Q/ Check continuity of the functions at their associated points showing in the table below and determine discontinuities.

No.	Functions	Points
1	$f(x) = sgn(x)$	at $x = 0\ ,\ 0.5\ ,\ 1$
2	$f(x) = sgn([x])$	at $x = 0\ ,\ 0.5\ ,\ 1$
3	$f(x) = [x]$	at $x = \boldsymbol{\sigma}$, where $\boldsymbol{\sigma}$ is a number between two integer numbers a and $a-1$
4	$f(x) = [x] - [-x]$	at $x = 0$
5	$f(x) = [x] - [x-1]$	at $x = 0\ ,\ 1$
6	$f(x) = \sqrt{x} - sgn(\sqrt{x})$	at $x = 1$
7	$f(x) = sgn(x^2+1) + sgn(x^2-1)$	at $x = 1$
8	$f(x) = \begin{cases} x+1, & x \geq 2 \\ x^2, & x < 2 \end{cases}$	at $x = 2$

Find the value of a and b that make $f(x) = \begin{cases} \boldsymbol{ax^2 + b}, & \boldsymbol{x \geq 1} \\ \boldsymbol{3ax - b}, & \boldsymbol{x < 1} \end{cases}$ continuous at $x = 1$.

Find the value of c that makes $f(x) = \begin{cases} \boldsymbol{x + c^2 + 1}, & \boldsymbol{x > 0} \\ \boldsymbol{5}, & \boldsymbol{x = 0} \\ \boldsymbol{x + c + 3}, & \boldsymbol{x < 0} \end{cases}$ continuous at $x = 0$.

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The solutions of the continuity					
The question		The function	Left Limit	Right Limit	Result
$f(x) = sgn(x)$	0	$sgn(0) = 0$	-1	1	Not continuous
	0.5	$sgn(0.5) = 1$	1	1	Continuous
	1	$sgn(1) = 1$	1	1	Continuous
$f(x) = sgn([x])$	0	$sgn([0])$=0	-1	0	Not continuous
	0.5	$sgn([0.5])$=0	0	0	Continuous
	1	$sgn([1])$=1	0	1	Not continuous
$f(x) = [x]$	σ	$[\sigma] = a - 1$	$a - 1$	$a - 1$	Continuous
$f(x) = [x] - [x - 1]$	0	$[0] - [0 - 1] = 1$	1	1	Continuous
	1	$[1] - [1 - 1] = 1$	1	1	Continuous
$f(x) = [x] - [-x]$	0	$[0] - [-0] = 0$	-1	1	Not continuous
$f(x) = \sqrt{x}\ sgn(x)$	1	$\sqrt{1}\, sgn(1) = 1$	1	1	Continuous
$f(x) = sgn(x^2 + 1) + sgn(x^2 - 1)$	1	$sgn(1^2 + 1) + sgn(1^2 - 1) = 1$	0	2	Not continuous
$f(x) = \begin{cases} x + 1, x \geq 2 \\ x^2, \quad x < 2 \end{cases}$	2	$2 + 1 = 3$	4	3	Not continuous

Find the values of $\boldsymbol{a}$ and $\boldsymbol{b}$ that make $f(x) = \begin{cases} ax^2 + b, & x \geq 1 \\ 3ax - b, & x < 1 \end{cases}$ continuous at $x = 1$.

Sol/:-

$$f(1) = a(1)^2 + b = \lim_{x \to 1-} (3ax - b) = \lim_{x \to 1+} (ax^2 + b)$$
$$a + b = 3a - b$$
$$2b = 2a$$
$$a = b$$

This means, that $f(x) = \begin{cases} ax^2 + b, & x \geq 1 \\ 3ax - b, & x < 1 \end{cases}$ is continuous when the values a and b are equal each other.

Find the value of $\boldsymbol{c}$ that makes $f(x) = \begin{cases} x + c^2 + 1, & x > 0 \\ 5, & x = 0 \\ x + c + 3, & x < 0 \end{cases}$ continuous at $x = 0$.

Sol/:-

$$f(0) = 5$$
$$\lim_{x \to 0-} (x + c + 3) = 0 + c + 3$$
$$\lim_{x \to 0+} (x + c^2 + 1) = 0 + c^2 + 1$$
$$\because \text{ The function is continuous } \Longrightarrow f(0) = \lim_{x \to 0} f(x)$$
$$\Rightarrow 5 = 0 + c + 3 \ \Rightarrow c = 2$$

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