

## Investigation of Adapted Power and Rate on MQAM for Fading Channels

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### Abstract

The optimal adaptive technique which discussed in this paper uses variable-rate and power transmission with reasonable complexity. The impacts of these effects on the capacity, then on the spectral efficiency are described in two fading environments. Two suboptimal techniques, channel inversion and truncated channel inversion are investigated.

**Index Terms:** Adaptive power and rate (AMC), MQAM, Fading channel and OFDM.

### Introduction

The fourth generation of wireless communication systems including the 4G mobile communication phone systems and networks must provide bandwidth-efficient, robust communication with low delay, supporting multiple users on broad band wireless channels. Since time varying channel conditions and thus time-varying channel capacity is an important features of 4G mobile communication systems and any future wireless communication systems, these systems should exhibit a high degree of adaptively on many levels in order to reach these goals. Adaptive modulation and coding (AMC) techniques are used to maximize average spectral efficiency while maintaining a given average or instantaneous bit error probability (BER). The same basic premise can be applied to MIMO channels[1], frequency-selective fading channels with equalization[2], OFDM[3], or CDMA[4], and cellular systems. The OFDM scheme in particular which is mandatory for all 4G mobile systems divides the available spectrum into a number of overlapping but orthogonal narrowband subchannels, and hence converts a frequency channel into a non-frequency selective channel [5]. OFDM is a promising modulation technique for future wireless systems[5]. Adaptation algorithms of OFDM vary according to the type of estimation

channel. There are two main kinds of channel estimation techniques for OFDM based systems, blind and non-blind. The blind estimation exploit the statistical behavior of the received signals and required a large amount of data[5]. For non-blind estimation, the adapted parameters require a feedback path between the transmitter and receiver[6]. Any adaptive strategy may take into account the estimation error and delay of channel side information feedback[7]. These techniques are used in current systems specifically, in WiFi and WiMax[8]. In general the modulation parameters to vary the transmission rate are fixed over a block or frame of symbols, where the frame size is a parameter of the design. Frames may also include pilot symbols for channel estimation and other control information. Quadrature amplitude modulation (QAM) is such a class of nonconstant envelope schemes that can achieve higher bandwidth efficiency than MPSK with the same average signal power[9].

In this paper, three scenarios for adapting MQAM are introduced. Variable rate and power (optimal techniques) where the data rate and transmitted power are adapted together in sending side. The spectral efficiency of the two suboptimal techniques, channel inversion with fixed rate and discrete rate adaptation are discussed in two fading channels, Rayleigh and Lognormal shadowing.

In section II, the scenario of variable- rate variable-power is described. Channel inversion with fixed rate scenario and discrete rate adaptation scenario are presented in section III and IV, respectively. Finally, the conclusion is presented in section V.

## System Model and Adaptive Schemes for MQAM

### Variable-Rate Variable-Power

In a scenario where channel conditions fluctuate dynamically, systems based on fixed modulation schemes do not perform well, because the difference in channel conditions must take into account. In such situation, a system that adapts to the worst case scenario would have to be built to offer an acceptable bit-error rate. To achieve a robust and a spectrally efficient communication over multi-path fading channels, variable rate, variable power, variable error portability, variable coding, and the techniques of hybrid (combination) of two or more from the above parameters are used, which adapts the transmission scheme to the current channel characteristics. In variable-rate modulation the data rate  $R[\gamma]$  is varied relative to the channel gain  $g$  ( or equivalently, the signal to noise ratio,  $\gamma$ ). This can be done by fixing the symbol rate of the modulation and using multiple modulation schemes or constellation sizes, or by fixing the modulation (e.g. MPSK or MQAM) and changing the symbol rate. It's variation is difficult to implement in practice due to a varying signal bandwidth and complication bandwidth sharing. In contrast, changing the constellation size or modulation type with a fixed symbol rate is fairly easy[7]. If the channel can be estimated properly, the transmitter can be easily made to adapt to the current channel conditions by altering the modulation schemes while maintaining a constant BER. The model of [7] (see Figure 1) is considered. This model assumed a linear modulation where the adaptation that takes place at a multiple of the symbol rate  $R_s = 1/T_s$ .

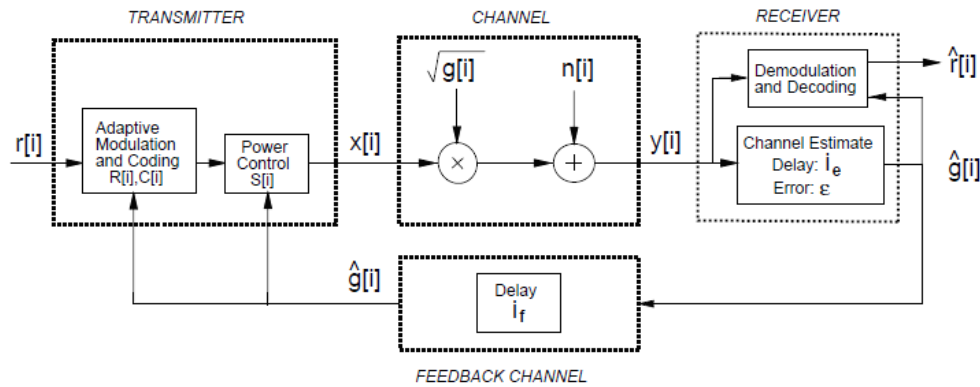


Figure 1: The system model

Consider a family of MQAM signal constellations with a fixed symbol rate  $T_s$ . The modulation with ideal Nyquist data pulses ( $\text{sinc}[t/T_s]$ ) is assumed, so the signal bandwidth  $B = 1/T_s$ . The flat-fading channel modeled as a discrete-time channel where each channel use corresponds to one symbol time  $T_s$ . The channel has stationary and ergodic time-varying gain  $g[i]$  that follows a given distribution  $p(g)$  and AWGN  $n[i]$ , with power spectral density  $N_0/2$ . Let  $\bar{S}$  denotes the average transmit signal power,  $B = 1/T_s$  denotes the received signal bandwidth, and  $\bar{g}$  denotes the average channel gain. The instantaneous received SNR is then  $\gamma[i] = \bar{S}g[i]/(N_0B)$ ,  $0 \leq \gamma[i] < \infty$ , and its expected value over all time is  $\bar{\gamma} = \bar{S}\bar{g}/(N_0B)$ . Since  $g[i]$  is stationary, the distribution of  $\gamma[i]$  is independent of  $i$ , and denoted by  $p(\gamma)$ . For  $M$ -ary modulation the data rate  $R[i] = \log_2 M[i]/T_s = B \log_2 M[i]$  bps. The spectral efficiency (the average transmitted data rate per unit bandwidth for a specified average transmit power and bit error rate (BER)) of the  $M$ -ary modulation is  $R[i]/B = \log_2 M[i]$  bps/Hz. The estimated SNR denoted as  $\hat{\gamma}[i] = \bar{S}\hat{g}[i]/(N_0B)$ , which is based on the power gain estimate  $\hat{g}[i]$ . The data rate of the modulation  $R(\hat{\gamma}[i]) = R[i]$  can be adapted. Similarly, the transmit power is adapted relative to  $\hat{\gamma}[i]$ . This adaptive transmitted power at time  $i$  denoted by  $S(\hat{\gamma}[i]) = S[i]$  and the received power at time  $i$  is then  $\hat{\gamma}[i]$  multiplied by  $S(\hat{\gamma}[i])/C$  and/or the  $C(\hat{\gamma}[i]) = C[i]$  relative to the estimate  $\hat{\gamma}[i]$ . The above are the common schemes for adapting the transmission. The time reference  $i$  relative to  $\gamma$  has been omitted, then the transmitted power, the data rate of the modulation and the coding parameters shorted to  $S(\gamma)$ ,  $R(\gamma)$ , and  $C(\gamma)$ , respectively. The capacity of a fading channel with bandwidth  $B$  and average power reduced coding parameters for some cutoff value  $\gamma_0$  and some distribution  $p(\gamma)$  for variable transmit power  $S(\gamma)$  the channel capacity is [11]:

$$C = \int_{\gamma_0}^{\infty} B \log_2 \left( \frac{\gamma}{\gamma_0} \right) p(\gamma) d\gamma \tag{1}$$

with average power constraint:

$$\int_{\gamma_0}^{\infty} \left( \frac{1}{\gamma_0} - \frac{1}{\gamma} \right) p(\gamma) d\gamma = 1 \tag{2}$$

For  $\gamma > 2$  the Equation (1) for Rayleigh channel can be approximated as [12]:

$$C = B \cdot \log_2(s) \cdot s^{-1/\gamma} (-E + \ln \gamma + \frac{1}{\gamma}) \quad (3)$$

where E is Euler constant (0.5772157).

For lognormal shadowing environment the probability density function is [7]:

$$p(\gamma) = \frac{10/\ln(10)}{\sqrt{2\pi} \sigma_{\gamma_{dB}} \gamma} \exp\left(-\frac{(10 \log_{10} \gamma - \bar{\gamma}_{dB})^2}{2\sigma_{\gamma_{dB}}^2}\right) \quad (4)$$

where  $\bar{\gamma}_{dB}$  is the mean of  $\gamma_{dB} = 10 \log_{10} \gamma$  and  $\sigma_{\gamma_{dB}}$  is the standard deviation of  $\gamma_{dB}$ .

From (1) the capacity of fading channels is limited by the available transmit power and bandwidth B as in AWGN channels. If  $\gamma[i] < \gamma_0$  at time i, then no power is allocated to the i-th data transmission. In particular when the channel is favorable, more power is allocated for transmission. Conversely, when the channel is not as good, less power will be transmitted. If the channel quality drops below  $\gamma_0$  the channel is not used (water-filling). Some modulation and coding scheme achieve this capacity.

The BER for MQAM is bounded for  $M \geq 4$  and  $0 \leq \bar{\gamma} \leq 30$  by [7]:

$$\text{BER} \leq 0.2 \exp(-1.5\bar{\gamma}/(M-1)) \quad (5)$$

The spectral efficiency for fixed M is  $\log_2 M$ , the number of bits/symbol. M with respect to an instantaneous  $\gamma$ :

$$M(\gamma) = 1 + \frac{1.5\gamma}{-\ln(\text{BER})} \frac{S(\gamma)}{S} \quad (6) \text{ or}$$

$$M(\gamma) = 1 + \gamma K \frac{S(\gamma)}{S} \quad (7)$$

where

$$K = -\frac{1.5}{\ln(\text{BER})} < 1 \quad (8)$$

The spectral efficiency can be maximized by maximizing the following equation:

$$E[\log_2 M(\gamma)] = \int \log_2 \left(1 + \frac{K\gamma S(\gamma)}{S}\right) p(\gamma) d\gamma \quad (9)$$

subject to the power constraint given by

$$\int S(\gamma) p(\gamma) d\gamma = S \quad (10)$$

The power adaptation policy that maximizes (9) has the same form as the optimal power adaptation policy of (1) that achieves spectral efficiency:

$$C/B = \int_{\gamma_K}^{\infty} \log_2 \left(\frac{\gamma}{\gamma_K}\right) p(\gamma) d\gamma \quad (11)$$

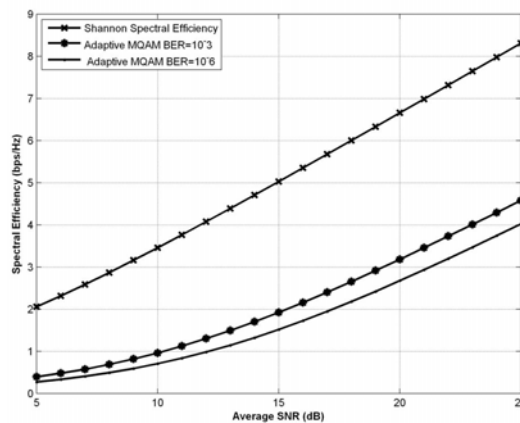
where  $\gamma_K = \gamma_0/K$ , C in Equation (11) is the Shannon capacity of fading channel when

the transmitter adapts to the channel variation using a constant-power variable-rate.

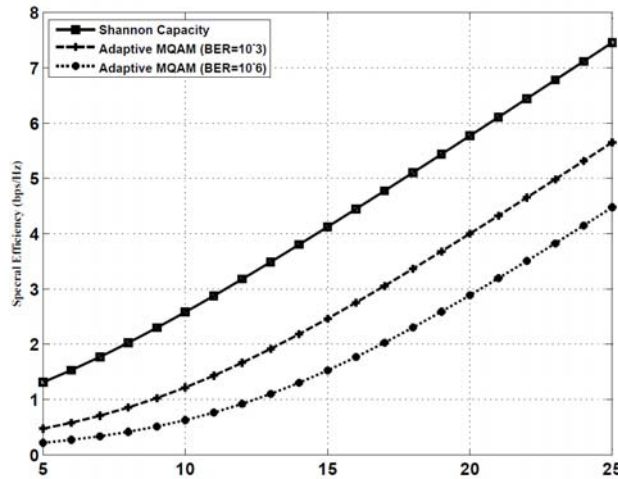
Comparing the power adaptation (1) and (11) we see that the spectral efficiency for both the optimal transmission scheme and [13] MQAM technique are the same, with power loss of  $K$ . With total power inversion policy (6) becomes:

$$R/B = \log_2 \left( 1 + \frac{-1.5}{\ln(\text{BER}) \overline{1/\gamma}} \right) \tag{12}$$

where  $\overline{1/\gamma}$  is the average of  $1/\gamma$ .



**Figure 2:** Efficiency in lognormal shadowing (mean=1,  $\sigma = 8$  dB).



**Figure 3:** Efficiency in Rayleigh fading

The average spectral efficiency (12) are plotted for adaptive MQAM at target BER's of  $10^{-3}$  and  $10^{-6}$  for both log-normal shadowing and Rayleigh fading in Figures 2 and 3, respectively. Note that the gap between the spectral efficiency of

variable-rate variable-power MQAM and capacity is the constant  $K$ , which from (7) is a simple function of the BER.

### Channel Inversion with Fixed Rate

Channel inversion adapts the transmit power to maintain a constant received SNR, then transmit a single fixed-rate MQAM modulation that achieves the target BER. The constellation size  $M$  that meets this target BER is obtained by substituting the channel inversion power adaptation  $S(\gamma)/\bar{S} = \sigma/\gamma$  with  $\sigma = 1/E[1/\gamma]$ , where  $E[x]$  is the expectation of  $x$ . Since the resulting spectral efficiency  $C/B = \log_2(M)$ , this yields the spectral efficiency of the channel inversion power adaptation as

$$C/B = \log_2 \left( 1 + \frac{-1.5}{\ln(\text{BER})E[1/\gamma]} \right) \quad (13)$$

The capacity with channel inversion (13) in Rayleigh fading is zero[14]. Eq. (13) with truncated channel inversion which maintain a constant received SNR unless the channel fading falls below a given cutoff level  $\gamma_0$ , at which point a signal outage is declared and no signal is sent. The capacity per unit bandwidth can be approximated in Rayleigh environment as[14]:

$$C/B = \log_2 \left( 1 + \frac{P}{E_1(\gamma_0/\bar{\gamma})} \right) \cdot e^{-\gamma_0/\bar{\gamma}} \quad (14)$$

where  $E_1(\cdot)$  is the exponential-integral function of first order[14]:

$$E_1(x) = \int_1^{+\infty} t^{-1} e^{-xt} dt \quad (15)$$

$E_1(x) = -E_i(-x)$ [14], where  $E_i(\cdot)$  is used in [12] as:

$$E_i(-x) = e^{-x} \cdot \sum_{k=1}^{\infty} (-1)^k \cdot \frac{(k-1)!}{x^k} + R_n \quad (16)$$

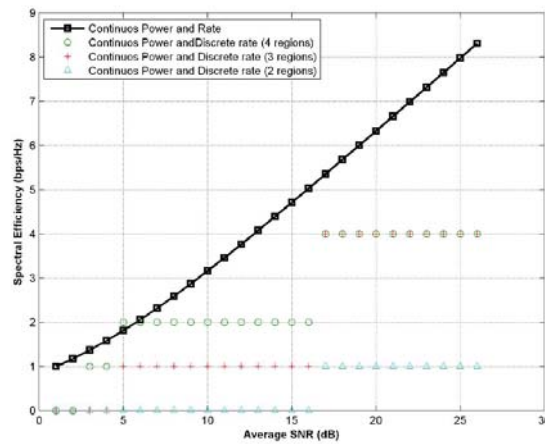
where  $R_n$  is the remainder.

### Discrete Rate Adaptation

The same model in the previous section is assumed with restriction of the adaptive MQAM to a limited set of constellations. Specifically, a set of square constellations of size  $M_0 = 0$ ,  $M_1 = 2$ , and  $M_j = 22(j-1)$ ,  $j = 2, \dots, N-1$  for some  $N$  is assumed. For simple implementation, square constellations for  $M > 2$  are assumed [12]. The spectral efficiency for this discrete-rate policy is just the sum of the data rates associated with each of the regions multiplied by the probability that  $\gamma$  falls in that region, Figure 4:

$$R/B = \sum_{j=1}^N \log_2(M_j) p(M_j \leq \gamma/\gamma_R^* < M_{j+1}) \quad (17)$$

where  $\gamma_R^*$  is a parameter which optimize to maximize spectral efficiency. The above result for fixedpower. The spectral efficiency can be maximized by adapting transmitted power for targeted BER.



**Figure 4:** Efficiency in Rayleigh fading of Discrete rate.

## Conclusion

For there is a constant gap about 2.5dB and about 6-7dB between the channel capacity and the maximum efficiency of adaptive MQAM which is a simple function of the target BER for Rayleigh and log-normal channels, respectively. Truncated channel inversion with fixed rate transmission has almost the same spectral efficiency as the optimal variable rate and power MQAM. This would tend to indicate that truncated channel inversion is more desirable in practice, as it achieves almost the same spectral efficiency as variable rate and power transmission but does not require varying the rate.

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