Set of numbers

- **Natural numbers** is denoted as **N** and can be written as: $\boldsymbol{N} = \{\mathbf{1}, \mathbf{2}, \mathbf{3} \ldots.\}$
- **Integers numbers** are denoted by the symbol **Z** and can be written as: $\boldsymbol{Z} = \{\cdots, \mathbf{-2}, \mathbf{-1}, \mathbf{0}, \mathbf{1}, \mathbf{2}, \cdots\}$
- **Rational numbers** are those numbers, which can be expressed as a division between two integers. The set of rational numbers is denoted as **Q**.
$$\boldsymbol{Q} = \left\{ \frac{\boldsymbol{p}}{\boldsymbol{q}} \middle| \boldsymbol{p}, \boldsymbol{q} \in \boldsymbol{Z}, \boldsymbol{q} \neq \mathbf{0} \right\}$$
- **Irrational numbers** decimal numbers, which are neither exact nor recurring decimals, are characterized by infinite no periodic decimal digits, i.e. that never end nor have a repeating pattern. For example $\pi, \sqrt{2}, \sqrt[3]{5}$.
- **Real numbers** the set formed by rational numbers and irrational numbers is called the set of real numbers and is denoted as **R**.
- **Complex number** can be represented by an expression of the form $\boldsymbol{a} + \boldsymbol{bi}$, where $\boldsymbol{a}$ and $\boldsymbol{b}$ are real numbers and $\boldsymbol{i}$ is a symbol with the property that $\boldsymbol{i}^2 = \mathbf{1}$.

Intervals

<u>**Definition**</u>**:** If **a** and **b** are real numbers **,** we define the Intervals as follows**:**

- **Finite Intervals**
  1. Open Intervals $(a, b) = \{x \in R: a < x < b\}$
  2. Closed Intervals $[a, b] = \{x \in R: a \leq x \leq b\}$
  3. Half open, half closed Intervals $[a, b) = \{x \in R: a \leq x < b\}$ and
     $(a, b] = \{x \in R: a < x \leq b\}$

- **Infinite Intervals:**
  1. $[\boldsymbol{a}, \infty) = \{\boldsymbol{x} \in \boldsymbol{R}: \boldsymbol{x} \geq \boldsymbol{a}\}$
  2. $(-\infty, \boldsymbol{a}] = \{\boldsymbol{x} \in \boldsymbol{R}: \boldsymbol{x} \leq \boldsymbol{a}\}$
  3. $(-\infty, \boldsymbol{a}) = \{\boldsymbol{x} \in \boldsymbol{R}: \boldsymbol{x} < \boldsymbol{a}\}$
  4. $(-\infty, \infty) = \{\boldsymbol{x} \in \boldsymbol{R}: \boldsymbol{x}\ \boldsymbol{is}\ \boldsymbol{real}\ \boldsymbol{number}\}$

Functions, Domain and Range

Definition: A relation $f: X \to Y$ is called a function if and only if for each element $x \in X$ there exist a unique element $y \in Y$ such that $y = f(x)$

Notes:

1. The set X of all possible input values is called the domain of f and it's denoted by D_f.
2. The set Y is called the co-domain of the function.
3. The set of all elements of Y such that $y = f(x)$ is called the Range of the function and represented by R.
4. The variable x in a function $y = f(x)$ is called an independent variable. While, the variable y is called a dependent variable.

Example: Verify the domains and associated ranges of the following functions

1. $y = x^2$
2. $y = x + 5$
3. $y = \frac{1}{x+2}$
4. $y = \frac{1}{\sqrt{2-x}} + 5$
5. $y = \sqrt{4 - x}$

Algebraic Combination of Function

If f and g are two functions with domains D_f and D_g respectively, then:

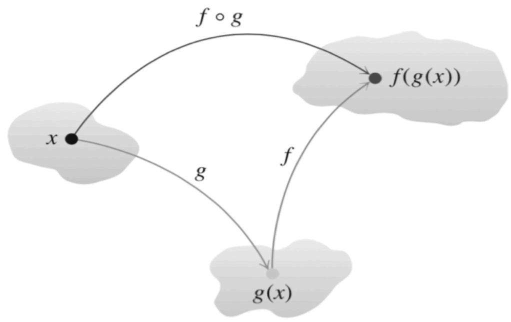
1. $(f + g)(x) = f(x) + g(x)$ with $D_{f+g} = D_f \cap D_g$
2. $(f - g)(x) = f(x) - g(x)$ with $D_{f-g} = D_f \cap D_g$
3. $(f \cdot g)(x) = f(x) \cdot g(x)$ with $D_{f \cdot g} = D_f \cap D_g$
4. $(\frac{f}{g})(x) = \frac{f(x)}{g(x)}|_{g(x) \neq 0}$ with $, D_{\frac{f}{g}} = D_f \cap D_g$ and $g(x) \neq 0$

Composite Function

If the Range of the function $f(x)$ is contained in the Domain of the function $g(x)$, then the composition $f \circ g$ is the function defined by $(f \circ g)(x) = f\big(g(x)\big)$

The domain of $f \circ g$ consists of the number x in the domain of g for which $g(x)$ lies in the domain of f.

$$x \xrightarrow{\;\;g\;\;} g(x) \xrightarrow{\;\;f\;\;} f(g(x))$$

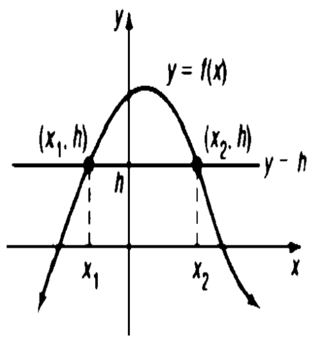


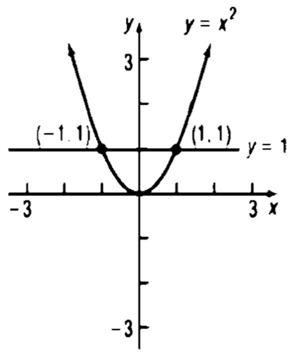
<u>The Inverse of the functions</u>

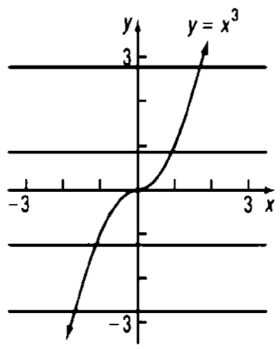
Definition: A function is said to be one – to– one function if and only if there is no two elements of the domain have the same image in the range.

i.e. $if\;\; f(x_1) = f(x_2) \;\Rightarrow\; x_1 = x_2 \quad or \quad if\;\; x_1 \neq x_2 \;\Rightarrow f(x_1) \neq f(x_2)$

<u>The Horizontal Line Test:</u> A function $y = f(x)$ is **one-to-one** if and only if its graph intersects each horizontal line at most one point.







Definition: Suppose that f is a one-to-one function on a domain D with range R. The **inverse function** f^{-1} is defined by $f^{-1}(b) = a$ if $f(a) = b$. The domain of f^{-1} is R and the range of f^{-1} is D.

How to find the inverse function of a one-to-one function *f*

1. Write $y = f(x)$.
2. Solve this equation for x in terms of y (if possible).
3. To express f^{-1} as a function of x, interchange x *and* y, the resulting equation is $y = f^{-1}(x)$