## Logical Semantics

By 'logical semantics' means the study of meaning with the aid of mathematical logic. The term is commonly used by logicians in a narrower sense than this: to refer to the investigation of the meaning, or interpretation, of expressions in specially constructed logical systems. Logical semantics in this narrower and more technical sense may be referred to, following Carnap (1942, 1956), as pure* semantics. It is a highly specialized branch of modern logic.

Constructed logical systems are frequently referred to as languages. But we will not adopt this usage. We will refer to them, instead, as calculi*, keeping the term 'language' for natural languages. This will enable us to oppose linguistic semantics* (a branch of linguistics) to pure semantics* (a branch of logic or mathematics).

The relationship between logic and language has long been, and still is, controversial. There are those who maintain that languages are of their nature imperfect and illogical, and therefore totally unsuited to systematic reasoning and scientific discussion: that it is hopeless and wrongheaded to attempt to correct their imperfections and that they should be replaced with logical calculi constructed especially for the purpose. Others have maintained that languages have their own internal logic, appropriate to the multifarious functions which they fulfil; that the criticisms directed against language should be turned instead against those philosophers and logicians who have failed to understand that this is so and have themselves confused language with the use, or misuse, of language; and that, in any case, the logical calculi constructed by mathematicians and logicians have been strongly influenced by the grammatical structure of particular languages and cannot therefore be regarded as independent ideal systems by reference to which language can be judged and found to be deficient.

Some of the major contributions to the development of mathematical logic have been made by scholars with a particular interest in epistemology and a commitment to empiricism. It was their view that an ideal language (to which actual languages might approximate in various degrees, but which they certainly did not realize) would directly reflect the structure of reality. Every simple expression of the language would have a single meaning and this could be described, either directly or by reduction, in terms of the relationship holding between the expression and the object or class of objects which the expression stood for, or named, in the external world. Sentences stood for facts, or states-ofaffairs; and in an ideal language they would be in structural correspondence with them.

One of the main problems encountered by logical atomism was that of accommodating as part of the real world such peculiar entities as negative facts and objects of belief. Most damaging ultimately was the increasing recognition

by philosophers, not only that language was used for many other purposes besides that of describing the world (whatever one might mean by 'the world'), but also that many of the other uses of language were philosophically, and indeed logically, interesting. Thus was born the movement commonly known as ordinary language philosophy (or, in a very special sense of the term, linguistic analysis).

The term 'proposition', like 'fact', has been the subject of considerable philosophical controversy. Some authors think of propositions as purely abstract, but in some sense objective, entities; others regard them as subjective or psychological; and there are certain logicians who avoid the term entirely, because they do not wish to adopt either of these alternatives. Further difficulties are caused by the use of 'proposition' in relation to 'sentence' and 'statement': some writers identify propositions with (declarative) sentences, others identify them with statements, and others with the meanings of (declarative) sentences; and there is little consistency in the way in which 'statement' is defined.

Our formulation of the relationship between sentences and propositions allows for the following possibilities: that different sentences of the same language may express the same proposition; that a sentence may express two or more propositions (so that it may be intended by the speaker, or writer, in one sense and taken by the hearer or reader, in another); and finally that not all the declarative sentences in a language will express propositions.

Propositions may be true or false; we will use T to stand for 'true' and F to stand for 'false'. These are the two possible truth-values* that a proposition may have in the standard interpretation of the propositional calculus: it is a two-valued* system. Furthermore, it is nonmodal, in the sense that it makes no use of the operators of necessity and possibility.

Logic is the study of the nature of valid inferences and reasoning. The logical tradition constitutes one of the major strands in the study of meaning, and some knowledge of its background is indispensable in linguistic semantics. In this chapter we will study some basic logical tools and concepts. Our aim is twofold:

◆ first, to understand the ways in which some types of meaning can be represented in logical symbolism

◆ second, to appreciate the advantages and disadvantages of this type of representation.

We begin by introducing the ideas of validity, soundness and logical form (6.1): these define the context and aims of a logical approach to language. In 6.2 we present an exposition of the basic principles of propositional logic, the logic of basic sentences, including a treatment of the principal logical operators: and, not, or and if . . . then. In 6.3 we discuss the extent to which these logical concepts overlap with the meanings of their ordinary language equivalents. Section 6.4 introduces predicate logic, the logic of expressions like some and all. In 6.5 we discuss the ways in which the concept of a model allows us to

describe reference using logical techniques. Section 6.6 contains a discussion of the sentence relations of entailment, presupposition and contradiction. This leads to a discussion of meaning postulates in Section 6.7, which use the sentence relations introduced in 6.6 as part of a non-decompositional approach to meaning. In 6.8 Russell's theory of descriptions is discussed. This is a proposal for the analysis of noun phrases containing the definite article, and provides an instructive example of the advantages and problems of applying logical tools to the analysis of natural language.

Validity, soundness and logical form

Logic may be defined as the study of valid reasoning and inference. On this definition, logic investigates the properties of valid chains of reasoning, and specifies the conditions which these chains must meet in order to be valid, in order to work as arguments. Consider the following exchange:

(1) A: Koko is a primate, so she likes daytime television.
B: What? I don't get it.
A: All primates like daytime television

Initially, B is unable to follow A's line of thought, and as a result A is forced to state the general principle on which her conclusion rests. This allows us to reconstruct A's original train of thought as the following argument or syllogism:

(2) 1. All primates like daytime television.
2. Koko is a primate.
therefore
3. Koko likes daytime television.

Argument (2) thus reveals the explicit logical structure of A's comment in (1). As Kneale and Kneale explain (1962: 12), the 'first tentative steps towards logical thinking are taken when men try to generalize about valid arguments and to extract from some particular valid argument a form or principle which is common to a whole class of valid arguments'. Given the meanings of the words all, like and is, the conclusion Koko likes daytime television just has to be true as long as we accept the truth of the proposition All primates like daytime television. It seems likely that it was in domains like mathematics, especially geometry, that the need to make the principles of valid reasoning explicit first arose (Kneale and Kneale 1962: 2); in modern times, the study of logic has been particularly undertaken in the attempt to symbolize the types of reasoning that underlie mathematical arguments.

Logic is important to linguistics for at least three reasons. First, the study of logic is one of the oldest comprehensive treatments of questions of meaning. When people first began to think systematically about the meanings of language and the relations between these meanings, it was logical concepts to which they often appealed for explanations. As a result, the tradition of logical analysis, which we can trace as far back as Aristotle, provides a rich body of reflection on meaning, and most scholars who have studied meaning in the Western tradition have had at least some knowledge of logical principles. The relevance of logic

to linguistics is far from simply historical, however. Logical concepts inform a wide range of modern formal theories of semantics, and are also crucial in research in computational theories of language and meaning. We will not be exploring formal theories in themselves here, but our exposition of some fundamental logical ideas will provide some background for those wanting to do so. Lastly, logical concepts provide an enlightening point of contrast with natural language. The basic logical concepts are accessible to practically anyone; indeed, many philosophers have seen in logical principles the universal 'laws of thought' which constitute the basic grounds of human rationality: for Immanuel Kant, for example, 'logic is the science that exhaustively presents and strictly proves nothing but the formal rules of all thinking' (1998 [1787]: 106). Yet, as we will see, logical meanings often differ strikingly from the types of meaning found in natural language. Studying logic therefore provides a window onto a body of apparently universal concepts with strikingly different behaviour from natural language, which provide a rigorous and enlightening way of disambiguating certain types of natural language expression.

Formal theories

A formal theory is one which offers an analysis of meaning in a technical, usually symbolic, metalanguage, according to principles which can be expressed in mathematical terms. A formal representation of meaning avoids the ambiguities contained in natural language by enforcing a strict correspondence between symbols and meanings: whereas natural languages always contain ambiguous or polysemous terms, in which a single form stands for several meanings (think of English step, match or get), a formal language has a strictly one-to-one relation with its meanings, so that each symbol of the formalism has one and only one interpretation.

As the above quotation from Kant suggests, the principles of valid argument have typically been taken, in the logical tradition, as the very principles governing rational human thought. Logic can be seen, from this perspective, as the science of the laws of rational thought. On this view, logic is the science which tries to specify all the conclusions that can validly be reached from a given set of propositions. It is logical principles which thus describe the process of valid reasoning.

The first two propositions in (2) are called the premises. An argument's premise may be defined as its starting-point, one of the propositions from which the conclusion follows. In (2), the last proposition is the conclusion. Note that the validity of arguments or of chains of reasoning has a special relationship to the words in which the premises and conclusion may be stated: substitute different words, and the argument may not be valid. None of the following arguments, for instance, is valid:

(3) a. Most primates like daytime television.
Koko is a primate.
therefore

Koko likes daytime television.

## Propositional logic

Propositional logic is the branch of logic which deals with relations between propositions. A proposition is something which serves as the premise or conclusion of an argument. In (2) above, Koko is a primate, All primates like daytime television, and Koko likes daytime television are all propositions. Propositions are either true or false. In English, we may think of propositions as roughly like positive or negative factual sentences. The parallel between sentences and propositions is not absolute, however. A sentence like (9) expresses an infinite number of different propositions, depending on the values of the deictic expressions I (my), you (your) and this afternoon:

(9) I want you to know that your behaviour this afternoon had nothing to do with my decision to drop out.

For each assignment of referents to the deictic expressions, a different proposition results. Similarly, 'Koko likes daytime television' can only be considered a proposition as long as the referent of the noun 'Koko' has been fixed. Only if we know who 'Koko' refers to can we know whether a proposition in which she is mentioned is true or not.

Strictly, the notion of a proposition belongs to logic. We can, however, see it in mental terms. A series of experiments by psychologists has shown that people are very bad at remembering the actual words of utterances.About twenty seconds after hearing or reading an utterance, all people remember is its content or gist: the actual words used usually can't be remembered accurately. Given this, the propositions discussed here would be one possible representation of this remembered content or gist.

Natural language is not a collection of brute propositional statements without any mutual interrelations: a single statement like (10a) or (10b) can serve as the basis for a whole series of additional statements, depending on the additional linguistic elements added to it. Some examples of these additional statements are given in (10c–h):

(10) a. Daryl Tarte grew up to publish a raunchy family saga in 1988.

b. Patsy Page is telling the truth.

c. Someone suspects that Daryl Tarte grew up to publish a raunchy family saga in 1988.

d. It is probable that Daryl Tarte grew up to publish a raunchy family saga in 1988.

e. Daryl Tarte did not grow up to publish a raunchy family saga in 1988.

f. Daryl Tarte grew up to publish a raunchy family saga in 1988, and Patsy Page is telling the truth.

g. Either Daryl Tarte grew up to publish a raunchy family saga in 1988, or Patsy Page is telling the truth.

h. If Daryl Tarte grew up to publish a raunchy family saga in 1988, then Patsy Page is telling the truth.

It is the italicized elements in (10c–h) which chiefly serve to insert the original propositions (10a–b) into a new, longer one. Among these elements, propositional logic attaches special importance to the four found in (10 e–h). In English, these four elements are expressed by the words and, or, not and if . . . then. We will refer to these as the propositional connectives or logical operators. These four differ from others, such as those in (10c–d), in that they are truth-functional.

This means that whether the larger propositions they are part of are true or not depends solely on the truth of the original basic propositions to which they have been added: the logical operators do not add anything true or false to the basic propositions themselves; all they do is generate additional propositions from the basic ones.

Let's demonstrate truth-functionality by considering the operator not. Let's grant that (10a) 'Daryl Tarte grew up to publish a raunchy family saga in 1988' is true. Then, (10e) 'Daryl Tarte did not grow up to publish a raunchy family saga in 1988' cannot be true: the two propositions are contradictory, and we cannot imagine a world in which they could be simultaneously possible. Conversely, if (10e) is true, then (10a) must be false. We can deduce the truth or falsity of one proposition from the other: if one is true, the other can only be false. Similarly, if (10a–b) are true, then (10f) must also be true. But if one or both of (10a–b) are false, then (10f) as a whole must likewise be false.

**Truth-conditional semantics** is an approach to semantics of natural language that sees meaning (or at least the meaning of assertions) as being the same as, or reducible to, their truth conditions. This approach to semantics is principally associated with Donald Davidson, and attempts to carry out for the semantics of natural language what Tarski's semantic theory of truth achieves for the semantics of logic

Truth-conditional theories of semantics attempt to define the meaning of a given proposition by explaining when the sentence is true. So, for example, because 'snow is white' is true iff (read 'if and only if') snow is white, the meaning of 'snow is white' is snow is white.

The first truth-conditional semantics was developed by Donald Davidson in Truth and Meaning (1967). It applied Tarski's semantic theory of truth to a problem it was not intended to solve, that of giving the meaning of a sentence.

Scott Soames has harshly criticized truth-conditional semantics on the grounds that it is either wrong or uselessly circular. Under its traditional formulation, truth-conditional semantics gives every necessary truth precisely the same meaning, for all of them are true under precisely the same conditions (namely, all of them). And since the truth conditions of any unnecessarily true sentence are equivalent to the conjunction of those truth conditions and any necessary truth, any sentence means the same as its meaning plus a necessary truth. For example, if "snow is white" is true iff snow is white, then it is trivially the case that "snow is white" is true iff snow is white and 2+2=4, therefore under truth-conditional semantics "snow is white" means both that snow is white and that 2+2=4.

Soames argues further that reformulations that attempt to account for this problem must beg the question. In specifying precisely which of the infinite number of truth-conditions for a sentence will count towards its meaning, one must take the meaning of the sentence as a guide. However, we wanted to specify meaning with truth-conditions, whereas now we are specifying truth-conditions with meaning, rendering the entire process fruitless.

Truth-conditional semantics is based on the notion that the core meaning of any sentence (any statement) is its truth conditions. Any speaker of the language knows these conditions. If a sentence is true (or false), what other sentences, expressing partly the same, partly different conditions, can be judged by this sentence? If a given sentence is true, does this make another sentence also true, or does it falsify the other sentence, or is there no truth relation? Matters of truth and logic are of more importance in truth-conditional semantics than meanings of lexemes per se.
A fundamental fact about declarative sentences is that they are either true or false1 (and since we use language to communicate information about the world, a listener will in general assume that a sentence they have just heard is true, and uses that fact to enrich their knowledge of the world). Thus (1) is true and (2) is false:
(1) Barack Obama moved into the White House on Jan. 20, 2009.
(2) John McCain moved into the White House on Jan. 20, 2009.
Hence, one basic notion used for the construction of meanings is a truth value—for now assume that there are just two such values: true and false. The claim that truth values are a fundamental part of meaning is also motivated by noting that—as shown by the examples above—speakers have intuitions about truth, given certain facts about the world, just like they do about acceptability. And these judgments can be used to test the adequacy of particular theories of meaning. Following standard practice, we use 1 for true and 0 for false. Thus

the set of truth values {1,0} and we will also refer to this set as t. Let us use [[Æ]] to mean the semantic value (i.e., the meaning) of a linguistic expression Æ. Then (temporarily) we can say that [[Barack Obama moved into the White House on Jan. 20, 2009]] = 1.

Some worries should immediately spring to mind. The most obvious is that something seems amiss in calling the meaning of (1) "true" even if we are willing to accept the fact that it is true. We will enrich the toolbox directly to take care of that. But there are other objections: does it really make sense to say that all declarative sentences are true or false? Clearly not—for some sentences the truth value depends on who is speaking (and on when the sentence is spoken). Take (3):

(3) I am President of the United States.

This is true if spoken by Barack Obama in 2011, but not if spoken by John McCain and not true if spoken by Barack Obama in 2006. So this has no truth value in and of itself. Nonetheless once certain parameters are fixed (time of utterance and speaker) it is either true or false. So we might want to think of the meaning of (3) as a function into {1,0}—it does yield a truth value but only once we fix certain parameters. But it seems inescapable that a declarative sentence is telling us something about the world, and so truth values are certainly one fundamental piece.

In fact, there are many parameters that need to be set in order to derive a truth value. Certain words like I, you, here, now, etc. quite obviously have the property that their value depends on when, where, and by whom these are spoken (these are called indexicals). There are also more subtle cases—such as (4) and (5)—where truth seems to depend on what is at issue in the discourse context in which these are uttered:

(4) Magic Johnson is tall.

(5) Every student got an A on their formal semantics midterm.

(4) might be true if we are comparing Magic Johnson to the general population (Magic Johnson is a former basketball player for the Los Angeles Lakers) but perhaps not true if we are comparing him to the set of basketball players. The context of utterance usually makes it clear what is the relevant comparison class. (5) may be true if we are restricting the interpretation of students to those students in my formal semantics class in 2011, but not if we mean every student in the world, or even every student at Brown or every student who has ever taken formal semantics from me.

So far, we have two reasons that it is oversimplified to say that the meaning of a sentence is true or false: (a) even once we do determine the truth value of a sentence we surely don't want to call that "meaning," and (b) often the truth value can't be determined until we know the context of utterance. Two further worries have to do with the fact that (c) there are vague sentences where some have the intuition that the truth value is something in between 1 and 0, and (d)

some sentences (even once we fix the context of utterance) seem to be neither true nor false (nor anything in between).
6) The present King of France is bald.
(7) He stopped going to aerobics class.
Surely (6) is not true, but do we want to say it is false?2 For the most part, we will make the expository simplification of assuming all sentences are true or false, although we will return to the issue of presupposition from time to time.
This still leaves the issue of vagueness. For example, even once we do fix the comparison class at issue in a sentence like (4) there remains some vagueness: is a 6 foot 9 inch basketball player tall for a basketball player? If so, exactly where does one draw the line between tall and not tall? There is a rich literature on the problem of modeling vagueness and some have attempted to model it using intermediate truth values (where a sentence can have any value between 1 and 0).
The study of truth or truth conditions in semantics falls into two basic categories: the study of different types of truth embodied in individual sentences (analytic, contradictory, and synthetic) and the study of different types of truth relations that hold between sentences (entailment and presupposition).
**Analytic Sentences.** An analytic sentence is one that is necessarily true simply by virtue of the words in it. For example, the sentence A bachelor is an unmarried man is true not because the world is the way it is, but because English language is the way it is. Part of our knowledge of ordinary English is that bachelor "means" an unmarried man, thus to say that one is the other must necessarily be true. We do not need to check on the outside world to verify the truth of this sentence. We might say that analytic sentences are "true by definition." Analytic sentences are sometimes referred to as linguistic truths, because they are true by virtue of the language itself.
**Contradictory Sentences**. Contradictory sentences are just the opposite of analytic sentences. While analytic sentences are necessarily true as a result of the words in them, contradictory sentences are necessarily false for the same reason. The following sentences are all contradictory: A bachelor is a married man, A blue gas is colorless, A square has five equal sides. In each case, we know the sentence is false because we know the meaning of the words in it: part of the meaning of bachelor is „unmarried"; part of the meaning of blue is „has color"; part of the meaning of square is „four-sided." It is not necessary to refer to the outside world in order to judge each of these sentences false. Consequently, contradictory sentences are sometimes referred to as linguistic falsities, because they are false by virtue of the language itself.
Synthetic Sentences. Synthetic sentences may be true or false depending upon how the world is. In contrast to analytic and contradictory sentences, synthetic sentences are not true or false because of the words that comprise them, but rather because they do or do not accurately describe some state of affairs in the

world. For example, the sentence My next door neighbor, Bud Brown, is married is a synthetic sentence. Note that you cannot judge its truth or falsity by inspecting the words in the sentence. Rather, you must verify the truth or falsity of this sentence empirically, for example by checking the marriage records at the courthouse. Other examples of synthetic sentences include Nitrous oxide is blue, Nitrous oxide is not blue, Bud Brown‟s house has five sides, and Bud Brown‟s house does not have five sides. In each case, the truth or falsity of the sentence can be verified only by consulting the state of affairs that holds in the world. Thus, synthetic sentences are sometimes referred to as empirical truths or falsities, because they are false by virtue of the state of the extralinguistic world.

**Entailment**

An entailment is a proposition (expressed in a sentence) that follows necessarily from another sentence. For example, Martina aced chemistry entails Martina passed chemistry, because one cannot ace chemistry without passing chemistry. The test for entailment is as follows; sentence (a) entails sentence (b) if the truth of sentence (a) ensures the truth of sentence (b) and if the falsity of sentence (b) ensures the falsity of sentence (a). Our example sentences pass both tests. First, the truth of sentence (a) ensures the truth of sentence (b). Note that if Martina aced chemistry, she necessarily passed chemistry. Second, the falsity of sentence (b) ensures the falsity of sentence (a). If Martina didn‟t pass chemistry, she necessarily didn‟t ace chemistry.

Note, however, that the relation of entailment is unidirectional. For instance, consider our example sentences again, but in the opposite order: (b) Martina passed chemistry and (a) Martina aced chemistry. In this case, sentence (b) does not entail (a) (if Martina passed chemistry, she did not necessarily ace chemistry – she may have made a C); and the falsity of (a) does not ensure the falsity of (b) (if Martina did not ace chemistry, it is not necessarily the case that she did not pass chemistry – she may, once again, have made a C). In short, then, it should be clear that the relation of entailment is unidirectional.

This is not to say, however, that there cannot be a pair of sentences such that each entails the other. Rather, when such a relation holds, it is called paraphrase. For example, the sentences Martina passed chemistry and What Martina passed was chemistry are paraphrases of each other. Note, incidentally, that entailment describes the same relationship between sentences that hyponymy describes between words. Likewise, paraphrase describes the same relationship between sentences that synonymy describes between words. These relations are illustrated in the following :

Thus, if sentence (a) Martina aced chemistry presupposes sentence (b) Martina took chemistry, the denial of sentence (a) Martina did not ace chemistry also presupposes sentence (b) Martina took chemistry. If Martina did not take chemistry, then Martina did not ace chemistry cannot be judged true or false.

The relationship between entailment and presupposition is illustrated in this figure. This figure should be read as follows: Martina aced chemistry entails Martina passed chemistry. Both of those sentences, in turn, presuppose Martina took chemistry.

**Syllogism**

A syllogism is a form of logical reasoning that joins two or more premises to arrive at a conclusion. For example: "All birds lay eggs. A swan is a bird. Therefore, a swan lays eggs." Syllogisms contain a major premise and a minor premise to create the conclusion, i.e., a more general statement and a more specific statement. In the example, the major premise is that all birds lay eggs. The minor premise is that a swan is a bird. The conclusion links these two propositions to conclude that if a swan is a bird it must lay eggs. Syllogistic arguments are generally presented in this three-line format.

**Anomaly**

It is a semantic feature conflict as in: Colourless green ideas sleep furiously. It may also come in the form of nonsensical word combinations or uninterpretable expressions as in: Lewis Carroll's poem "Jabberwocky" since jabberwocky does not exist in the lexicon of the language. In literary contexts, semantic violations make up anomalies when breaking the semantic rules helps to create the desired images as in : a grief ago (in place of a year ago, or last grief).

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