sum of transverse Lyapunov exponents



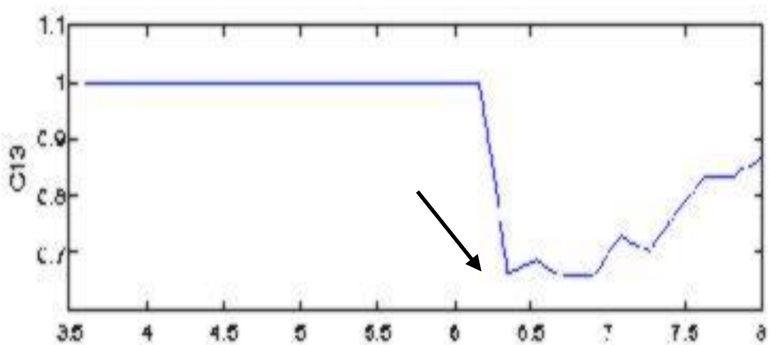
Synchronization as a function of coupling strength

Correlations

$\tau = 60$



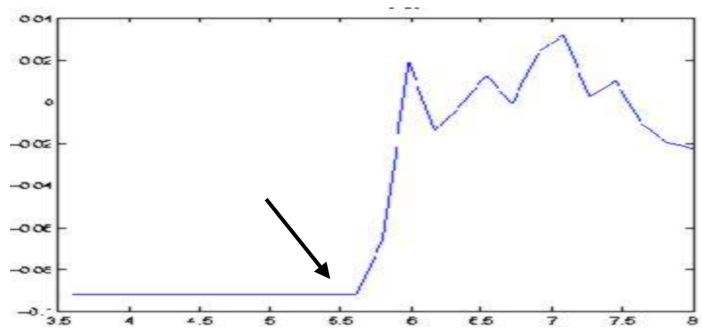
$\tau = 120$



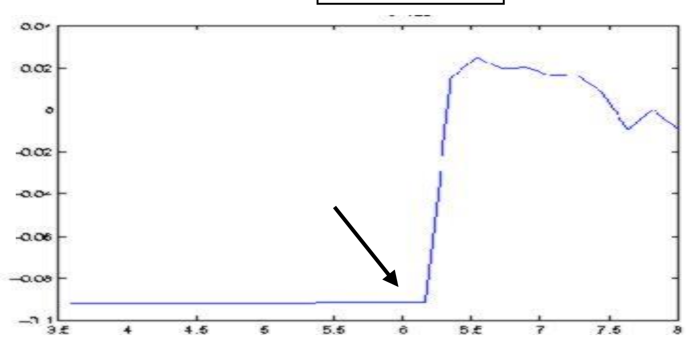
Coupling strength $\delta_1 = \delta_2$

Sum of Lyapunov exponents

$\tau = 60$



$\tau = 120$



Coupling strength

Laser Results

- Synchronization on the time scale of the delay, similar to **generalized synchronization** of driven dissipative systems
  - Outer lasers become a function of the middle one
- Improved synchronization with increased **dissipation**
  - "washes out" the difference in initial conditions
- Improved synchronization for longer **delays**
  - Need sufficiently long times to average out fluctuations
- Less synchronization with increase in **coupling strength**
  - Greater amplitude fluctuations, requiring longer delays for the outer lasers to synchronize

Discussion

- Synchronization phenomena observed in many systems (chaotic and regular)
  - **Chaotic Lasers**
  - Limit-cycle oscillators
- Phase-locking
- Complete synchronization
- Generalized synchronization

Conclusion

basic ideas from synchronization
useful in studying a wide variety of nonlinear
coupled oscillator systems

References

- A.S. Landsman and I.B. Schwartz*, "Complete Chaotic Synchronization in mutually coupled time-delay systems",* PRE 75, 026201 (2007), *http://arxiv.org/abs/nlin/0609047*
- “A.S. Landsman and I.B. Schwartz*, "Predictions of ultra-harmonic oscillations in coupled arrays of limit cycle oscillators”,* PRE 74, 036204 (2006), *http://arxiv.org/abs/nlin/0605045*
- A.S. Landsman, I.B. Schwartz and L. Shaw, “Zero Lag Synchronization of Mutually Coupled Lasers in the Presence of Long Delays”, to appear in a special review book on “Recent Advances in Nonlinear Laser Dynamics: Control and Synchronization”, *Research Signpost,* Volume editor: Alexander N. Pisarchik