

# Lagrange Interpolation in MATLAB

## Derivation of Lagrange Interpolation:

Consider a given set of  $k+1$  points,  $(x_0, y_0)$ ,  $(x_1, y_1)$ ,  $(x_2, y_2)$ , ...,  $(x_k, y_k)$  where each points are distinct.

Let's assume a function  $f(x_j)$  such that  $f(x_j) = y_j$ ,  $j = 0, 1, 2, \dots, k$

Observing the following points

Mathematically,  $f[x_0, x_1, x_2, x_3, \dots \dots \dots x_n] = 0$

By using the second property of divided difference, it can be written that

$$\frac{f_0}{(x_0 - x_1) \dots (x_0 - x_n)(x_0 - x)} + \frac{f_n}{(x_n - x_0) \dots (x_n - x_{n-1})(x_n - x)} + \dots + \frac{f_x}{(x - x_0) \dots (x - x_n)} = 0$$

Simplifying this equation, we get

$$f(x) = \frac{(x - x_1) \dots (x - x_n)}{(x_0 - x_1) \dots (x_0 - x_n)} f_0 + \dots + \frac{(x - x_0) \dots (x - x_{n-1})}{(x_n - x_0) \dots (x_n - x_{n-1})} f_n$$

This can be represented as:

$$\sum_{i=0}^n \left( \prod_{\substack{j=0 \\ j \neq i}}^n \frac{x - x_j}{x_i - x_j} \right) f_i$$

```
function y0 = lagrange_interp(x, y, x0)
% x is the vector of abscissas.
% y is the matching vector of ordinates.
% x0 represents the target to be interpolated
% y0 represents the solution from the Lagrange interpolation
y0 = 0;
n = length(x);
for j = 1 : n
    t = 1;
    for i = 1 : n
        if i~=j
            t = t * (x0-x(i))/(x(j)-x(i));
        end
    end
    y0 = y0 + t*y(j);
end
```

```
function y0 = lagrange_interp(x, y, x0)
```

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y0 = 0;
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n = length(x);
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for j = 1 : n
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    t = 1;
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    for i = 1 : n
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        if i~=j
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            t = t * (x0-x(i))/(x(j)-x(i));
```

```
        end
```

```
    end
```

```
    y0 = y0 + t*y(j);
```

```
end
```

$$f(x) = \frac{(x-x_1) \dots (x-x_n)}{(x_0-x_1) \dots (x_0-x_n)} f_0 + \dots$$

Ex: Consider the curve  $y = x^2 - 4x + 3$ . We now that points  $x = 1 : 10$ . What are the values of  $y$  when  $x = 50$

```
x(1)=1;  
for i =1:10;  
y(i)=x(i)^2 - 4*x(i) +3;  
x(i)=x(i)+1;  
end
```

```
>> x0 = 50;  
>> y0 = lagrange_interp(x,y,x0)
```