International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 2 (2018) pp. 922-928

Simplified Analysis of Continuous Beams

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Abstract

The analysis of continuous beams and frames to determine the bending moments and shear is an essential step in the design process of these members. Furthermore, the evaluation of the maximum deflection is a mandatory step in checking the adequacy of the design. There are many computer programs available to perform these tasks. However, a hand spot checks for moments at selected points still necessary. Also, a quick determination of moments, even they are approximate, is usually required for simple structures and preliminary evaluation of complicated ones. The aim of the present work, is to develope a simple and reasonably accurate method to determine moments and deflection for continuous beams. The slope-deflection method and a beam analysis code are implemented to analyze a large number of continuous beams of equal spans length. Beams of various span numbers and loading distribution are investigated. The method of superposition is used to represent a continuous beam by the appropriate single-span beams (each span by two propped cantilevers and one simply supported beam). Simple expressions are presented to determine the equivalent load on each of the substituent beams. From which, the bending moment, shear force and deflection at any location can be calculated by the method of superposition. The validity of the suggesetd method are examined by applying it to several cases of contionuous beams.

The presented method is found to give exact values for beams of two and three spans. While for the purpose of simplicity and getting compact expressions, approximate results with errors less than 0.5% are obtained for beams of four and more spans.

Keywords: Continuous beams, closed-form solution, structural analysis, equivalent single span beams, approximate bending moment.

INTRODUCTION

In both of the analysis and design processes of continuous beams, it is of significant importance to find the bending moment and deflection. Therefore, different methods are developed to achieve this aim. Some of these methods yield exact values, but they usually involve extended mathematical operations. On the other hand, others use simple formulas, but approximate values are obtained. The current practice of structural engineering uses the computer-aided analysis codes including finite element method to analyze complicated statically indeterminate structures, which when skillfully used, can give almost exact results. However, the use of simple approximate methods still necessary in many cases as a spot check tool for checking the results of computer codes and for obtaining approximate values of the member forces, which are necessary for the preliminary analysis, used to estimate the initial member sizes to be used in rigorous extensive analysis. More explanations about the reasons of the importance of the approximate methods of structural analysis are explained by McCormack [1].

Benscoter [2] developed an iterative method to determine the bending moments at internal supports of the continuous beams. His method started at first by representing each span by a single span simply supported beam. Then the end slopes at the simple supports together with the flexibility of each span are determined. The next step is to determine the rotation dislocation, which is the difference between the end slopes of the adjacent spans at their common supports. The bending moment at each internal support of the continuous beam is proportional to the value of the angular dislocation at that support and the stiffness values of the two spans on its both sides. The value of the bending moment at the internal support of each span is then modified due to the carryover moment from the bending moment of the other internal support of the same span. The final step is to continue in iterations like that used in the Hardy Cross moment distribution method [3]. Zuraski [4] developed a closed form analysis to determine the support bending moments for symmetric continuous beams. His analysis adopted the conjugate beam method to derive expressions for the span end moments, which depends on the ratio of the length of the loaded span to that of the considered span and the number of spans between them. The method was mainly devoted to the analysis of continuous highway bridge beams. In his paper, Harrison [5] presented a simplified finite element program that can be executed on a microcomputers to analyze plane frames and continuous beams. The software can implimented to determine the bending moments, deflection, and draw the influence lines. The continuous beam can be of variable cross-section and subjected to point or trapezoidal distributed load.. Jasim and Karim [6] used moment distribution method to derive closed-form expressions to determine the exact values of member end moments of continuous beams and frames. The method is based on the series solution of the moment distribution terms obtained from the successive iterations. The final expressions need no iteration and can be used irrespective of the type of loading. Dowell [7] suggested a method that can be used as a spot-check tool to determine the exact member-end-moments for continuous beams and bridge structures. The method is also based on the series solution of the distributed moments and carry over factors. Dowell and Johnson [8] extend the closed from solution of continuous beams and bridge frames to include deep beams to take into consideration the effect of shear deformation. Series and multiple products expressions were

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 2 (2018) pp. 922-928

used to find exact results as those obtained by stiffness method. Adam et al [9] used the method of moment distribution to analyze continuous beams of various number of spans and different span lengths subjected to a uniformly distributed load on all spans. They determined the values of negative bending moments and the results were filtered and presented in tables and charts giving the values of coefficients for the negative moments. The discrepancy of the results from the exact values may be up to about 9%. In other work, Adam [10] summarizes the bending moment values at supports of continuous beams in a form of charts. Beams of up to four spans with a uniformly distributed load and different length of spans were considered. There are also many other attempts to formulate closed-form solutions for the analysis of cases of continuous beams subjected to vibration, beams stiffened with FRP, and skew curved beam. [11, 12, 13]

The present work is an attempt to make use of the results of the exact methods of analysis to derive formulas, although approximate, but simple to be used for preliminary analysis and design purposes. For this aim, the method of superposition of Jasim and Atalla [14], which was originally developed, for continuous composite beams is generalized to include any continuous beam. In this method, each span of the continuous beam is substituted by three single-span beams, namely; propped cantilever having right end fixed (RP), propped cantilever having left end fixed (LP), and simply supported beam (SS). Each of these substituted single span beams, has the same length as that of the actual considered span, as described in Fig.(1). The load on the substituent two propped cantilever beams are determined such that the bending moments at the fixed ends of RP and LP equal to the bending moment at the right and left supports of the considered span, respectively.

Since the bending moments at the two supports of any span in a continuous beam are functions of the loads on all spans, thus, the loads on the propped cantilevers are also functions of the loads on all spans of the continuous beam and not only of the considered span alone. The load on the third substituent beam, i.e. the simply supported beam is determined such that the sum of the loads on the three substituent beams equals the load on the actual considered span.

It is worth to note that for any continuous beam, the bending moment at the fixed end of RP for any span must equal the bending moment at the fixed of LP of the next adjacent right span. This is because that the bending moments in their fixed supports must equal the bending moment in the actual common support between the considered spans of the continuous beam. Furthermore, it is obvious that the bending moment in LP for the first left span as well as the bending moment in RP for the last right span must be zero, since the moments at the exterior supports are zero.

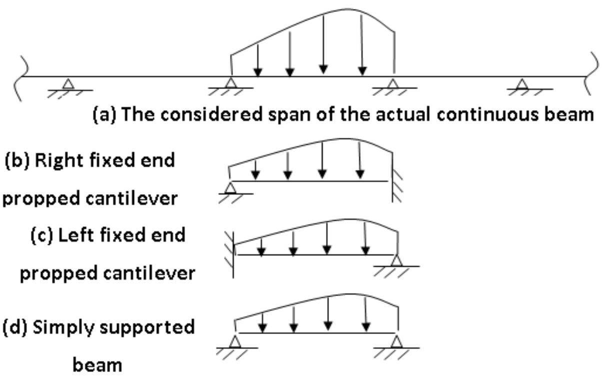

Figure 1: The considered span of a continuous beam and its substituents single span beams.

METHOD OF ANALYSIS

The following steps can summarize the method of analysis:

1. For the beam shown in Fig. (1), the load on the first span only is considered (assuming the other spans temporarily unloaded), and the classical methods is then used to determine the moment at each support. For each span, the equivalent load on RP is found such that the bending moment at the fixed end of the RP equals the bending moment at the right support of that span.
2. The same steps are repeated for the loads on the other spans, each considered separately.
3. The total equivalent load on the RP for each span due to loads on all loaded spans is the sum of loads on RP for each separate case, i.e.

$$L\,(RP)_m = \sum_{k=1}^{n} L(RP)_{mk} \qquad (1)$$

   where $L\,(RP)_m$: is the total load on RP of span m due to load on all spans.

   $L(RP)_{mk}$: the total load on RP of span m due to load on span k only.

   n: the number of spans in the continuous beam.

4. Recall that the bending moment in the fixed end of LP at any span must equal the bending moment in the fixed end of the left adjacent RP. This leads to the conclusion that the total equivalent load on LP for each span is

$$L\,(LP)_m = L\,(RP)_{m-1} \qquad (2)$$

5. The total equivalent load on SS for each span is determined such that the sum of loads on the three substituent single span beams equals the load on the considered span of the continuous beam. Thus,

$$L\,(SS)_m = L_m - [L(RP)_m + L(LP)_m] \qquad (3)$$

   where L_m is the actual load on span m of the continuous beam.

6. Since the moments at the external supports are zero, this leads to

$$L(LP)_1 = L(RP)_n = 0 \qquad (4)$$

DERIVATION OF LOAD EXPRESSIONS:

To find the expressions of the load for each substituent beam for the various cases of continuous beams, the above method of analysis is applied to the various cases of continuous beams of various spans number and loading distributions. For each case, the total equivalent load on each of the single-span substituent beams is determined. The procedure can illustrated by considering the case of two spans beam, shown in figure (2).

The first step is to consider the load on the first span, i.e., span abalone. From which bending moment (B.M.) at support B can prove to be ($-\omega_1 L^2/16$). For this span, the toads on substituent beams for span ab is first considered.

The load on RP is determined such that the B.M. at its fixed end equals the B.M. at support B. Hence,

$-(L(RP)_{11}) \times L^2 / 8 = -\omega_1 L^2/16$

which yields, $L(RP)_{11} = 0.5\ \omega_1$ (5-a)

From equation (4), the load on LP is zero, i.e.

$L(LP)_{11} = 0$ (5-b)

The load on SS is determined from eq.(3) as,

$L(SS)_{11} = \omega_1 - 0.5\ \omega_1 - 0$

$L(SS)_{11} = 0.5\ \omega_1$ (5-c)

The loads on the substituent beams for span bc due to load on span ab is determined using the same procedure to obtain.

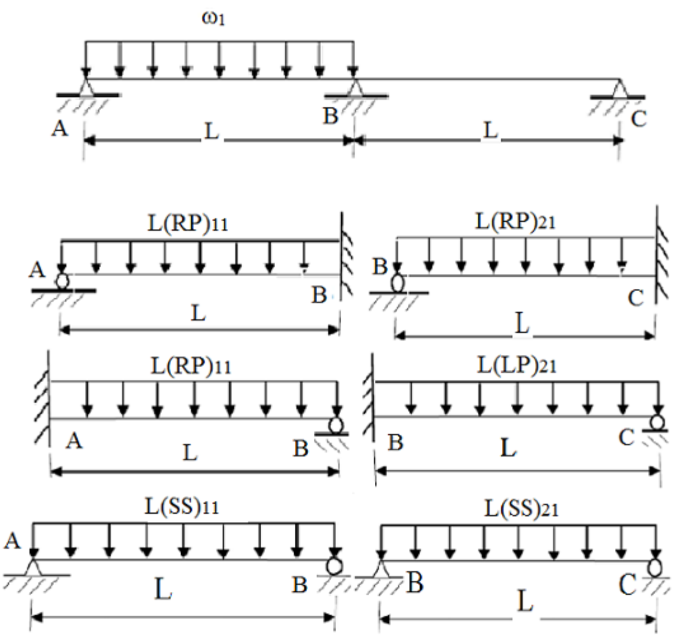

Figure 2: Derivation of load expressions for the case of two spans continuous beam

$L(RP)_{21} = 0$ (5-d)

$L(LP)_{21} = 0.5\ \omega_1$ (5-e)

$L(SS)_{11} = -0.5\omega_1$ (5-f)

Since span bc is not loaded, the above expressions represents the total loads on the substituent spans for the two spans beam shown in figure (2).

In general, the loads of the substituent single span beams for continuous beams of various span number and loads on their various spans can be in determined by a similar procedure. The results are summarized as follows:

FOR CASE OF TWO SPANS BEAM:

$L(RP)_1 = 0.5\ L_1 + 0.5\ L_2$ (6-a)

$L(RP)_2 = 0$ from Eq. (4)

FOR CASE OF THREE SPANS BEAM:

$L(RP)_1 = 8/15\ L_1 + 6/15\ L_2 - 2/15\ L_3$ (6-c)

$L(RP)_2 = -2/15\ L_1 + 6/15\ L_2 + 8/15\ L_3$ (6-d)

$L(RP)_3 = 0$ from Eq. (4)

FOR THE CASE OF FOUR AND MORE SPANS BEAM:

The values of bending moments for a continuous beam of more than three spans when the load is applied on one of its spans are found to be slightly affected by the number of spans of that beam. This means that, for example, when the load is on the first span of a continuous beam (of four or more spans), the bending moments at first, second, and other interior supports are approximately the same irrespective of the number of spans of that beam. This finding can be exploited to subtend the continuous beams of four and more spans in one case. For the purpose of determining the equivalent loads on RPs, a continuous beam of ten spans is taken as a representative case. In the present case, the development of simple approximate expressions is found to be more beneficial than exact complicated one.

By using a procedure similar to that used in the preceding cases, the load is considered to be applied on one span only, with the other spans are free of loads. The bending moments at the interior supports are then determined by any of the classical methods of analysis of continuous beams. In this work, the free version of DTBeams program is utilized [16]. The equations of moments at the interior supports number m, i.e. M_m as ratios of the moment at the right support of the loaded span k, i.e. M_k for the various cases of location of the loaded span are detailed in the following:

<u>when the load is on span 1 (k = 1):</u>

$M_{m,1} = M_1 \times (-0.268)^{(m-1)}$

<u>when the load is on span 2 (k = 2):</u>

$M_{m,2} = M_2 \times (-0.268)^{(m-2)}$ for $m \geq 2$

$M_{m,2} \approx M_2$ for $m = 1$

when the load is on span 3 (k = 3):

$M_{m,3} = M_2 \times (- 0.268)^{(m - 3)}$ for m ≥ 3

$M_{m,3} = M_2 \times (- 0.268)^{(2 - m)}$ for m = 2,1

when the load is on span 4 (k = 4):

$M_{m,4} = M_2 \times (- 0.268)^{(m - 4)}$ for m ≥ 4

$M_{m,4} = M_2 \times (- 0.268)^{(3 - m)}$ for m = 3,2,1

In general, the moment in right support of span m due to the load on span k can be determined by using the general equation below:

$$M_{m,k} = M_k \times (- 0.268)^{T} \quad (7\text{-a})$$

where:

$M_{m,k}$: the bending moment at the right support of span m due to load on span k.

M_k : the bending moment at the right support of span k due to load on that span.

and $T = m - k$ for m ≥ k (7-b)

or $T = k - m - 1$ for m < k (7-c)

It is worth to mention that values very close to the factor (0.268) in the above equations can be determined by using the deformations compatibility relations. In fact, the value of this factor abruptly reduces to 0.25 when determining the moment in the last internal support away from the loaded span. These findings are in general agreement with the results of equations developed by Zuraski [5]. The discrepancy in the values of moments due to the difference between the used value of the factor in eq.(7-a) which is 0.268 with the actual value at the last internal support which is 0.250, is obviously of minor importance since the value of the bending moment at the that support will be very small and of negligible effect. So that, for simplicity, the factor is fixed at a single value of 0.268 for all spans.

The other task is to determine the bending moment at the right support of the loaded span, M_k which would be used to determine the equivalent load on the right fixed end propped cantilevers RP that substitute the loaded and other spans. As mentioned above for the case of continuous beams of more than three spans, the development of approximate yet simple expressions rather than exact complicated ones is more useful. In this context, to simplify the derivation, the load is assumed as uniformly distributed. The expressions of bending moment at the right support of the loaded span, i.e., M_k are as following:

When the load is on span 1 (k=1):

$$M_1 = - 0.067\, w_1 \times (L_1)^2 \quad (7\text{-d})$$

When the load is on any of the other spans (k > 1):

$$M_k = - 0.053\, w_k \times (L_k)^2 \quad (7\text{-e})$$

Substituting equations (7-d and e) in equations (7- a), yields

a. For k = 1

$$M_{m,1} = - 0.067\, w_1 \times (L_1)^2 \times (- 0.268)^{T} \quad (7\text{-f})$$

b. For k > 1

$$M_{m,k} = - 0.053\, w_k \times (L_k)^2 \times (- 0.268)^{T} \quad (7\text{-g})$$

The final step is to determine the equivalent load on RP substituent beam for each span by equating the bending moment at the right support of that span with the bending moment at the fixed end of RP beam. Thus, the value of the equivalent load on RP for span number m when only the load on span number k is active is

a. For k = 1

$$L(RP)_{m,1} = 0.536 \times (- 0.268)^{T} \quad (7\text{-h})$$

b. For k > 1

$$L(RP)_{m,k} = 0.424 \times (- 0.268)^{T} \quad (7\text{-i})$$

The total equivalent load $L(RP)_m$ for span number m due to all loads is determined from eq.(1). The other loads, i.e. $L(LP)_m$ and $L(SS)_m$ are determined from eqs. (2 and 3).

Although the preceding derivation considers the case when the load is uniformly distributed, in fact the same equations result if another case of the load is considered. The applications will examine different loading types to prove the validity of these equations.

APPLICATIONS:

In the following article, the suggested equations are applied to three examples of continuous beams to verify their accuracy.

CASE STUDY 1: Two – spans continuous beam

In the first application of the proposed method a case of two-spans continuous beam shown in Fig.(3) is considered. The analysis is started by replacement of each span by the three single span substituent beams, i.e RP, LP, and SS. By using the expressions derived above, the load on each of the substituent beams is determined.

From eq. (6-a), $L(RP)_1 = 0.5L_1 + 0.5\, L_2$

thus, $L(RP)_1 = 0.5\, w + 0.5\, P$

From eq. (4), $L(LP)_1 = 0$

From eq.(3), $L(SS)_1 = L_1 - [L(RP)_1 + L(LP)_1]$

thus, $L(SS)_1 = w - [w/2 + P/2] = w/2 - P/2$

Using the same equations, the loads on substituent beams for the second span can be determined, to be

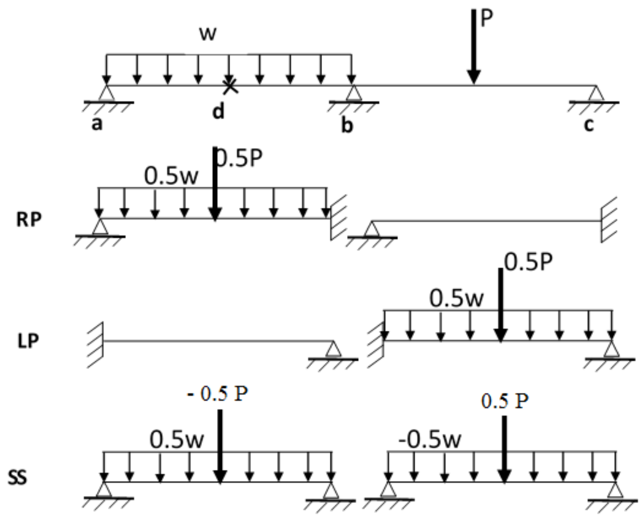

Figure 3: Continuous beam of two spans

$L(RP)_2 = 0$

$L(LP)_2 = LRP1 = w/2 + P/2$, and

$L(SS)_2 = P - \{w/2 + P/2\} = P/2 - w/2$

The accuracy of the method is examined by determining some values of bending moment and deflection at selected locations by using the equations for loads and moments developed in the current work and compare them with those determined by exact methods. In this work, the moment at a certain location is determined by the method of superposition for the corresponding values determined for each of the substituent beams. By similar principle, the deflection at any point in one of the spans of the continuous beam is determined by the sum of deflection values at the same point for the three substituent beams of that span. For this purpose, the bending moment at middle support (M_b) and the deflection at midpoint (y_d) of the left span are considered.

The bending moment (M_b) equals the bending moment at fixed end of RP_1, since the bending moment at the right supports of LP_1 and SS_1 are zero. The bending moment at fixed the end of RP_1 is the sum of moments of propped cantilever loaded with a distributed load and propped cantilever loaded with a point load at mid-point, thus

$M_b = -(0.5w)L^2/8 - 3(0.5P)L/16 = -wL^2/16 - 3PL/48$

This value exactly equals the value of moment at mid-support of a continuous beam loaded as the beam shown in Fig.(3).[15]

Now, the moment at midpoint of the left span, M_d, is to be determined which is the sum of bending moments at midspans of RP_1, LP_1, and SS_1, which are $M(RP_1)_d$, $M(LP_1)_d$, and $M(SS_1)_d$,respectively. Thus

$M_d = M(RP_1)_d + M(LP_1)_d + M(SS_1)_d$

$$M_d = [\,(0.5w)L^2/16 + 5(0.5P)L/32] + [0] + [\,(0.5w)L^2/8 + (-0.5P)L/4]$$

$M_d = 3wL^2/32 - 3PL/64$

Again, this value exactly equals the value determined by the analytic methods [15].

The third test, is the deflection at the midpoint of the left span Y_d.

$Y_d = Y(RP_1)_d + Y(LP_1)_d + Y(SS_1)_d$

$$Y_d = [\,(0.5w)L^4/192EI + 7(0.5P)L^3/768EI] + [0] + [5(0.5w)L^4/384EI + (-0.5P)L^3/48EI]$$

$Y(L/2) = 7wL^4/768EI - 9PL^3/1536EI$

Which is identical to the exact value.

CASE STUDY 2: Three – spans continuous beam

In this second case the three-spans beam shown in Fig.(4) is considered.

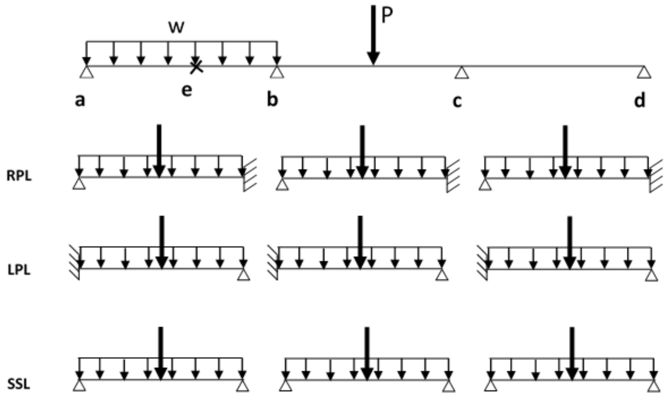

Figure 4: Continuous beam of three spans

By using the equations derived in section 4 for the case of three spans, the loads on the substituent beams can be determined to be:

$L(RP)_1 = 8/15\ w - 2/15\ P$, $L(RP)_2 = -2/15w + 8/15\ P$, $L(RP)_3 = 0$

$L(LP)_1 = 0$, $L(LP)_2 = 8/15w - 2/15\ P$, $L(LP)_3 = -2/15\ w + 8/15\ P$

$L(SS)_1 = 7/15\ w\ 2/15\ P$, $L(SS)_2 = -6/15\ w - 6/15\ P$, $L(SS)_3 = 2/15\ w + 7/15\ P$

The bending moment at point d is to be checked.

$M_b = M(RP_1)_b + M(LP_1)_b + M(SS_1)_b$

$M_b = (8/15\ w)\ L^2/8 + 3(-2/15\ P)\ L/16 = wL^2/15 - PL/40$

The deflection at point e is to be checked.

$Y_e = Y(RP_1)_e + Y(LP_1)_e + Y(SS_1)_e$

$$Y_e = [(8/15\ w)\ L^4/192EI + (-2/15\ P)\ 7L^3/768EI\ ] + [0] + [(7/15\ w)\ 5L^4/384EI + (2/15P)\ L^3/48]$$

$Y_e = (51\ wL^4 + 9PL^3)/5760EI$

The above results are identical to the exact values. [15]

CASE STUDY 3: Six – spans continuous beam

The six spans continuous beam shown in Fig.(5) is considered as an example for the case continuous beam of four or more spans. The beam is loaded as shown and each span is of 4m length.

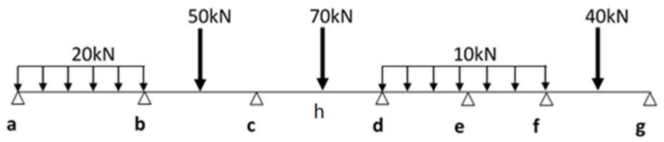

Figure 5: Continuous beam of six spans

In this case, the bending moment at points c and d and the deflection at the point h will be determined. One advantage of the present method is that, when deflection, moment, or shear at certain span are required, there will no necessity to analyze the whole beam, and instead, that span is only analyzed. So that, the calculations for the loads on the substituent beams for third span are only conducted.

The load on RP for the third span, i.e. $L(RP)_3$, is determined using eq.(1), thus

$$L(RP)_3 = \sum_{k=1}^{6} L(RP)_{3k}$$

From eq. (7-h),

$L(RP)_{31}$= [0.536(-0.268)$^{3-1}$] x 20 = +0.77 kN/m,

From eq. (7-i) and eq. (7-b),

$L(RP)_{32}$=[0.424(-0.268)$^{3-2}$] x 50 = -5.68 kN

$L(RP)_{33}$= [0.424(-0.268)$^{3-3}$] x70 = +29.68 kN

From eq. (7-i) and eq. (7-c),

$L(RP)_{34}$= [0.424(-0.268)$^{4-3-1}$] x 10 = +4.24 kN/m

$L(RP)_{35}$= [0.424(-0.268)$^{5-3-1}$] x 10 = -1.14 kN/m

$L(RP)_{36}$= [0.424(-0.268)$^{6-3-1}$] x 40 =+1.22 kN

Thus, $L(RP)_3$= + 3.87 kN/m + 25.22 kN

The loads on RP of the third span, i.e. $L(LP)_3$ is determined using eq. (2), hence

$L(LP)_3 = L(RP)_2$

Using similar procedure similar to that used in calculating $L(RP)_3$

$L(RP)_{21}$= - 2.87 kN/m

$L(RP)_{22}$= + 21.2 kN

$L(RP)_{23}$= + 29.68 kN

$L(RP)_{24}$= - 1.14 kN/m

$L(RP)_{25}$= + 0.30 kN/m

$L(RP)_{26}$= - 0.33 kN

$$L(LP)_3 = L(RP)_2 = \sum_{k=1}^{6} L(RP)_{2k} = -3.71 \text{ kN/m} + 50.55 \text{ kN}$$

Finally, the load on SS for the third span, i.e. $L(SS)_3$ is determined using eq. (3),

$L(SS)_3$ = 70 kN - [(+ 3.87 kN/m + 25.22 kN) + (- 3.71 kN/m + 50.55 kN)]

$L(SS)_3$ = - 0.16 kN/m – 5.77 kN

The bending moment at point c is first determined. This moment equal the bending moment at the fixed end of LP_3, which in turn equals the moment comes from the distributed load w and the point load P. Thus,

$$M_c = -\frac{w\,L^2}{8} - \frac{3\,P\,L}{16}$$

$$M_c = -\frac{(-3.71)\times 4^2}{8} - \frac{3\,(+50.55)\times 4}{16}$$

$$M_c = -30.49\ kN.m$$

The exact value computed using DTBeam continuous beam analysis code [16], is

$(M_c)_{\text{exact}}$ = - 30.54 kN.m,

The error in the calculated value using the suggested method is 0.16 %.

Now, the bending moment at point d is checked, which in this case equals the bending moment at the fixed end of RP_3.

$$M_d = -\frac{(+3.87)\times 4^2}{8} - \frac{3\,(+25.22)\times 4}{16}$$

$$M_d = -26.66\ kN.m$$

The exact value is $(M_d)_{\text{exact}}$ = - 26.73 kN.m

The error is 0.26%

The deflection at point h is examined, which equals the sum of deflections of the three single span beams substituted the third span. Each of them comes from the effects of the distributed and point loads. Thus,

$$y_h = y(LP_3)_{L/2} + y(RP_3)_{L/2} + y(SS_3)_{L/2}$$

In which y_h is the deflection of the continuous beam at point h, $y(LP_3)_{L/2}, y(RP_3)_{L/2}, and\ \ y(SS_3)_{L/2}$ are the deflection of LP, RP, and SS beams of the third span at their midponts.

$$y(LP_3)_{L/2} = \frac{w\,L^4}{192EI} + \frac{7\,P\,L^3}{768EI}$$

$$y(LP_3)_{L/2} = \frac{24.341}{EI}\ (m)$$

Similarly,

$$y(RP_3)_{L/2} = \frac{+19.872}{EI}\ (m)$$

and

$$y(SS_3)_{L/2} = \frac{5w\,L^4}{384EI} + \frac{P\,L^3}{48EI}$$

$$y(SS_3)_{L/2} = \frac{-8.227}{EI}\ (m)$$

The deflection at point h is the sum of the above three values, i.e.

$$y_h = \frac{+35.986}{EI}\ (m)$$

The exact value is

$$(y_h)_{exact} = \frac{+36.064}{EI}\ (m)$$

The error is 0.22%

The above results show that the error in results using the suggested method are less than 0.5% for moment and deflection calculations.

CONCLUSIONS

An elastic analysis of continuous beams of equal spans is conducted to determine the bending moment and deflection at various locations. Three single span beams are suggested to substitute each span of the continuous beam and the load on each of them is determined by equations developed for this purpose. The equations are simple to be applied to various loading and number of spans. These equations together with the substituent single span beams can be used to determine the bending moment, shear force and deflection at any location in the continuous beam without the need to analyze the whole beam.

The application of the method on selected typical cases of continuous beams shows that the errors in the calculated values of bending moment and deflection are less than 0.5%. The method can be considered as a spot check tool for the results from other method and can be used in the preliminary analysis to estimate member dimensions that would be used in the rigorous analysis.

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