Modeling of Short Duration Rainfall Intensity Duration Frequency(SDR-IDF) Equation for Basrah City

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Abstract:

The Rainfall Intensity-Duration-Frequency (IDF) relationship is one of the most commonly used tools in water resources engineering, either for planning, designing and operating of water resource projects or for various engineering projects against floods. The objective of this study is to develop an empirical formula to estimate rainfall intensity for any duration and any return period with minimum effort. Daily rainfall data for years 1980-2010 from Iraqi Metrological Organization and Seismology was used in this study. Hershfeild's method was used to estimate the short duration rainfall intensity from daily rainfall data. Various distribution functions were used for analysis and Chi-Square goodness to fit test were used to identify the best statistical distribution among them. Study showed that Log Pearson type III is the best probability distribution and the best (IDF)empirical formula was in the form [i=a/(b=td)].

Keyword: Chi-Square tests, Probability Distribution Function, Return Period, Short Duration rainfall.

تمثيل معادلة الشدة المطرية –التكرار – الاستدامة القصيرة لمدينة البصرة

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الخلاصة:

العلاقة بين الشدة المطرية وتكرار الشدة المطرية والاستدامة هي إحدى أكثر الأدوات المستعملة في هندسة مصادر المياه، سواء في التخطيط أو في تشغيل مصادر المياه او في مختلف المشاريع الهندسية لمقاومة الفيضانات. الهدف من هذه الدراسة هو تمثيل معادلة تجريبية لتخمين الشدة المطرية لأي تكرار وأي استدامة بجهد قليل. استخدمت في هذه الدراسة معلومات مطرية ل٣١ سنة كما استخدمت طريقة هيرشفيلد لإيجاد الاستدامة القصيرة للشدة المطرية. استخدمت مختلف الدوال التوزيعية الإحصائية لتحليل المعلومات واستخدمت طريقة ال Chi-Square كاختبار لتشخيص التوزيع الإحصائي المناسب. ولقد بينت الدراسة ان Log-Pearson typeIII هي ألافضل وان أفضل معادلة للشدة المطرية هي بالشكل [i=a/b+td].

Introduction:

The design of storm water drainage system is a critical issue in urban areas since it depends on estimation of rainfall intensity which varies with time and location. The fall in estimating the rainfall intensity can cause flood of these areas. The rainfall intensity is usually estimated based on rainfall duration and return period. The formula that represents the mathematical relationship between rainfall intensity, duration and return period is called Intensity Duration Frequency (IDF) formula.

In order to formulate IDF relationship for specific site, like Basrah city in south of Iraq, historical series of the maximum rainfall intensity data at a higher time resolution (in order of minutes) is required. Due to instrumental limitation short duration rainfall data in Basrah city are not available. The aim of this study is to put an empirical formula for IDF relationship in Basrah city.

Many sets of IDF relationships have been constructed for several parts of the globe: Hershfiled (1961) developed various rainfall contour maps to provide the design rain depths for various return periods and durations. Bell (1969) proposed a generalized IDF formula using the one hour, 10 years rainfall depths; P_1^{10}, as an index. Chen (1983) further developed a generalized IDF formula for any location in the USA using three base rainfall depths; P_1^{10}, P_{24}^{10},P_1^{100}, which describe the geographical variation of rainfall. Kouthyari and Garde (1992) presented a relationship between rainfall intensity and P_{24}^{2} for India.

Estimation of Short Duration Rainfall

Daily rainfall data for the study area (Basrah city) were available for a period of 30 years. From this database, the maximum values were extracted for each year and were converted into shorter duration values (5, 10, 15, 30, 60, and1440min) using Hershfeild's method, this method gives the relationship as ratios as follows (Brian,2009):

- The ratio of 60- minutes maximum rainfall to 24-hour maximum rainfall is 0.6.
- The ratio of 30- minutes maximum rainfall to 1-hour maximum rainfall is 0.79.
- The ratio of 15- minutes maximum rainfall to 1-hour maximum rainfall is 0.57.
- The ratio of 10- minutes maximum rainfall to 1-hour maximum rainfall is 0.45.
- The ratio of 5- minutes maximum rainfall to 1-hour maximum rainfall is 0.29.

Empirical IDF Equation

IDF is a mathematical relationship between the rainfall intensity (i), the duration (td) and the return period (T) (Burlando 1996, Koutsoyiannis 1998). Equations (1)-(2) are the forms of IDF empirical equation which were used in this study.

$$i = [a / (b + td)] \quad \text{-----------------------(1)}$$

$$i = x.(td)^{-y} \quad \text{----------------------------(2)}$$

where, i is the rainfall intensity in mm/hr, td is the rainfall duration in min. and a, b, x, and y are the fitting parameter.

These empirical equations are widely used in various hydrological applications. These equations indicate that for a given return period the rainfall intensity decrease with the increase in rainfall duration. Least-square method was applied to find the parameter a, b, x, and y for the IDF empirical formula. Correlation coefficient (R) was estimated to find the best fit for IDF empirical equation. For a specific return period the equation that gives R value nearer to 1 have the best fit.

Generalized IDF Formula

To put a formula for IDF, rainfall data for at least 25 years is required. The first step of constructing the IDF curve for Basrah city is to assess the local rainfall data and determine the maximum rainfall depth associated with each year. Then, statistical analysis for each duration is run. The mean and standard deviation are determined as functions of duration. Two arrays were obtained: one for the mean depth of rainfall (μ) as a function of rainfall duration and the other one is for the standard deviation of the rainfall depth (σ).

The second step is to fit a Probability Distribution function (PDF) or a Cumulative Distribution Function(CDF) to each group comprised of the data values for a specific duration. It is possible to relate the maximum rainfall intensity for each time interval with the corresponding return period from the cumulative distribution function. Given a return period (T), its corresponding cumulative frequency (F) will be:

$$F = 1-(1/T) \text{ or } T = (1/1-F) \quad \text{----------------- (3)}$$

Once a cumulative frequency is known, the maximum rainfall intensity is determined using chosen theoretical distribution function (e.g. Normal, Gumbel, Pearson type III distributions).

Before explaining the third step, frequency analysis using frequency factors will be explained. The magnitude X_T of a hydrologic event may be represented as the mean μ plus the departure ΔX_T of the variate from the mean:

$$X_T = \Delta X_T + \mu \quad \text{-----------------------------------(4)}$$

The departure may be taken as equal to the product of the standard deviation σ and a frequency factor K_T; that is, $\Delta X_T = K_T \sigma$. The departure ΔX_T and the frequency factor K_T are functions of the return period and the type of probability distribution to be used in the analysis. Equation (4) may therefore be expressed as,

$$X_T = K_T \sigma + \mu \quad \text{-------------------------------------(5)}$$

Probability Distribution:

In this study the maximum rainfall intensity for various return periods were estimated using different theoretical distribution functions. Normal (Hazen 1914), Two Parameter Log Normal, Three Parameter Log Normal (Kite, 1977), Gamma Distribution Two Parameter, Pearson Type III, Log Pearson Type III, and Extreme Value Type I (Gumbel) (Chow, 1964;Yevejevik,1972). were used for probability distribution of the daily rainfall data.

Normal Distribution: If X is the sum of n independent and identically distributed random variables with finite variance, then with increasing n the distribution of X becomes normal regardless of the distribution of random variables the Probability Distribution function PDF for normal distribution is.

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{-------(6)}$$

Where μ is the mean and σ is the standard deviation. A standard normal distribution is a normal distribution with mean (μ) = 0 and standard deviation (σ) = 1.Normal distribution is transformed to standard normal distribution by using the following formula:

$$z = \frac{X - \mu}{\sigma} \quad \text{----------------------(7)}$$

Where z is called the standard normal variable.

Two Parameter Log Normal: If the PDF of X is skewed, it's not normally distributed. If the PDF of Y = log (X) is normally distributed, then X is said to be log normally distributed.

$$f(x)=\frac{1}{x\sigma\sqrt{2\pi}}\exp\left(-\frac{(y-\mu_y)^2}{2\sigma_y^2}\right) \qquad x>0, and\ y=log\ x$$

--------(8)

Three Parameter Log Normal: A combination of the normal distribution and the modified logarithmic transformation results in the three parameter log normal distribution. This distribution has an additional parameter is the location parameter ζ, which is the lower limit of the variable.

$$Y=\ln(x-\zeta) \leftrightarrow x=\zeta+e^{y}$$ --------------(9)

$$f(x)=\frac{1}{x\sigma\sqrt{2\pi}}\exp\left(-\frac{(y-\mu_y)^2}{2\sigma_y^2}\right) \qquad x>0, and\ y=log\ x$$

------(10)

Gamma Distribution: In hydrology, the interarrival time (time between stochastic hydrologic events) is described by exponential distribution,

$$f(x)=\lambda e^{-\lambda x} \qquad x\geq 0;\ \lambda=\frac{1}{x}$$ --------------(11)

Gamma distribution – a distribution of sum of β independent and identical exponentially distributed random variables.

$$f(x)=\frac{\lambda^{\beta}x^{\beta-1}e^{-\lambda x}}{\Gamma(\beta)} \qquad x\geq 0;\Gamma=gamma\ \ function$$

------(12)

Pearson Type III : Named after the statistician Pearson, it is also called three-parameter gamma distribution. A lower bound is introduced through the third parameter (ε).

$$f(x) = \frac{\lambda^{\beta}(x-\varepsilon)^{\beta-1}e^{-\lambda(x-\varepsilon)}}{\Gamma(\beta)} \qquad x \geq \varepsilon; \Gamma = gamma\ \ function$$

---(13)

Log-Pearson Type III: If log X follows a Person Type III distribution, then X is said to have a log-Pearson Type III distribution.

$$f(x) = \frac{\lambda^{\beta}(y-\varepsilon)^{\beta-1}e^{-\lambda(y-\varepsilon)}}{\Gamma(\beta)} \qquad y = log\ x \geq \varepsilon$$

------------(14)

Extreme value Type I (Gumbel) distribution: If M1, M2…, Mn be a set of daily rainfall or stream flow, and let X = max(Mi) be the maximum for the year. If Mi are independent and identically distributed, then for large n, X has an extreme value type I or Gumbel distribution.

$$f(x) = \frac{1}{\alpha}\exp\left[-\frac{x-u}{\alpha} - \exp\left(-\frac{x-u}{\alpha}\right)\right]$$

$$\alpha = \frac{\sqrt{6}s_x}{\pi} \qquad u = \bar{x} - 0.5772\alpha$$

-----------(15)

Frequency Analysis for Extreme Events:

ExtreamVlue type(I) PDF and CDF

$$f(x) = \frac{1}{\alpha}\exp\left[-\frac{x-u}{\alpha} - \exp\left(-\frac{x-u}{\alpha}\right)\right]$$

$$\alpha = \frac{\sqrt{6}s_x}{\pi} \qquad u = \bar{x} - 0.5772\alpha$$

---------------(15)

$$F(x) = \exp\left[-\exp\left(-\frac{x-u}{\alpha}\right)\right]$$

----------------------(16)

Define a reduced variable y

$$y = \frac{x-u}{\alpha}$$

$$F(x) = \exp[-\exp(-y)]$$

$$y = -\ln[-\ln(F(x))] = -\ln[-\ln(1-p)] \quad where\ p = P(x \geq x_T)$$

$$y_T = -\ln\left[-\ln\left(1-\frac{1}{T}\right)\right]$$

If you know T, you can find y_T, and once y_T is know, x_T can be computed by

$$x_T = u + \alpha y_T$$

Chi-Square Test:

To identify a specific theoretical distribution for the available data it is important to do a test. The aim of the test is to find how a good fit is between the observed and the prediction data. Chi –square test is one of the most widely used tests to find the best fit theoretical distribution of any specific data set which is represented by equation(17).

$$X^2 = \sum_{i=1}^{n} (Oi - Ei) / Ei \quad \text{--------------(17)}$$

Where, Oi and Ei represent the observed and expected frequencies respectively. If the observed frequencies are close to the corresponding expected frequencies, the X^2 value will be small, indicating a good fit; otherwise it will be a poor fit.

Results and Discutions:

In order to put an empirical formula for intensity- duration- frequency relationship in Basrah city, the available data required from the General Iraqi Meteorological Authority and Seismology includes 24-hour rainfall data from 1980 – 2010 for Basrah city were considered as presented in Table (1).

Various short duration rainfalls like (5, 10, 15, 30, 60, and 1440 min.) were estimated from annual maximum 24 hours rainfall data using Hershfield's method as shown in Table (2). These estimated short duration rainfall data were used in various probability distribution methods to determine the rainfall and their corresponding return period. Table (3) shows the X^2 values of different probability distribution for 5, 10, 15, 30, 60, and 1440minutes duration rainfall.

Table (1): Maximum Daily Rainfall Recorded in Basrah City During the period 1980-2010.

Year	Daily Rainfall (mm)	Year	Daily Rainfall (mm)
1980	28.1	1996	29.4
1981	21.3	1997	30.8
1982	15.1	1998	22.1
1983	18.3	1999	73.6
1984	45.4	2000	33.0
1985	73.2	2001	26.6
1986	58.5	2002	15.7
1987	9.60	2003	17.5
1988	M	2004	17.0
1989	17.5	2005	26.0
1990	11.6	2006	27.5
1991	57.0	2007	37.0
1992	21.9	2008	18.0
1993	22.9	2009	20.0
1994	28.5	2010	6.5
1995	21.0		

It was found in the study that X^2 value of all probability function was increasing with increasing the rainfall duration. The study shows that Log-Pearson Type III probability distribution function gives the best estimation of the distributed predicted rainfall data as it has the smallest value of X^2 , 0.0372, as compared to other distribution functions. Rainfall intensity values (in mm / hr) for various short durations and return periods in Basrah city were estimated using Log- Pearson Type III probability distribution function and they are presented in Table (4). Table (4) showed that maximum intensities occur at short duration with large variations with return period, while with long duration there is no much difference in intensities with return period and the maximum intensity occur at return period 100 years with duration of 5 minutes and minimum intensity occur at return period 2 years with duration of 1440 minute.

Table (2): Annual Time Series of Maximum Rainfall Amount in Basrah City

Year	Maximum rainfall amount in mm during indicated durations				
	5min.	**10min.**	**15min.**	**30min.**	**60min.**
1980	4.89	7.59	9.61	13.32	16.86
1981	3.71	5.75	7.29	10.10	12.78
1982	2.63	4.08	5.17	7.16	9.06
1983	3.18	4.94	6.26	8.68	10.98
1984	7.90	12.26	15.53	21.52	27.24
1985	12.74	19.76	25.04	34.7	43.92
1986	10.18	15.80	20.01	27.73	35.1
1987	1.67	2.59	3.28	4.55	5.76
1988	-	-	-	-	-
1989	3.05	4.73	5.99	8.30	10.5
1990	2.02	3.13	3.97	5.5	6.96
1991	9.92	15.39	19.50	27.02	34.2
1992	3.81	5.91	7.49	10.38	13.14
1993	3.99	6.18	7.83	10.86	13.74
1994	4.96	7.7	9.75	13.51	17.1
1995	3.65	5.67	7.18	9.95	12.6
1996	5.12	7.94	10.10	13.94	17.64
1997	5.36	8.32	10.54	14.60	18.48
1998	3.85	5.97	7.56	10.48	13.26
1999	12.81	19.87	25.17	34.89	44.16
2000	5.74	8.91	11.29	15.64	19.8
2001	4.63	7.18	9.10	12.61	15.96
2002	2.73	4.24	5.37	7.44	9.42
2003	3.05	4.73	5.99	8.3	10.5
2004	2.96	4.59	5.82	8.06	10.2
2005	4.53	7.02	8.89	12.32	15.6
2006	4.79	7.43	9.41	13.04	16.5
2007	6.44	9.99	12.65	17.54	22.2
2008	3.13	4.86	6.16	8.48	10.8
2009	3.48	5.4	6.84	9.48	12
2010	1.13	1.76	2.22	3.08	3.9
Mean (μ)	4.935	7.656	9.7	13.441	17.012
Standard deviation (σ)	2.75	4.43	5.67	7.91	10.05
Coeff. of variation (C_V)	0.56	0.58	0.585	0.589	0.591
Coeff. of skewness (Cs)	1.12	1.16	1.17	1.18	1.18

Table(3):Chi-Square value of various probability distribution functions for different rainfall duration.

Rainfall duration (min.)	X^2 for different probability distribution functions						
	Normal	Log-Normal two parameter	Log-Normal-three parameter	Gamma distribution two parameter	Pearson Type III	Log-Pearson Type III	Gumbel
5	0.105	0.0197	0.0152	0.0185	0.0185	0.016	0.0149
10	0.020	0.033	0.023	0.031	0.031	0.027	0.0215
15	0.023	0.039	0.031	0.043	0.043	0.031	0.0278
30	0.030	0.058	0.045	0.062	0.062	0.048	0.0466
60	0.037	0.069	0.082	0.083	0.083	0.05	0.055
1440	0.061	0.114	0.097	0.142	0.142	0.051	0.0932
Average value of X^2	0.0461	0.0555	0.0488	0.0632	0.0632	0.0372	0.0432

Table(4):Rainfall intensity at different rainfall durations and return periods in Basrah city.

Rainfall duration (min.)	Rainfall intensity in mm / hr at indicated return periods					
	2 years	5years	10years	25years	50years	100years
5	48.72	79.8	106.68	148.92	184.92	228.48
10	37.8	61.92	82.62	115.2	143.04	176.58
15	31.68	52	69.4	96.88	120.36	148.68
30	22.06	36.1	48.16	67.18	83.38	102.92
60	13.87	22.76	30.39	42.42	52.7	65.1
1440	.963	1.579	2.11	2.94	3.65	4.513

After estimating the rainfall intensity a plot of rainfall duration versus rainfall intensity for different return period was done. Figure (1) represents the rainfall (IDF) curve of Basrah for different return period.

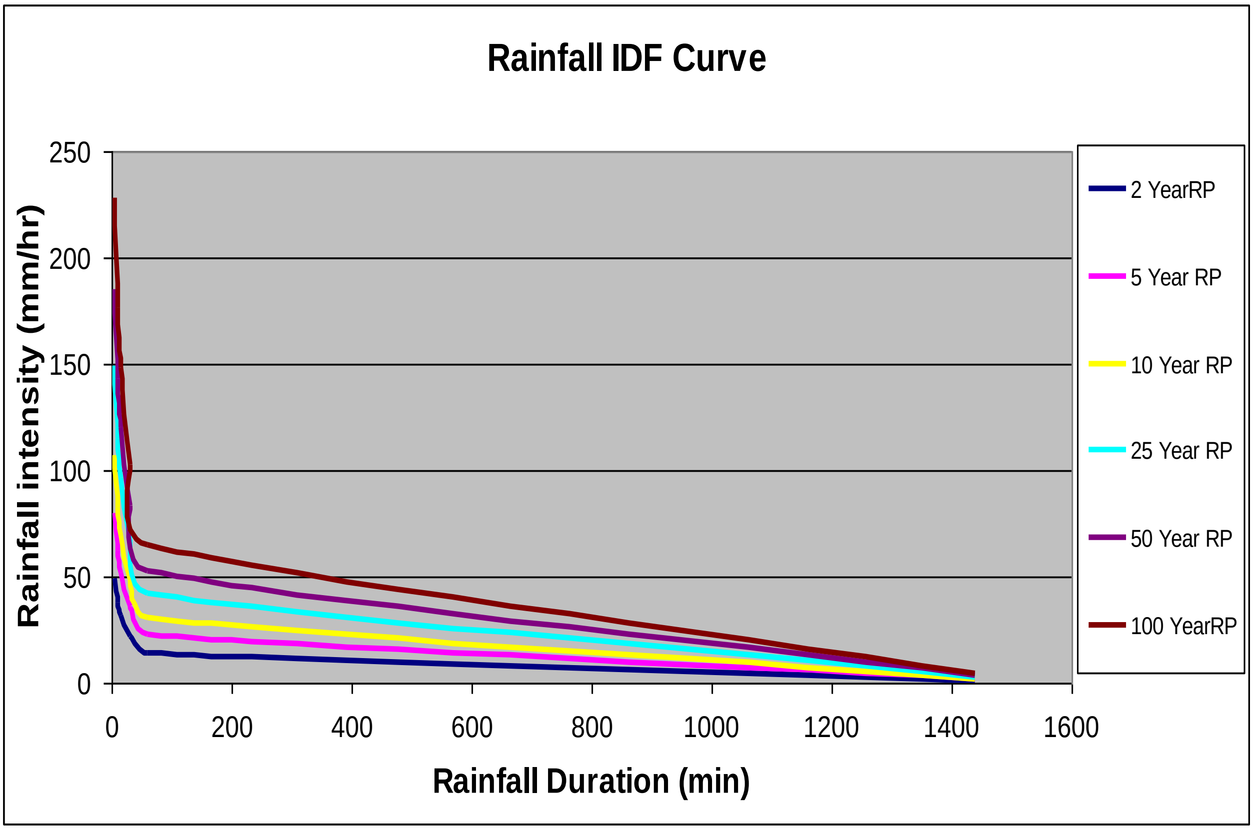

Fig.(1) Intensity- Frequency- Duration Curves for Basrah City.

Rainfall IDF empirical equation constants a, b, x, and y were calculated for different return period by least –square method as shown in Table (5). IDF empirical equation was formed by putting the value of a, b, x, and y in the mentioned equation format(1)-(2) for each return period separately as shown in Table (6).

Table (5): Parameter values of rainfall IDF empirical equations for various return periods.

Return period (year)	IDF empirical equation parameters			
	a	**b**	**x**	**y**
2	1416.41	31.4	205.59	0.715
5	2322.47	31.42	334.2	0.711
10	3103.61	31.48	449.78	0.715
25	4324.23	31.39	622.3	0.711
50	5368.45	31.37	781.63	0.715
100	6638.1	31.44	950.61	0.711

Table (6):Rainfall (IDF) empirical equation for respective return period and their correlation coefficient(R).

Return period (year)	$i=a/(b+td)$		$i=x^*(td)^{-y}$	
	Equation	Correlation coefficient	Equation	Correlation coefficient
2	1416.41/31.4+td	0.988	$205.59^*td^{-.715}$	.703
5	2322.47/31.42+td	0.988	$334.2^*td^{-.711}$	.705
10	3103.61/31.48+td	0.989	$449.78^*td^{-.715}$	.703
25	4324.23/31.39+td	0.988	$622.3^*td^{-.711}$	.705
50	5368.45/31.37+td	0.988	$781.63^*td^{-.715}$	.703
100	6638.1/31.44+td	0.988	$950.61^*td^{-.711}$	.705

Correlation coefficient (R) was, also, calculated for each form of IDF empirical equation and their corresponding return period. It was observed in the study that the IDF empirical form [i=a/(b+td)] had the best fit rather than the equation form [$i=x^*(td)^{-y}$] because it had R nearest to 1.

Conclusions:

In this study short duration rainfall intensity –duration- frequency empirical equations were developed for Basrah city. Among the various available probability distribution functions Log- Pearson Type III distribution had the best approximation rainfall intensity for various return periods because it had the smallest x^2 value which is 0.0372. The study showed that[i=a/b+td] was the best form of IDF empirical equation for Basrah city because it had a correlation coefficient R of 0.988 which is nearest to 1. These IDF equations will help to estimate the rainfall intensity for any specific return period in Basrah city in a short time and more easily. The study, also, showed that maximum intensities occur at short duration with large variations with return period, while with long duration there is no significant difference in intensities with return period and the maximum intensity occur at return period 100 years with duration of 5 minute while minimum intensity occur at return period 2 years with duration of 1440 minute.

List of symbols:

symbol	definition
X^2	chi-square goodness to fit
Oi	observed frequencies
Ei	expected frequencies
i	rainfall intensity(mm/hr)
td	rainfall duration (min)
a, b, x, and y	fitting parameter
F	cumulative frequency
T	return period
μ	mean
σ	standard deviation
K_T	frequency factor
X_T	The magnitude of a hydrologic event
ΔX_T	the departure of the variant from the mean
z	standard normal variable
ζ	location parameter
Γ	gamma function
ε	lower bound
β	sum of independent exponentially distribution

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