Gauss Elimination Method

Derivation of Gauss Elimination Method:

Consider the following system of linear equations:

$A_1x + B_1y + C_1z = D_1$ . . . . (1)
$A_2x + B_2y + C_2z = D_2$ . . . . (2)
$A_3x + B_3y + C_3z = D_3$ . . . . (3)

In order to apply Gauss elimination method, we need to express the above three linear equations in matrix form as given below:

A = $\begin{bmatrix} A1 & B1 & C1 \\ A2 & B2 & C2 \\ A3 & B3 & C3 \end{bmatrix}$

B = $\begin{bmatrix} D1 \\ D2 \\ D3 \end{bmatrix}$

Arrange matrices A and B in the following form (augmented matrix):

$$\left\{\begin{array}{ccc|c} A1 & B1 & C1 & D1 \\ A2 & B2 & C2 & D2 \\ A3 & B3 & C3 & D3 \end{array}\right\}$$

Now, perform the following elementary row operations till it is reduced to echelon form by:

1) Exchanging or swapping two rows
2) Adding the certain multiple of one row to another row
3) Multiplying a row by non-zero number

This procedure is repeated until the augmented matrix is reduced to following echelon form:

$$\left\{\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & \mathbf{b} \\ 0 & 0 & 1 & c \end{array}\right\}$$

Thus, the solution of above system of linear equation is (a, b, c) i.e. x = a, y = b and z = c.

Ex: Use the Gauss Elimination Method to find the solution for these equations:

$3x + 4y + 6z = 3$ . . . . (1)

$2x - 6y - 7z = 10$ . . . . (2)

$-x + 8y + 8z = 12$ . . . . (3)

```matlab
%%  Gauss Elimination Method
a = [3 4 6 3; 2 -6 -7 10; -1 8 8 12];

[m,n]=size(a);

%% Exchanging or swapping two rows
%% Adding the certain multiple of one row to another row

for j=1:m-1
    for z=j+1:m
        if a(j,j)==0
            t=a(j,:);a(j,:)=a(z,:);
            a(z,:)=t;
        end
    end
    for i=j+1:m
       a(i,:)=a(i,:)-a(j,:)*(a(i,j)/a(j,j));
    end
end

%% Multiplying a row by non-zero number
x=zeros(1,m);
 for s=m:-1:1
    c=0;
    for k=2:m
        c=c+a(s,k)*x(k);
    end
 x(s)=(a(s,n)-c)/a(s,s);
 end
 disp('Gauss elimination method:');
a
x'
```

```
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ANOTHER METHOD TO SOLVE LINEAR EQUATION

Ex: Consider the system of equations:

$3x + 4y + 6z = 3$ . . . . (1)

$2x - 6y - 7z = 10$ . . . . (2)

$-x + 8y + 8z = 12$ . . . . (3)

Find the solution for x , y ,z to the system of equations.

```
A = [3 4 6; 2 -6 -7; -1 8 8];
B = [3; 10; 12];
X = A\B
```

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Find the solution for x , y ,z to the system of equations.

```
syms  x y z
    eq1 = ‘3*x + 4*y + 6*z = 3';
    eq2 = ‘2*x - 6*y - 7*z = 10';
    eq3 = ‘-1*x + 8*y + 8*z = 12';
    [x,y,z] = solve(eq1, eq2, eq3)
```